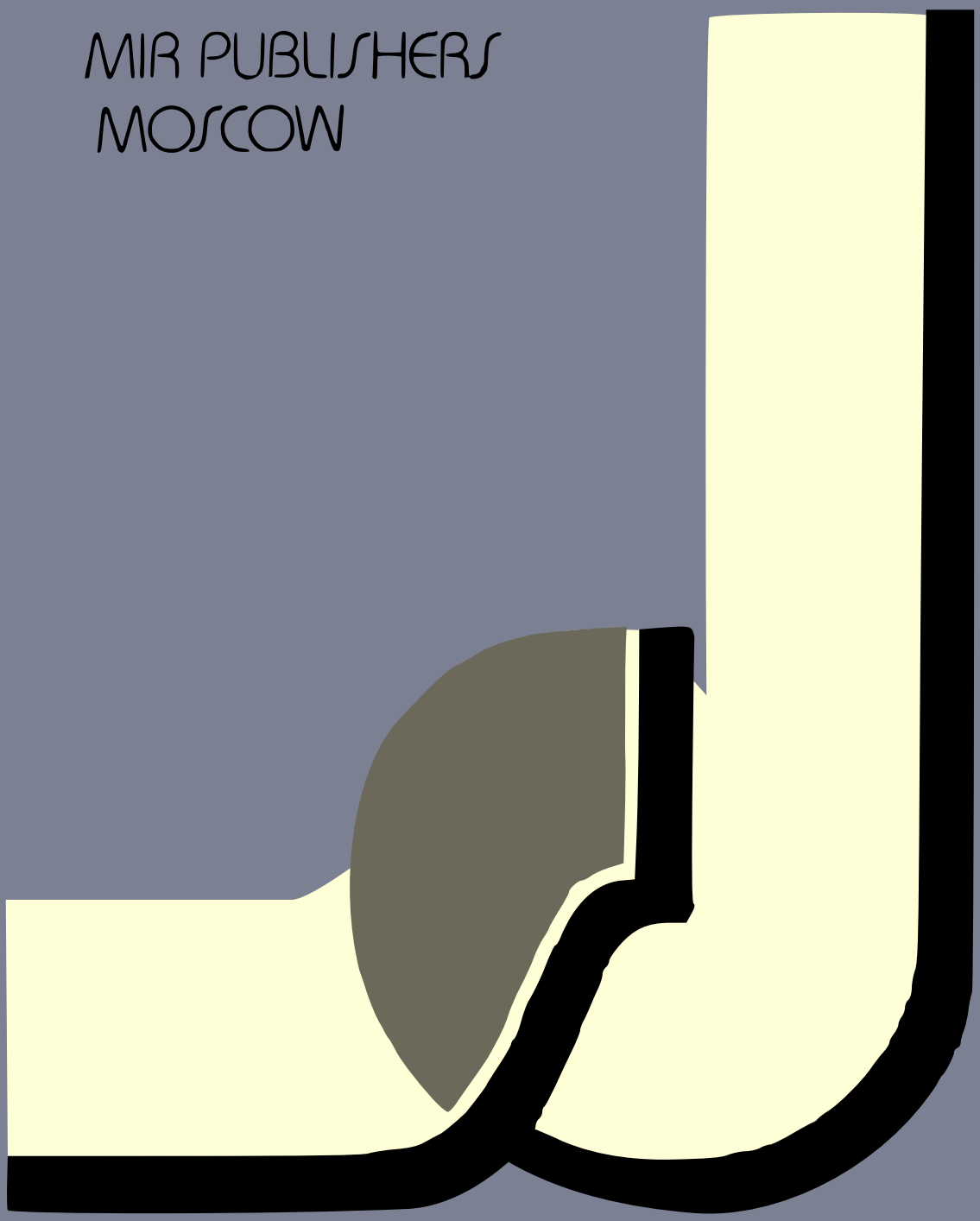
V. N. Baikov
S. G. Strongin

# structural design

MIR PUBLISHERS
MOSCOW



В. Н. Байков, С. Г. Стронгин

# СТРОИТЕЛЬНЫЕ КОНСТРУКЦИИ

Стройиздат · Москва

V. N. Baikov S. G. Strongin

# structural design

Translated from the Russian
by
Alexander Kuznetsov

MIR PUBLISHERS MOSCOW

*First published 1982*
*Revised from the 1980 second Russian edition*

## The Russian Alphabet and Transliteration

| | | | | | |
|---|---|---|---|---|---|
| А а | a | К к | k | Х х | kh |
| Б б | b | Л л | l | Ц ц | ts |
| В в | v | М м | m | Ч ч | ch |
| Г г | g | Н н | n | Ш ш | sh |
| Д д | d | О о | o | Щ щ | shch |
| Е е | e | П п | p | ъ | ″ |
| Ё ё | e | Р р | r | ы | y |
| Ж ж | zh | С с | s | ь | ′ |
| З з | z | Т т | t | Э э | e |
| И и | i | У у | u | Ю ю | yu |
| Й й | y | Ф ф | f | Я я | ya |

## The Greek Alphabet

| | | | | | |
|---|---|---|---|---|---|
| Α α | Alpha | Ι ι | Iota | Ρ ρ | Rho |
| Β β | Beta | Κ κ | Kappa | Σ σ | Sigma |
| Γ γ | Gamma | Λ λ | Lambda | Τ τ | Tau |
| Δ δ | Delta | Μ μ | Mu | Υ υ | Upsilon |
| Ε ε | Epsilon | Ν ν | Nu | Φ φ | Phi |
| Ζ ς | Zeta | Ξ ξ | Xi | Χ χ | Chi |
| Η η | Eta | Ο ο | Omicron | Ψ ψ | Psi |
| Θ ϑ θ | Theta | Π π | Pi | Ω ω | Omega |

*На английском языке*

© Стройиздат, 1980
© English translation, Mir Publishers, 1982

*Printed in the Union of Soviet Socialist Republics*

## CONTENTS


Preface . . . . . 8

*Chapter One.* **Basics of Structural Design** . . . . . 9

I.1. Limit States . . . . . 9
I.2. Design Factors . . . . . 10
I.3. Characteristic and Design Loads and Strength . . . 11
I.4. Limit-State Design . . . . . 13
I.5. Units of Measure . . . . . 15

*Chapter Two.* **Reinforced Concrete as a Construction Material** 16

II.1. General . . . . . 16
II.2. Concrete . . . . . 19
II.3. Reinforcing Steel . . . . . 25
II.4. Main Properties of Reinforced Concrete . . . . . 35

*Chapter Three.* **Analysis and Design of Reinforced Concrete Members** . . . . . 37

III.1. General . . . . . 37
III.2. Design of Prestressed Concrete Structures . . . . 40
III.3. Practical Hints for Design and Engineering . . . 47
III.4. Engineering Hints for Prestressed Members . . . 51

*Chapter Four.* **Members in Axial Compression** . . . . . 55

IV.1. Constructional Features . . . . . 55
IV.2. Design Under Accidental Eccentricity . . . . . 57

*Chapter Five.* **Reinforced Concrete Members in Axial Tension** 62

V.1. Constructional Features . . . . . 62
V.2. State of Stress and Design of Nonprestressed Members 63
V.3. State of Stress and Design of Prestressed Members 67

*Chapter Six.* **Reinforced Concrete Members in Bending** . . . . 72

VI.1. Constructional Features . . . . . 72
VI.2. State of Stress in Bending Members . . . . . 80
VI.3. Normal-Section Strength Analysis of Members . . 85

VI.4. Inclined-Section Shear Strength Analysis . . . . 98
VI.5. Incipient-Cracking Resistance of Prestressed Members . . . . . 108
VI.6. Sag Analysis . . . . . 114
VI.7. Crack-Opening Analysis . . . . . 118

*Chapter Seven.* **Reinforced Concrete Members in Eccentrical Compression and Tension** . . . . . 120

VII.1. Constructional Features of Members in Eccentrical Compression . . . . . 120
VII.2. Design of Members in Eccentrical Compression 123
VII.3. Members in Eccentrical Tension . . . . . 132

*Chapter Eight.* **Masonry and Reinforced Masonry Structures** 135

VIII.1. Masonry Materials and Strength . . . . . 135
VIII.2. Design of Masonry Members in Compression . . 138
VIII.3. Reinforced Masonry Structures . . . . . 142
VIII.4. Design of Masonry Structures . . . . . 145
VIII.5. Worked Examples for Design of Masonry Members 147

*Chapter Nine.* **Metal Structures** . . . . . 152

IX.1. Materials for Metal Structures . . . . . 152
IX.2. Joints in Metal Structures . . . . . 157
IX.3. Design and Proportioning of Beams . . . . . 164
IX.4. Design and Proportioning of Columns . . . . . 175
IX.5. Design and Proportioning of Trusses . . . . . 188
IX.6. Prestressed Steel Structures . . . . . 191

*Chapter Ten.* **Design of Buildings** . . . . . 193

X.1. Principles of Building Layout . . . . . 193
X.2. Reinforced Concrete Floors . . . . . 196
X.3. Reinforced Concrete Column Footings . . . . . 219
X.4. Prefab Reinforced Concrete One-Storey Industrial Buildings . . . . . 227
X.5. Buildings for Water Supply and Sewage Disposal Systems and Boilers . . . . . 237

*Chapter Eleven.* **Special Structures for Water Supply and Sewage Disposal Systems** . . . . . 246

XI.1. General . . . . . 246
XI.2. Construction Types of Circular Reinforced Concrete Tanks . . . . . 249
XI.3. Design of Circular Tanks . . . . . 260
XI.4. Construction Types of Reinforced Concrete Rectangular Tanks . . . . . 273
XI.5. Design of Rectangular Tanks . . . . . 285
XI.6. Construction Types and Design of Steel Circular Tanks . . . . . 292

XI.7. Reinforced Concrete Pipes and Wells for Water Supply and Sewage Disposal Systems . . . . . 296
XI.8. Construction Types and Design of Water Towers 300

*Chapter Twelve.* **Design of Reinforced Concrete Structures. Worked Examples** . . . . . 306

XII.1. General . . . . . 306
XII.2. Nonprestressed Ribbed Roof Slab . . . . . 306
XII.3. Prestressed Ribbed Roof Slab . . . . . 317
XII.4. Square Roof Slab . . . . . 324
XII.5. Roof Girder . . . . . 329
XII.6. Column and Footing . . . . . 337
XII.7. Wall of a Precast Reinforced Concrete Rectangular Tank . . . . . 342
XII.8. Wall of a Precast Circular Tank . . . . . 348

*Chapter Thirteen.* **Structures for Heat Supply Systems** . . . . 355

XIII.1. General . . . . . 355
XIII.2. Construction Types of Conduits and Headers . . 357
XIII.3. Servicing Chambers, Compensating Niches, and Supports for Heat Pipelines . . . . . 367
XIII.4. Heat Pipelines Laid without Conduits . . . . . 378
XIII.5. Overhead Heat Pipelines . . . . . 379
XIII.6. Design Principles for Heat Pipeline Structures 386

Appendices . . . . . 395

Index 413

## PREFACE

This is the English translation of the second Russian edition, revised and enlarged in comparison with the first (1970) edition.

The book presents a systematic and step-by-step approach to the physical and mechanical properties of concrete, reinforcing steel, reinforced concrete, masonry, and structural steel.

The theoretical basis and design principles for reinforced concrete, masonry, and steel structures have been brought in line with new standards which were put in force in the Soviet Union since January, 1977. The material is presented with emphasis on Soviet practice in highly industrialized precast reinforced concrete, type-design structural elements, and high-strength materials.

A good deal of attention is given to special engineering structures for water supply, sewage disposal, and heat supply systems, including tanks, settlers, aeration filters, aeration tanks, water towers, buried conduits and headers, and heat-pipeline structures.

Carefully worked design examples are shown throughout the book, and an appendix gives a selection of design charts associated with the basic Soviet codes of practice.

Intended as textbook for students of building, civil, and structural engineering, the book will also be of interest to students in other departments and practising engineers.

CHAPTER ONE

# BASICS OF STRUCTURAL DESIGN

## 1.1. Limit States

After a structural member has appropriately been analyzed, it is necessary to determine the internal forces caused by the external loads it is to carry, to select its cross-sectional area required to withstand those forces, to choose the cross-sectional area of junction elements (if several members are assembled to form a single entity), and to determine the necessary volume of reinforcement (if the member is made of reinforced concrete). This is done to ensure an adequate behaviour of the member during the entire service life at a minimum cost and minimum material consumption.

A structure or an element may become unfit for service, if:

(1) the internal forces in it exceed its ultimate strength, so that the material fails at the most heavily loaded sections, or some of the component members or the entire structure loses stability;

(2) the loads imposed on it lead to excessive deformations (such as sag, vibrations, or settlement) or cracks which may open up wider than it is allowed.

In the course of structural design, as this procedure is customarily called, it is advisable to employ the load values and stress-strain relations recommended by relevant building codes. In practice, however, the loads that a structure is expected to carry are frequently smaller or, which is worse, greater than those established by the codes.

At present, most structures are designed using what is known as the *limit-state approach*—it clearly establishes the limit states of structures and sets up a system of design coefficients which guarantee that a structure will not attain such states under the worst load combinations and at the minimum strength of the materials.

The term 'limit state' refers to a boundary condition beyond which a structure or element is considered to be unfit for use. This may arise in many ways, but the principal limit states to be considered are those of

(1) ultimate strength, and

(2) serviceability.

Design in terms of ultimate strength (in this text it will also be referred to as design in terms of the first category of limit states) should be carried out for all types of structures, whereas design in terms of serviceability (which will also be called design in terms of the second category of limit states) should be done only for structures whose deformation may cause failure before the internal forces exceed the load-carrying capacity. This applies to large-span slabs and beams whose cross-sectional area depends on the limit of sag rather than the necessary strength.

Design in terms of serviceability for reinforced concrete structures should provide for a cross section that will not only prevent a structure from exceeding the limit of sag, but will also help to keep the width of cracks within the specified limits or even to avoid cracking altogether.

## I.2. Design Factors

These are employed to allow for the variability in load and the mechanical characteristics of materials. They include:

(1) *overload factors* $n$ (that is, partial safety factors in terms of load);

(2) *safety factors* $k$ (that is, partial safety factors in terms of materials);

(3) *service factors* $m$ which permit the designer to take into account some aspects of the material behaviour that cannot be allowed for explicity in the course of design.

According to a relevant standard, overload and safety factors ($n > 1$* and $k > 1$) are adopted, as already men-

* If a likely decrease in load is more dangerous for a structure than an increase, $n$ is taken to be smaller than unity. For example, when analyzing a water tower or a retaining wall for resistance to tilt, its self-weight (which adds to this resistance) should be multiplied by $n = 0.9$.

tioned, to account for the variability in load and the mechanical characteristics of the materials, respectively. The greater the variability, the higher the values of $n$ and $k$, and vice versa.

Figure I.1 shows a frequency distribution curve for the strength of materials. The strength is laid off as abscissa, and the ordinate is the frequency with which this value is observed. In most cases, the strength shows a definite average or mean value. Referring to the figure, the strength may move up or down in magnitude from the mean value, and the number of cases in which a given strength is observed decreases with increasing deviation from the mean. The shape of the frequency distribution curve helps to evaluate the variability of the quantity in question. For example, if the curve extends along the Y-axis (curve *a* in Fig. I.1), the variability is low; if the curve runs at a small slope (curve *b* in Fig. I.1), the variability is high.

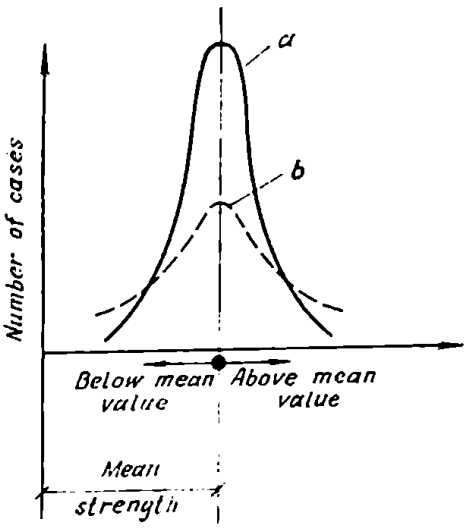



Fig. I.1. Frequency distribution curves

## I.3. Characteristic and Design Loads and Strength

The loads that may be imposed on a structure in service are specified in relevant standards. Such loads are known as *characteristic loads* and designated by $q^{ch}$.

*Design loads*, labelled $q$, are obtained by multiplying the characteristic values by appropriate overload (partial safety) factors $n$:

$$q = q^{ch} n$$

Some values of $q^{ch}$ and $n$ are given in Appendix II.

The loads that act upon a structure during the entire service life are termed *dead loads*. They include the self-weight of the structure, the weight of the members it carries, and the weight of the soil above the structure (in the case of buried structures). The loads that vary in service in both magnitude and point of application are called *live loads*. They are constituted by people, furniture, and equipment (so-called floor loads), the pressure exerted by liquids and particulate materials stored in tanks and bins, snow and wind loads, crane loads, etc.

*Special loads* refer to earthquake loads, loads caused by the nonuniform settlement of the foundation, etc.

Live loads may, in turn, be subdivided into *long-time* (the pressure of liquids and particulate materials in tanks, the weight of stationary equipment in industrial buildings, floor loads in store-houses, archives, libraries, and the like, and the snow load reduced by 70 kgf $m^{-2}$ (700 N $m^{-2}$), some of the crane load, etc.), and *short-time* (some of the load constituted by people, spare parts, and materials in attendance and maintenance areas, the remainder of the snow and crane loads not included in the long-time loads, the wind loads, etc.).

Any structure should be designed for different *load combinations*. It is customary to distinguish *basic load combinations* which refer to the simultaneous action of dead, long-time and short-time live loads, and *special load combinations* which include the loads entering a basic load combination and one special load. If a basic load combination contains only one short-time live load (the most critical one in a given case), the latter should be taken into account without any reduction. If a basic load combination includes two or more short-time live loads, they (or the respective internal forces) should be multiplied by the combination factor $n_{comb} = 0.9$. In the case of special load combinations, the design values of the short-time loads (or the respective internal forces) must be multiplied by $n_{comb} = 0.8$, and the special load should be taken without any reduction.

Relevant standards specify that the main parameter defining the strength of a material shall be its characteristic strength $R^{ch}$ (measured in kgf $m^{-2}$ or MPa and equal to the proof strength). This is established with allowance for

the statistical variability of strength and is taken as the least observable ultimate strength. The minimal level of confidence (confidence coefficient), as set by appropriate specifications for the characteristic strength, is 0.95.

The *design strength* of a material, $R$ (measured in the same units), is obtained by dividing the respective characteristic strength by an appropriate safety factor, $k > 1$. Sometimes, the design strength is multiplied by a service factor, $m \leqslant 1$, which accounts for the utilization of the strength of materials in a given method of design, the behaviour of the entire structure, the effect of service conditions, etc.,

$$R = (R^{ch}/k)\, m$$

The values of the characteristic strength, safety factors, service factors, and design strength of the materials used in reinforced concrete, metalwork, and masonry structures can be found in appropriate standards and specifications.

In practice, use is generally made of the design strength of materials.

## I.4. Limit-State Design

**Design in Terms of Ultimate Strength** (the first category of limit states). Here, we are concerned with the maximum load-carrying capacity of a structure within the limits of strength of the materials used, with allowance for a likely increase in load variability and a decrease in strength. Therefore, the left-hand side of the design relations contains the force caused in a member by design loads (with overload factors applied), and the right-hand side is the force that the member can bear when the stress in the material is equal to the design strength (that is, with safety and service factors applied). If the left-hand side does not exceed the right-hand side in magnitude, the load-carrying capacity of the structure is adequate.

As an example, for a single-material member subjected to axial tension, the design expression takes the form

$$\Sigma N^{ch} n \leqslant F\, (R^{ch}/k)\, m$$

Designating the total design load by $N$ and recalling that

$$(R^{ch}/k)\, m = R$$

we may re-write the above equation to obtain the compact formula generally used for design purposes:

$$N \leqslant FR$$

Given the value of $N$, the necessary cross-sectional area can be found by equating the left- and right-hand sides of the design equation.

**Design in Terms of Serviceability** (the second category of limit states). Here, we are concerned with the appearance of a structure, the effects of deflection or deformation on other elements, and its durability. If a structure is to be designed so as to avoid excessive deformation (say, sag), the left-hand side of the design expression is the strain $f$ developed in the structure, determined according to the load system used, and the right-hand side is the limit of strain $f^{ch}$ specified by relevant standards on the basis of past experience:

$$f \leqslant f^{ch}$$

If a reinforced concrete structure is to be designed so as to avoid cracking, with allowance for likely strength variations and, for some structures*, with allowance for likely load variations, the left-hand side of the design expression is the force $N^{ch}$ produced by characteristic loads (or $N$, due to design loads), whereas the right-hand side is the force $N_{cr}$ developing in the member immediately before cracking occurs in the concrete, taken with the relevant safety and service factors applied:

$$N^{ch} \leqslant N_{cr} \quad \text{or} \quad N \leqslant N_{cr}$$

If a structure is allowed to crack in service, the crack width should appropriately be limited. In this case, the left-hand side of the design equation gives the crack width $a_{cr}$ found according to the load system adopted (with allowance for likely variations in the stress-strain behaviour of

* These are r.c. structures supposed to have a crack resistance high enough to meet the requirements of the first category of limit states (see Chap. III).

the materials used), and the right-hand side is the limiting crack width $a_{cr}^{ch}$ specified by relevant standards:

$$a_{cr} \leqslant a_{cr}^{ch}$$

## I.5. Units of Measure

According to a relevant USSR building code, structural design is carried out in the system of units in which mass is in kg and t, force is in kgf and tf, moment of force is in kgf m, kgf cm or tf m, linearly distributed load is in kgf $m^{-1}$, kgf $cm^{-1}$ or tf $m^{-1}$, and surface load, pressure, stress and modulus of elasticity is in kgf $cm^{-2}$, kgf $mm^{-2}$ or tf $m^{-2}$. This system of units does not allow for variations in the free-fall variations $g$ in different geographical localities and uses the same unit for stress and surface load. Because of this, use is sometimes made of the International System of Units (SI) where mass is in kg and t, force, load and weight in N and kN, moment of force in N m, surface load in N $m^{-2}$, and pressure, stress, and modulus of elasticity in Pa and MPa.

The present text uses the units as specified by the USSR building code. The SI units can be derived, using the table in Appendix I (where $g = 9.8$ m $s^{-2}$ is rounded off to 10 m $s^{-2}$).

CHAPTER TWO

# REINFORCED CONCRETE AS A CONSTRUCTION MATERIAL

## II.1. General

Reinforced concrete is a combination of concrete proper and reinforcing steel. Although they markedly differ in physical and mechanical properties, concrete and reinforcement work together well in structural members.

Concrete is strong in compression and weak in tension. Reinforcing steel is usually strong in both tension and compression. This is why the two materials are put together to form reinforced concrete.

In bending members, the high compressive strength of concrete is employed in the compression zone, and the high tensile strength of reinforcing steel in the tension zone where the concrete is weak and is likely to crack (Fig. II.1*a*). Reinforced concrete beams are congested with a small amount (as compared with the cross-sectional area) of reinforcement, but the contribution of the reinforcement is so large that the load-carrying capacity of the beams is increased several-fold. Also, reinforcement is used to advantage in compressive members (Fig. II.1*b*), so that their load-bearing capacity is more than half again as that of their plain-concrete counterparts.

In a member or structure, a strong bond exists between the hardened concrete and the reinforcing steel, so they deform together under load. The bond does not deteriorate with time, nor is it sensitive to variations in temperature and humidity, because the two materials have very close coefficients of expansion.

Hardened concrete presents an excellent environment for reinforcing steel. Under normal conditions, the steel remains safe and sound for an indefinitely long time.

Reinforced concrete has good durability, offers high resistance to fire and atmospheric effects, carries large

static and dynamic loads, possesses sufficient density, water and gas tightness, and stands well exposure to radiation.

Concrete may develop cracks at relatively small tensile stresses—a feature generally undesirable. Cracked concrete markedly reduces the rigidity of reinforced concrete structures, has inadequate water tightness, and, if the cracks are very wide, exposes the steel to corrosion.

The cracking resistance of reinforced concrete members can greatly be improved by prestressing. This consists in applying initial compressive stresses to those areas in a concrete member which, when loaded, work in tension.

There are two basic methods of prestressing, namely *pretensioning* and *posttensioning*. In the former (Fig. II.2*a*), the tendons are installed in a bed, form or tray where they are anchored to an abutment at one end and tensioned at the other. After the member has been concreted and the concrete has attained the necessary strength, the tendons are released from the abutments. Being bonded to the concrete, the reinforcing steel is unable to recover its original length and transfers the prestressing force to the concrete by bond resistance. In a loaded prestressed beam, the concrete begins to work in tension only after all of the compressive prestress has been counteracted. As a result, the cracking resistance of the concrete is improved.

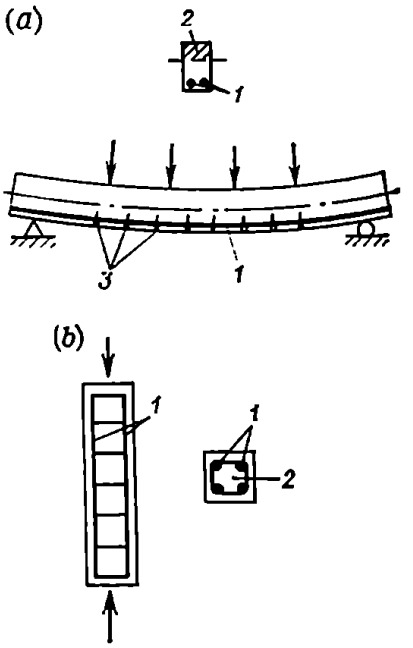



Fig. II.1. Reinforced concrete members subjected to (*a*) bending and (*b*) compression
*1*—reinforcing steel; *2*—concrete in compression; *3*—cracks

In posttensioning (Fig. II.2*b*), the tendons are tensioned against and anchored to the concrete after it has developed an adequate strength. Here, the tendons are inserted in ducts or raceways, tensioned, anchored, and grouted in

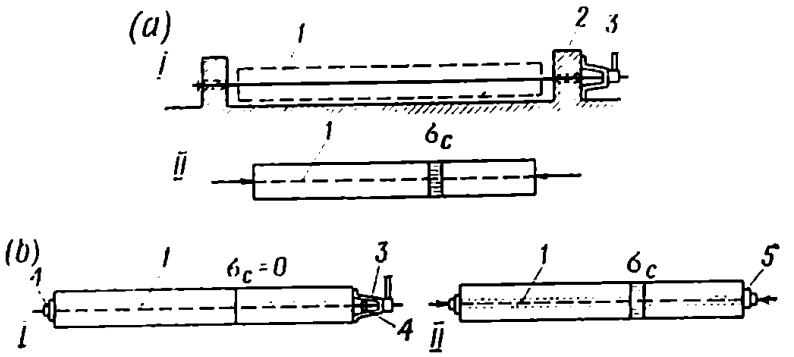



Fig. II.2. Prestressed concrete members (axial prestress)
(*a*) pretensioning; (*b*) posttensioning; *I*—member before transfer of prestress; *II*—member after transfer of prestress; *1*—prestress reinforcement; *2*—abutments; *3*—tensioning device; *4*—anchorage; *5*—embedded shim

the sequence listed. After the grout in the ducts has attained the necessary strength, the prestressed member is ready for use.

As already mentioned, prestressing increases the cracking resistance of reinforced concrete elements several-fold, markedly reduces the width of cracks, and improves the stress-strain behaviour of the members.

According to the manner of manufacture, reinforced concrete structures may be classed into *precast*, *cast in-situ*, and mixed (that is *precast/cast in-situ*).

As their name implies, *precast structures* are manufactured at reinforced concrete yards by high-output machines, with all the manufacturing conditions closely controlled and with the high quality of products maintained at a reasonable material consumption. At the site, prefabricated members are erected by high-power mechanisms. Precast products are best adapted to streamlined manufacture and can be assembled in a minimum period of time.

*In-situ reinforced concrete structures* call for scaffolding and formwork in which the reinforcement is installed and concrete is placed on the site. This can hardly be done at an adequate level of mechanization. Also, concrete should be held in the forms during the entire period of hardening and kept warm in winter. Therefore, in-situ structures are more labour-consuming, take more time to

construct, and are inferior in quality. Because of this, in-situ reinforced concrete is mostly confined to heavy structures and situations where its use is warranted economically.

*Precast/in-situ reinforced concrete structures* are midway between precast and in-situ types and utilize the advantages offered by each. They are advisable for platforms designed to carry superheavy loads and floors intended to withstand dynamic forces.

## II.2. Concrete

In its inner structure, concrete is a nonuniform material whose bulk is constituted mostly by inert aggregates (which may be coarse, such as broken stone or gravel, and fine, say, sand) bonded into a single entity by hardened cement produced by a reaction of cement powder with water (both fixed and free).

In the hardened cement, chemical reactions keep going on for a fairly long time. So, its strength builds up with time, too, and its volume undergoes a change (the concrete shrinks or, if special brands of cement are used, expands).

Concrete is not perfectly voidless, even if a concrete mix is thoroughly compacted. To make the mix workable, the amount of water used to prepare the mix must always exceed that just necessary for the hydration reaction to take place. On evaporating, the excess water leaves behind numerous pores and capillaries in the hardened cement and voids between the coarse-aggregate grains and reinforcing steel. Because of this, concrete is permeable to moisture and gas—a fact that should be kept in mind when designing plumbing structures. The porosity of concrete can be reduced by cutting down the initial amount of water in the concrete mix, by better compaction of the fresh mix during concreting, and by adding chemically active substances which swell when moistened.

The most important mechanical characteristic of concrete is its *compressive strength.* It depends on the strength of the hardened cement, the quality of aggregates, and the density of concrete itself. In turn, the strength of the hardened cement increases with increasing activity (brand number)

of cement and decreasing water-cement ratio of the mix. Heavy concretes generally used in plumbing structures employ dense aggregates which are stronger than the hardened cement, the strength of concrete being higher if the aggregate grains have a ragged surface as it improves the bond with the hardened cement. From this point of view, broken stone is more attractive than gravel.

A relevant Soviet standard requires that the compressive strength of concrete be determined on test specimens in the form of cubes with a 150-mm edge after 28 days of curing for in-situ structures (a shorter curing period is specified for precast structures, depending on the manner of manufacture and curing). The test result gives the brand of the concrete in terms of its ultimate strength in kgf $cm^{-2}$ (MPa).

According to compressive strength, the following brands are produced in the Soviet Union:

—heavy concretes, M-100, M-150, M-200, M-250, M-300, M-350, M-400, M-450, M-500, M-600, M-700, and M-800;

—porous-aggregate concretes, M-35, M-50, M-75, M-100, M-150, M-200, M-250, and M-300.

The cube crushing strength of concrete is given by

$$\bar{R} = N_{cr}/F$$

where $N_{cr}$ is the crushing load in kgf (N), and $F$ is the cross-sectional area of the cube in $cm^2$.

In compression, the initial dimensions of a cube test specimen decrease in the direction of the acting force and increase in the lateral direction (Fig. II.3). Tension tests have shown that the ultimate relative strain in tension is one-tenth to one-twentieth of that in compression. A compression test cube fails due to a lateral expansion resulting in cracks. If, before testing, the ends of the specimen are capped with neat cement paste or other material (Fig. II.3*a*), the lateral strain will meet no resistance, and the rupture cracks will be vertical (parallel to the direction of compression). If no measures have been taken to avoid friction between the platen and seat of the compression-testing machine and the cube faces (Fig. II.3*b*), the cracks will be inclined. In the latter case, the compressive strength of concrete is 2 to 2.5 times as high as in the former case.

A relevant USSR standard requires that the cube crushing strength of concrete be determined without capping.

In concrete prisms, the effect of friction at the prism ends decreases with increasing height-to-side ratio. When the ratio is four-to-one or more, the effect of friction is insignificant. Experiments have shown that the prism crushing strength of concrete is by 20% to 30% lower than that of cube specimens.

The axial tensile strength of concrete is one-tenth to one-fifteenth of its compressive strength. Experiments with concrete test specimens subjected to bending have shown that in this case the tensile strength is by about 70% higher than its axial tensile strength.

If a concrete structure works mostly in tension, one needs to know the tensile strength of the concrete in addition to its compressive strength. According to their tensile strength (in kgf $cm^{-2}$), Soviet-made concretes are classed as follows:

—heavy concretes, P-10, P-15, P-20, P-25, P-30, P-35, and P-40.

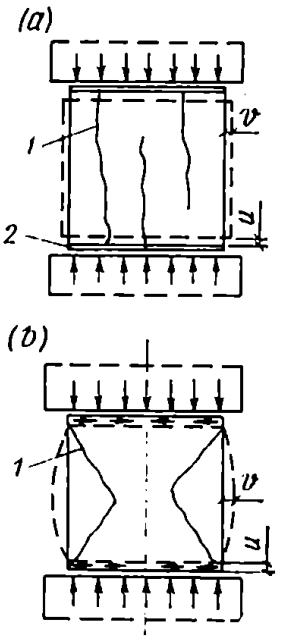



Fig. II.3. Concrete cube failure (*a*) with cube faces capped; (*b*) without capping; *1*—cracks; *2*—capping; *u*—contraction in direction of compressive forces; *v*—expansion in lateral direction

Brand numbers in terms of frost resistance are assigned to concretes used in structures subjected to freezing and thawing cycles in cold climates and in exterior structures in localities with frequent freeze-thaw changes, and also to porous-aggregate concrete used in filler walls. They are designated Mpa 50, Mpa 75, Mpa 100, Mpa 150, Mpa 200, Mpa 300, Mpa 400, and Mpa 500. The numerals give the number of freeze-thaw cycles that a given concrete will withstand in water-saturated conditions.

Brand numbers in terms of water resistance are assigned to concretes used in structures subjected to water pressure, such as tanks and pressure pipes. These brands are designated by the Russian letter "B"; they range from B-2 to B-12. The numerals give limit of water pressure (in kgf $cm^{-2}$) at which the flow of water through channels in the concrete is still prevented.

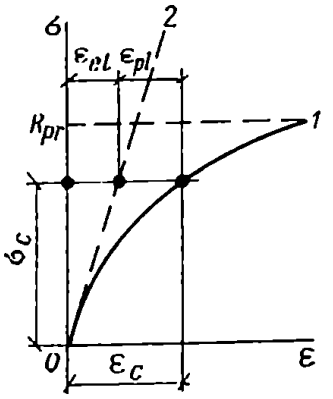



Fig. II.4. Stress-strain curve plotted for short-duration single loading
*1*—total elasto-plastic strain; *2*—elastic strain

Concrete is a nonelastic material. As a consequence, its stress-strain ($\sigma - \varepsilon$) behaviour is far from linear (Fig. II.4). The total strain $\varepsilon_c$ is composed of elastic (instantaneous or reversible) strain $\varepsilon_{el}$ and plastic (time-varying or irreversible) strain $\varepsilon_{pl}$. Under short-duration single loading, the share of plastic strain increases with increasing stress. As the duration of loading, $t$, increases, strain in concrete departs from elastic still more. Referring to Fig. II.5*a*, the strain increases with increasing time of loading for the same value of stress, $\sigma_1$.

An increase in plastic strain under a sustained stress is called *creep*. This is a characteristic feature of concrete (Fig. II.5*b*). In real structures, creep strain gradually subsides and approaches asymptotically a certain limiting value, $\varepsilon_{cr}^{lim}$.

The manner in which creep develops in concrete is governed by the inner structure of the hardened cement. The amount of creep increases with increasing amount of cement and initial amount of water in the concrete mix. Also, it depends on the age of concrete at the time of loading (the younger the concrete, the higher the creep), the level of stress in concrete (it rises with rising stress), the type of cement used (it decreases in the following sequence: slag Portland cement, ordinary Portland cement, high-strength Portland cement, and alumina cement).

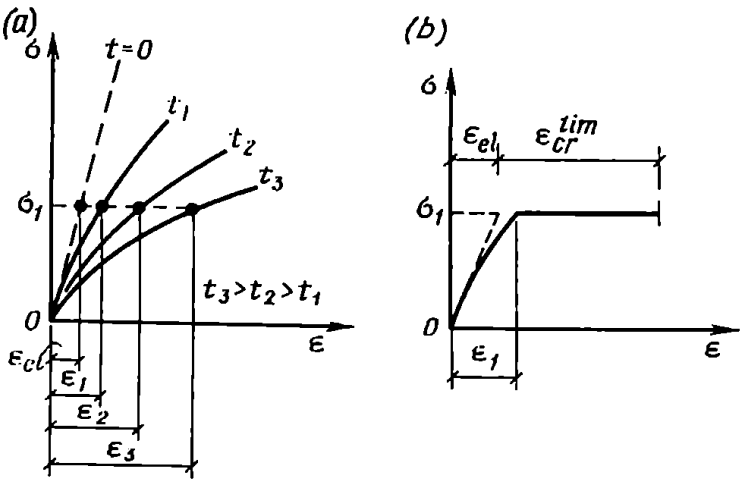



Fig. II.5. Stress-strain curves for concrete
(*a*) different test duration; (*b*) long-time constant stress

Creep in concrete is most significant during the first several months after loading and may keep increasing for several years until it reaches a maximum value, $\varepsilon_{cr}^{lim}$. The value of $\varepsilon_{cr}^{lim}$ may be two or three times as great as elastic strain. Creep in concrete has a pronounced effect on the stress-strain behaviour of reinforced concrete structures.

The elastic properties of concrete are expressed in terms of the *initial tangent modulus of elasticity* $E_c$. It is determined by prism crushing tests within the initial portion of the stress-strain curve (see Appendix III).

The *tangent modulus of elasticity* for concrete in compression, $E_c'$, describes the total strain in concrete (including creep) and is a variable quantity. However, variations in $E_c'$ are difficult to describe analytically, and design calculations use an average quantity known as the *secant modulus of elasticity*. This is likewise designated by $E_c'$ and defined as the slope of the chord drawn through a point on the stress-strain curve corresponding to a given stress. The relation between $E_c$ and $E_c'$ is found from Fig. II.4, using the following expressions:

$$\sigma_c = E_c \varepsilon_{el} \quad \text{and} \quad \sigma_c = E_c' (\varepsilon_{el} + \varepsilon_{pl})$$

Hence,

$$E_c^e = [\varepsilon_{el}/(\varepsilon_{el} + \varepsilon_{pl})]\, E_c = \nu E_c$$

where

$$\nu = \varepsilon_{el}\,(\varepsilon_{el} + \varepsilon_{pl})$$

is the ratio of elastic to total strain in concrete. According to experimental data for compressive members, $\nu$ may vary from 1 to 0.15. Relevant standards specify the values of $\nu$ with allowance for air humidity.

Concrete has a good fire resistance. It can withstand fire much longer than most construction materials, without losing any of its load-carrying capacity or other useful properties.

Ordinary heavy concrete may also be used in structures subjected for a long time to elevated temperatures from 50° to 200°C. It uses Portland cement or slag Portland cement as the binder and granites, dolomites, sienites, limestone, and other similar materials as the aggregate.

If a structure is to be used under sustained exposure to elevated temperatures exceeding 200°C, it should be made of heat-resistant concrete. According to the temperature it is to withstand, heat-resistant concrete may use alumina cement, Portland cement, or liquid glass (with appropriate hardeners and fine admixtures) as the binder, and chromite, diorite, basalt, diabase, andesite, grog, broken brick, and slag, as the aggregate.

Concrete is a very durable material. Under normal conditions, its inner structure and strength remain unchanged for an infinitely long time. However, in corrosive environments, such as acid gases acting in combination with an elevated air humidity, liquid alkalis and acids, salt solutions, solvents, oil, sugar solutions, corrosive dust combined with an elevated humidity, coal, most ores, salts, slags and the like, concrete deteriorates rather rapidly.

The resistance offered by concrete to corrosive environments can be enhanced by (1) increasing its density, for which purpose concrete is carefully proportioned with the use of an appropriate type of cement, and thoroughly consolidated during concreting; (2) reducing its permeability through the use of concretes with a low water-cement ratio

and consolidating admixtures; and (3) using appropriate acid-resistant binders and aggregates.

According to the type of environment, the corrosion resistance of heavy concretes may be improved in one of several ways. As a rule, the surface of concrete is floated, impregnated with high-molecular-weight compounds, coated with special varnishes, paints, or epoxy resins, or faced with chemically stable ceramic tile or plasto-concrete based on polymer binders and admixtures, or impregnated with polymer compounds.

In addition to heavy concretes, civil structures use no-cement concretes based on dense aggregates (such as lime concrete, slag-binding agent concretes, and gypsum concretes), porous-aggregate concretes using ceramsite, agloporite, slag pumice, pearlite, tuf, and similar materials as the aggregate, and cellular concretes. These concretes have not found wide use in load-bearing structures of plumbing systems, although porous-aggregate concretes of various types are used to insulate gas conduits.

According to their bulk unit weight, concretes are customarily classed into heavy (with a bulk unit weight of $\rho_c > 1\,800$ kg m$^{-3}$) and light-weight (with a bulk unit weight of $\rho_c \leqslant 1\,800$ kg m$^{-3}$). A more detailed classification includes superheavy concretes (with a bulk unit weight of more than 2 500 kg m$^{-3}$), heavy concretes (with a bulk unit weight of 2 200 to 2 500 kg m$^{-3}$), medium-heavy concretes (with a bulk unit weight of 1 800 to 2 200 kg m$^{-3}$), light-weight concretes (with a bulk unit weight of 500 to 1 800 kg m$^{-3}$), and super-light concretes (with a bulk unit weight of under 500 kg m$^{-3}$).

## II.3. Reinforcing Steel

Reinforcing steel is primarily the tensile component of reinforced concrete. Also, it is used to reinforce compression areas in members. The necessary amount of reinforcement is determined by appropriate design calculations.

In Soviet practice, reinforcing steel is classed into *load-bearing reinforcement* for which the steel ratio is found by calculations, and *erection reinforcement* for which the steel ratio is adopted from constructional and other considera-

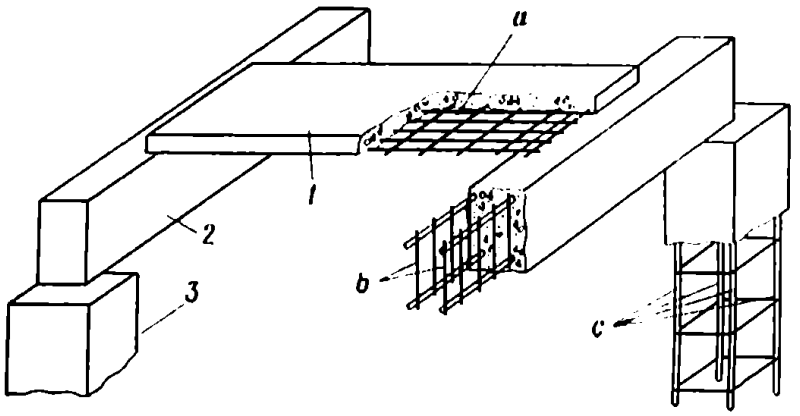



Fig. II.6. Reinforced concrete members and reinforcing steel
*a*—wire fabric; *b*—bar mats; *c*—reinforcing cage; *1*—slab; *2*—beam; *3*—column

tions. Erection reinforcement serves to hold the load-bearing reinforcing bars in place and to distribute stresses uniformly among the bars of load-bearing reinforcement in the course of erection. In addition, erection reinforcement may carry some loads, such as those due to earthquakes and thermal variations, which generally are not taken into account in design.

Load-bearing and erection reinforcement is combined into welded- or tied-wire fabric or bar mats which are placed in reinforced concrete members according to their behaviour under load (Fig. II.6).

Reinforcing steel may come in the form of hot-rolled bars and cold-drawn wire.

The term 'bar' includes reinforcing bars of any diameter, irrespective of how they are dispatched to the job—in bars or hanks. After manufacture, hot-rolled reinforcing steel may be strengthened by heat treatment or cold drawing.

In a structure, steel reinforcing bars and wire may be utilized either as prestressed or as non-prestressed (that is, not deliberately stressed before service loads are applied).

The principal forms of standard concrete reinforcement are *plain bars* (or wire) and *deformed bars* (or wire). Standard bars (or wire) are deformed so as to increase the bond between the reinforcement and the concrete (Fig. II.7).

According to their main mechanical characteristics, Soviet-made hot-rolled reinforcing bars are divided into

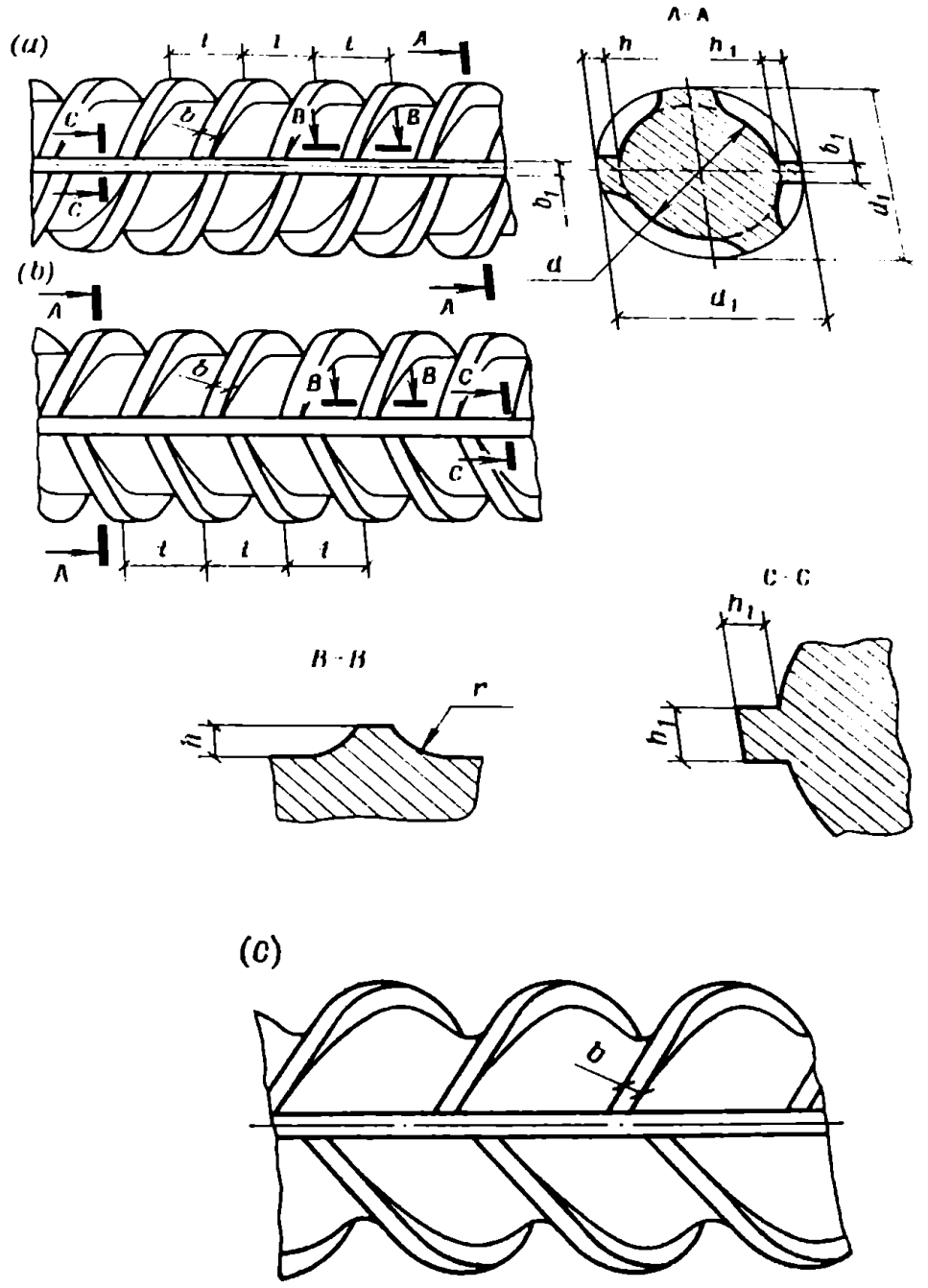



Fig. II.7. Soviet-made deformed reinforcing bars
(*a*) class A-III; (*b*) class A-III; (c) class A-IV and A-V

six classes, designated A-I, A-II, A-III, etc. (Table II.1). The Russian letter "т" in the designation stands for thermal treatment. Each class may include several steel grades

**Table II.1. Classification of Soviet-made Reinforcing Steel**

| Type and class of reinforcing steel | Steel grade | Diameter, mm | Ultimate strength, kgf cm$^{-2}$ (MPa) | Yield point, kgf cm$^{-2}$ (MPa) |
|---|---|---|---|---|
| Hot-rolled bars | | | | |
| Plain: —class A-I | Ст3сп3, Ст3пс3 | 6-40 | 3 800 (380) | 2 400 (240) |
| | Ст3кп3 | 6-40 | | |
| | ВСт3сп2 | 6-40 | | |
| | ВСт3пс2 | 6-40 | | |
| | ВСт3кп2 | 6-40 | | |
| | ВСт3пс2 | 6-18 | | |
| Deformed: | | | | |
| —class A-II | ВСт5сп2 | 10-40 | 5 000 (500) | 3 000 (300) |
| | ВСт5пс2 | 18-40 | | |
| | 18Г2С | 40-80 | | |
| | 10ГТ | 10-32 | | |
| —class A-III | 35ГС, 25Г2С | 6-40 | 6 000 (600) | 4 000 (400) |
| —class A-IV | 80С | 10-18 | 9 000 (900) | 6 000 (600) |
| | 20ХГ2Ц | 10-22 | | |
| —class A-V | 23Х2Г2Т | 10-22 | 10 500 (1 050) | 8 000 (800) |
| Thermally hardened bars: | | | | |
| —class Ат-IV | — | 10-25 | 9 000 (900) | 6 000 (600) |
| —class Ат-V | — | 10-25 | 10 500 (1 050) | 8 000 (800) |
| —class Ат-VI | — | 10-25 | 12 000 (1 200) | 10 000 (1 000) |
| Ordinary wire plain, class B-I | — | 3-5 | 5 000 (500) | — |
| —deformed, class Bp-I | — | 3-5 | 5 250-5 500 (525-550) | — |
| High-strength wire plain, class B-II | — | 3-8 | 19 000-14 000 (1 900-1 400) | — |
| —deformed, class B-II | — | 3-8 | 18 000-13 000 (1 800-1 300) | — |
| Seven-wire strands —class K-7 | — | 4.5-15 | 19 000-16 500 (1 900-1 650) | — |

which have the same mechanical characteristics, but differ in chemical analysis (see Table II.1).

The grade designations give an approximate chemical analysis of a given steel. For example, the first two-digit number in the designation "25Г2С" indicates the carbon content in points (hundredths of a per cent), the Russian letter "Г" indicates that the steel is alloyed with manganese, the numeral "2" indicates that the manganese content may be as high as 2%, and the Russian letter "С" stands for silicon. The Russian letter "Х" in the designation "20ХГ2Ц" stands for chromium.

Class A-IV and class A-V hot-rolled reinforcing steel and thermally hardened bars of classes Ат are intended for prestressed structures.

Reinforcing wire is divided into two classes. One incorporates ordinary class B-I plain (cold-drawn, low-carbon) and class Bp-I deformed wire intended for non-prestressed applications, and the other refers to high-strength class B-II (multi-pass drawn, carbon) plain and class Bp-II deformed wire intended for prestressed applications.

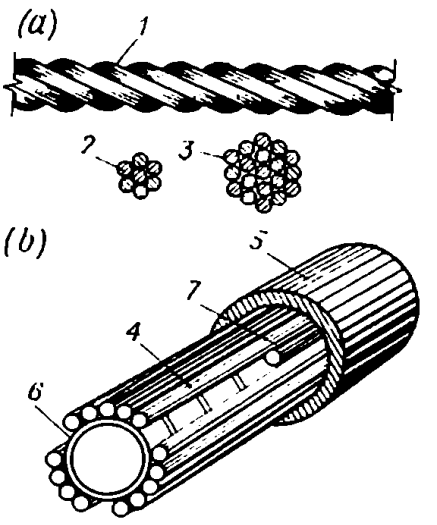



Fig. II.8. Prestressing tendons (*a*) wire strands; (*b*) cables; *1*—seven-wire strand, side view; *2* and *3*—cross sections of *7*- and 19-wire strands; *4*—cable wire; *5*—duct former; *6*—spiral; *7*—short bar

Prestressed concrete structures are reinforced with wires combined into strands and cables. As a rule, use is made of class K-7 and class K-19 stranded wire (Fig. II.8*a*) having seven or nineteen wires, respectively, class $K \times N$ multi-wire strands (where $N$ is the number of minor strands in a major one), and reinforcing cables shown in Fig. II.8*b*. A prestressing cable consists of straight wires or strands running parallel to one another. In most cases, they are spaced apart around a circumference, using spirals or spiders, so as to let grout inside the cable, and fastened by anchors at their ends. Such cables can be single- and multi-layer.

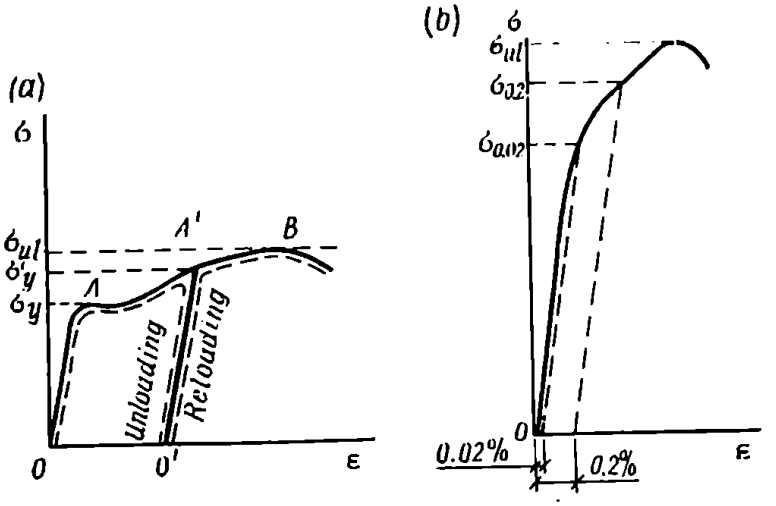



Fig. II.9. Stress-strain curves for reinforcing steel in tension
(*a*) steel with definite yield; (*b*) steel without definite yield

The mechanical characteristics of reinforcing steel are determined experimentally from stress-strain ($\sigma$ — $\varepsilon$) diagrams which are plotted on the basis of tension tests (Fig. II.9). Reinforcing steel with a well defined yield (Fig. II.9*a*) shows a considerable elongation after rupture. The first unit stress at which strain increases without a considerable increase in load is called the *yield point*, $\sigma_y$, and is measured in kgf cm$^{-2}$ (MPa). The behaviour of reinforcing steel having no well-defined yield point is characterized in terms of the proof yield strength, $\sigma_{0.2}$, that is, the stress at which the permanent set is 0.2%. It is likewise measured in kgf cm$^{-2}$ (MPa).

The yield point of reinforcing steel can artificially be raised by drawing. Hot-rolled reinforcement is cold-drawn until the stress in it exceeds the yield point, $\sigma'_y > \sigma_y$. Cold drawing (or cold working) changes the crystal lattice so that the reinforcing steel hardens. In all subsequent loadings, the behaviour of this steel is represented by the curve *O'A'B* showing an increased yield point (see Fig. II.9*a*).

Another important characteristic of reinforcing steel is the *ultimate tensile stress*, $\sigma_{ul}$, that is, the stress (or load) at or just before rupture.

Reinforcing steel has a high *elastic limit*, or the stress after which its stress-strain behaviour is no longer linear. This is a valuable property of steel because it permits the application of high prestressing forces. The *proof elastic limit*, $\sigma_{0.02}$ (in kgf cm$^{-2}$ or MPa), is taken as the stress at which the permanent set is 0.02%.

Reinforcing steel is inherently very ductile, so it will not suffer brittle rupture under load. The ductility of reinforcing steel is stated in terms of the total percentage of elongation, $\delta$ (%), which is found by dividing the increase in the bar length as measured across the fracture, by the original bar length, and the uniform percentage of elongation, $\delta_{un}$, which is determined by dividing the increase in the bar length, leaving out the neck region, by the original bar length. The percentage of elongation for class A-I, A-II, and A-III reinforcement is 25%, 19%, and 14%, respectively. For other classes, it ranges between 8% and 4%.

In the Soviet Union, reinforcing steel is classed into standard or gauge sizes according to nominal diameter. For deformed bars, a standard size is the diameter of plain bars having the same cross-sectional area. For deformed ordinary or high-strength reinforcing wire, a standard size is the wire diameter prior to deformation. For wire strands, a standard size is the diameter of the circumscribed circle (see Table II.1 and Appendix VIII).

As a rule, reinforcing steel for nonprestressed structures comes in the form of welded-wire fabric and bar mats. Longitudinal and transverse wires in fabric and mats (usually at right angles to one another) are joined by resistance spot welding (Figs. II.10 and II.11). According to a Soviet standard, welded-wire fabric is made of ordinary class B-I cold-drawn low-carbon steel wire from 3 to 5 mm in diameter, and class A-III hot-rolled low-alloy deformed bars from 6 to 9 mm in diameter. Welded-wire fabric comes in rolls and mats. The maximum diameter of longitudinal wires in rolls is 7 mm. The maximum width of fabric in mats, $B$, is 2.5 m, and the maximum length, $L$, is 9 m. The maximum width of welded-wire fabric in rolls is 3.5 m. The length of this type of fabric is limited by the weight of rolls, which may range from 100 to 500 kg. The dimen-

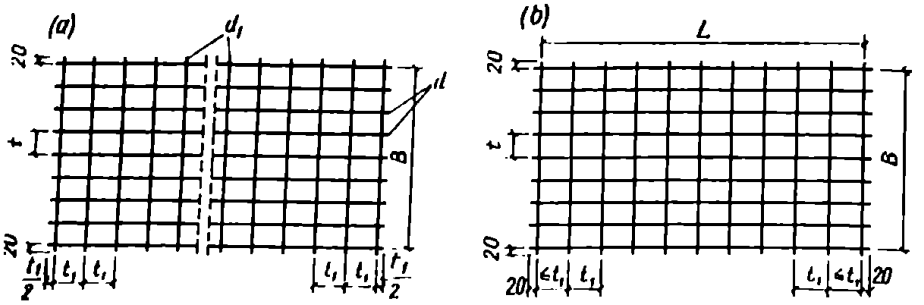



Fig. II.10. Welded-wire fabric
(a) in rolls; (b) in mats

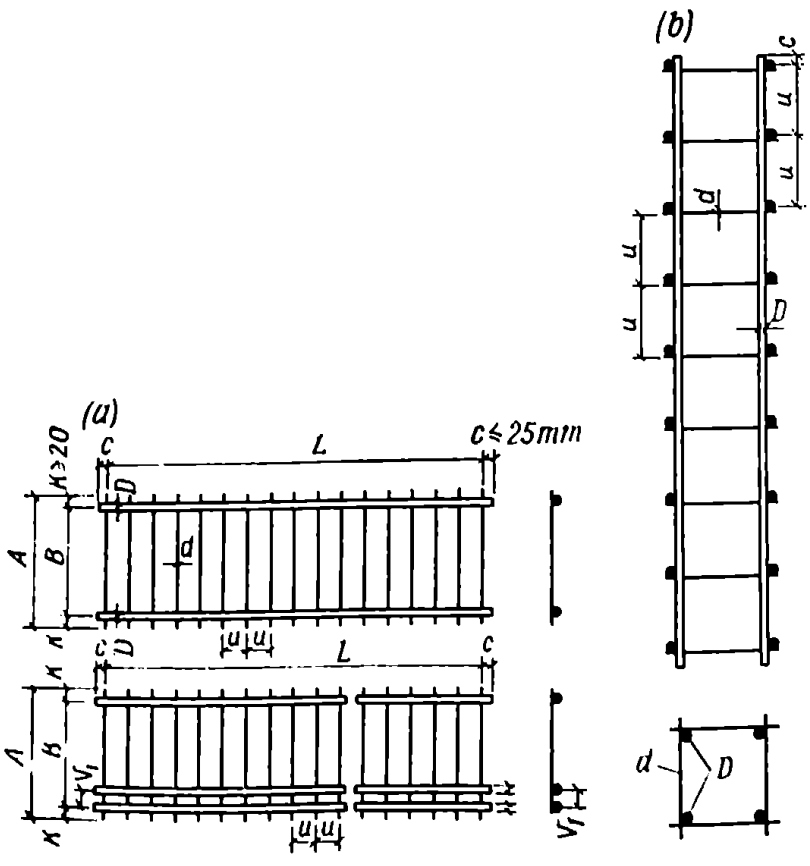



Fig. II.11. (a) Welded-bar mats and (b) reinforcing cage

sions $B$ and $L$ are taken between the centres of the outer wires. Type designations of welded-wire fabric contain the main characteristics of the product. The numerator gives the type of fabric in terms of $t/t_1/d/d_1$, and the denominator gives the width and length of the fabric, $B \times L$. For example, a welded-wire fabric mat 1 100 mm wide ($B$) and 5 900 mm long ($L$), fabricated from longitudinal wires 4 mm in diameter ($d$) and spaced 150 mm apart ($t$), and transverse wires 3 mm in diameter ($d_1$) spaced 250 mm apart ($t_1$) will be designated as follows

$$\frac{150/250/4/3}{1\,100 \times 5\,900}$$

Type designations for welded-wire fabric in rolls give only the width $B$.

Welded-bar mats are fabricated from longitudinal and transverse bars. Sometimes, welded-bar mats are combined into reinforcing cages (Fig. II.11). In designing welded-bar mats, it is important to observe the limitations imposed by the welding process to be used.

**Diameters of Longitudinal and Transverse Bars for Resistance Spot Welding**

| Longitudinal (transverse) bar diameter, mm | 3-12 | 14; 16 | 18; 20 | 22 | 25-32 | 36; 40 |
|---|---|---|---|---|---|---|
| Minimum transverse (longitudinal) bar diameter, mm | 3 | 4 | 5 | 6 | 8 | 10 |

Among other things, it is not recommended to fabricate welded-bar mats of reinforcing steel having poor weldability (class A-IV). The centre-to-centre distance $u$ between bars must be a multiple of 50 mm. Preferably, reinforcing bars should be joined by resistance spot welding or electroslag welding in reusable copper back-up plates. In the field, reinforcing bars may be arc-welded in a pool of molten metal within a permanent back-up plate. Sometimes, rein-

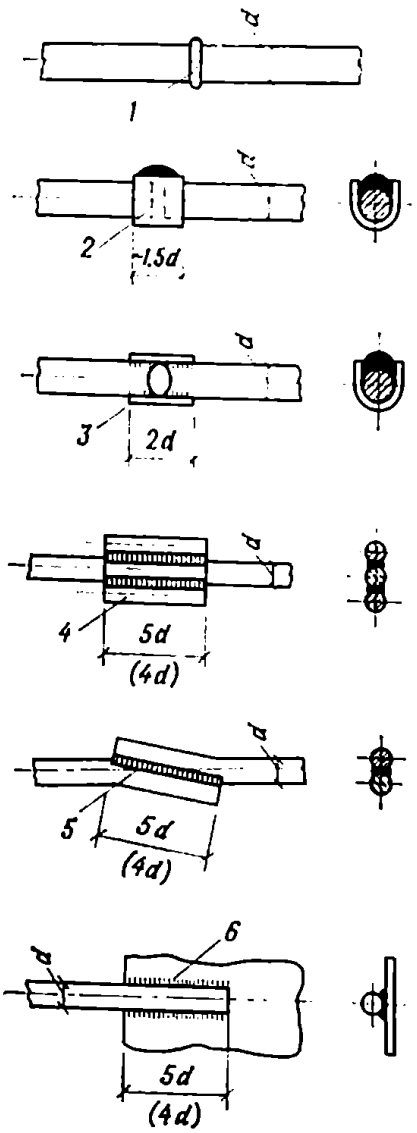



Fig. II.12. Welded joints in reinforcing bars

*1*—butt resistance welding; *2*—welding in pool of molten metal contained within permanent back-up plate; *3*—welding in pool of molten metal contained within permanent back-up plate, with fillet welds; *4*—arc welding with splice bars (four side fillet welds); *5*—arc lap welding (two side fillet welds); *6*—arc welding of bars to steel shapes (two side fillet welds)

forcing bars may be arc-welded using splice bars on either side or a lap-joint. The main types of welded joints for reinforcing steel are shown in Fig. II.12.

Reinforcing steel hardened by heat treatment (classes Ат) or drawing (classes Bp) may not be welded, because heat-treated steel would be tempered and cold-drawn wire would be annealed, thereby losing their strength. This is why these classes of reinforcing steel are used at the job in the "as-supplied" form. Sometimes, heat-treated bars may be joined using crimping clips.

Class A-I, A-II, and A-III reinforcing bars may be lap-joined without welding, if not all of the reinforcement is utilized in service. The necessary lap, $l_1$, is determined according to the anchorage recommended in the following section. This is, however, the least desirable type of joint, because it requires additional steel and is imperfect in construction

Sometimes, concrete structures are reinforced with rolled I-beams, channels, or angles. This is known as stiff reinforcement.

## II.4. Main Properties of Reinforced Concrete

Reinforced concrete is a happy combination of the physical and mechanical properties of concrete and steel. The reinforcement strengthens the concrete towards the action of loads, temperature variations, shrinkage, and creep. The concrete protects the reinforcing steel against exposure to fire and corrosion.

In reinforced concrete members, the reinforcing steel and the concrete work together. Owing to the bond between the materials, the reinforcing steel does not slip in the concrete under load. Bonding strength is tested by the "pull-out" and "press-in" tests. Experiments show that the bearing stress is nonuniformly distributed along the embedded length of a bar. The tangential stress gradually transfers the normal stress from the embedded bar to the concrete (Fig. II.13). The average bearing stress is given by

$$\tau_{b,av} = P/(\pi\, d l_{em})$$

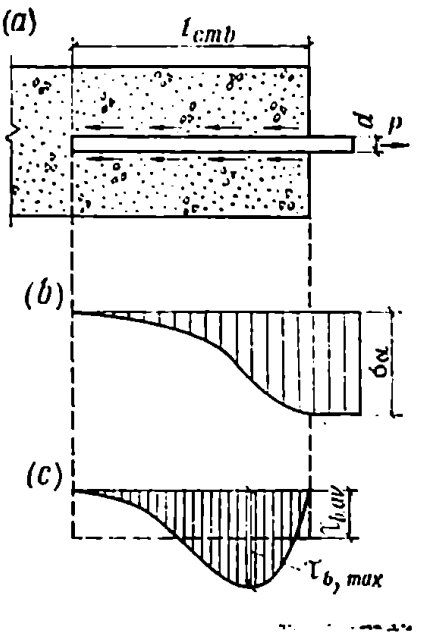



Fig. II.13. Bond between reinforcing steel and concrete
(a) test specimen; (b) normal stress diagram for steel; (c) steel-surface bonding force diagram

For plain bars, the average bearing stress ranges between 25 and 35 kgf cm$^{-2}$ (2.5 and 3.5 MPa). The bond strength increases with decreasing water-cement ratio, improved consolidation of the mix during concreting, and age.

When a bar is pressed into concrete, the bond strength is greater than when it is pulled out, because the concrete resists the lateral expansion of the compressed bar. With deformed reinforcing bars, the resistance to slip is two or three times the figure for plain reinforcing steel.

In reinforced concrete structures subjected to external loads the reinforcing steel and concrete are deformed together

before the advent of cracking, and the distribution of forces among them depends on their cross-sectional areas and moduli of elasticity. After the onset of cracking in the tension area of the concrete, the state of stress becomes more complicated (see Sec. V.2 and VI.6).

The bond between the reinforcing steel and the concrete restrains free shrinkage of the concrete. The restrained shrinkage deformation leads to initial balanced stresses owing to which the reinforcing steel is subjected to compressive stress and the concrete to tensile stress. Under certain conditions, the tensile stress in the steel may give rise to cracks. The total strain in reinforced concrete is markedly smaller than in plain concrete and depends on the amount of reinforcement.

As with shrinkage, reinforcing steel restrains free creep deformations. In a loaded reinforced concrete member, the restrained creep leads to a redistribution of stress between the steel and the concrete. In a member subjected to a sustained compressive force, the stress in the concrete gradually decreases and that in the steel increases. Owing to the restrained deformation of the concrete, the amount of creep in reinforced concrete structures is by a factor of 1.5 to 2 smaller than it is in plain concrete structures.

Variations in the initial length of reinforced concrete structures due to concrete shrinkage and temperature variations might bring about a state of internal stress and cause damage to individual members. To avoid this, large structures are divided into strain blocks by movement (expansion and shrinkage) joints.

If a structure is repeatedly and regularly exposed to temperatures from 50° to 200°C, its strength and rigidity somewhat decrease. Because of this, the design characteristics of the concrete and steel must be derated. If a structure is intended for exposure to elevated temperatures exceeding 200°C, its load-bearing members should be made of heat-resistant concrete.

Like plain concrete, reinforced concrete can serve for a very long time under normal exposure conditions. In corrosive environments, its service life may sharply decrease mainly due to the corrosion of concrete. Corrosion-preventive measures for reinforced concrete are given in standards.

CHAPTER THREE

# ANALYSIS AND DESIGN OF REINFORCED CONCRETE MEMBERS

## III.1. General

In Soviet practice, reinforced concrete structures are calculated for design loads and the respective load combinations, assuming the design strength for concrete (Appendices III and IV) and for reinforcing steel (Appendices VI and VII), as established by relevant standards for the first and second groups of limit states.

Depending on the exposure conditions and type of reinforcing steel used, the requirements for the crack resistance of reinforced concrete structures may be divided into three categories.

Under the first category, no cracks are allowed in structures. This applies to members subjected to the pressure of liquids or gases (such as tanks or pipes) and using high-strength reinforcement. For them, crack-resistance analysis is carried out with the loads multiplied by the overload factor $n > 1$ (as in strength analysis).

Under the second category, short-time opening of limited-width cracks is allowed, provided that they close tightly after the load has been removed. This applies to outdoor structures and structures reinforced with high-strength steel. For them, crack-resistance analysis is carried out, taking the overload factor $n > 1$, and crack-closure analysis, taking the overload factor $n = 1$.

Under the third category, short- and long-time opening of limited-width cracks is allowed. This applies to structures reinforced with bars. In crack-resistance analysis, the loads are multiplied by the overload factor $n = 1$.

Relevant standards specify the characteristic values of axial cube crushing strength $R^{ch}$, prism crushing strength $R^{ch}_{pr}$, concrete tensile strength $R^{ch}_{ten}$, and the design values of

$R$, $R_{pr}$ and $R_{ten}$ for the first group, and $R_{prII}$ and $R_{tenII}$ for the second group of limit states.

The data given in this text apply only to heavy concrete as it is most commonly used in plumbing structures.

The characteristic cube crushing strength of concrete is related to its brand number M (that is, its axial compressive strength $\bar{R}$ measured in kgf cm$^{-2}$ or MPa) as follows:

$$R^{ch} = \bar{R}\,(1 - 1.64v)$$

where $v$ is the variability of strength (for heavy concrete, $v = 0.135$), and 1.64 is a coefficient which ensures that the probability of occurrence for the characteristic strength is at least 0.95.

The characteristic prism crushing strength of concrete is given by the following empirical formula:

$$R_{pr}^{ch} = R^{ch}\,(0.77 - 0.000\,1\,\bar{R})$$

but it is taken not less than $0.72R^{ch}$.

The values of the characteristic strength of concrete can be found in Appendix III.

The design strength of concrete, $R_{pr}$ and $R_{ten}$ (see Appendix IV), is found by dividing $R_{pr}^{ch}$ and $R_{ten}^{ch}$, respectively, by the appropriate safety factors. For the first group of limit states, the safety factors are $k_{c,\,com} = 1.3$ in compression and $k_{c,\,ten} = 1.5$ in tension (when no particular limit is set on the characteristic axial tensile strength). For the second group of limit states, $k_{c,\,com} = k_{c,\,ten} = 1$.

For the first group of limit states, the effect that manufacture and exposure conditions and the manner of load application have on concrete is accounted for by the service factor $m_c$. In the USSR, eleven values of $m_c$ are set up by an appropriate standard, the most important of them being:

—for naturally hardened and heat-cured heavy concrete in structures serving under conditions favourable for strength build-up (under water, in wet soil, or at a relative humidity of over 75%), $m_{c1} = 1$;

—for all other cases, $m_{c1} = 0.85$;

—for autoclaved concrete, $m_{c10} = 0.85$.

For the second group of limit states, the design strength of concrete is determined, assuming that the service factor for concrete is $m_c = 1$.

The characteristic strength of reinforcement (see Appendices VI and VII) is established with allowance for the statistical variability of strength and is taken as the least observable value of the physical yield point or that of the proof yield point for reinforcing bars, and as the least observable ultimate strength for reinforcing wire. The minimum level of confidence for the characteristic strength of reinforcing steel required by relevant specifications is 0.95.

The design tensile strength of reinforcing steel, $R_s$, is found by dividing the characteristic strength $R_s^{ch}$ by the respective safety factor $k_s$. For the first group of limit states, $k_s$ ranges between 1.1 and 1.25 for reinforcing bars and between 1.55 and 1.75 for reinforcing wire. For the second group of limit states, $k_s = 1$ for all classes of reinforcing bars and wire.

The design compressive strength of steel, $R_{s,\,com}$, is taken equal to the respective tensile strength of the steel, $R_s$, but not greater than 4 000 kgf cm$^{-2}$ (400 MPa).

The effect of service conditions (type of anchorage, type of reinforcement, load cycling, etc.) is accounted for by an additional service factor $m_s$. For example, along inclined cracks not all stirrups are stressed to capacity (see Chap. VI), so a special service factor $m_{s,\,st} = 0.8$ and a strength $R_{s,\,st}$ are used.

The design strength $R_s$, $R_{s,\,com}$, and $R_{s,\,st}$ (see Appendices IV and V) should be multiplied by the steel service factor $m_s$. The most important of the six values of $m_s$ specified by relevant USSR standards are:

—in the stress transmission zone $l_{transm}$ of non-anchored longitudinal reinforcement or in the anchorage zone $l_{an}$ of nonprestressed steel (irrespective of its class),

$$m_{s3} = l_x/l_{transm} \quad \text{or} \quad m_{s3} = l_x/l_{an}$$

where $l_x$ is the distance from the beginning of the stress transmission zone to the section in question, and $l_{an}$ and $l_{transm}$ are as defined by Eqs. (III.26) and (III.27);

—when high-strength longitudinal reinforcing steel works under stresses exceeding the proof yield point, $m_{s4}$ (in more detail, this point is discussed in Chap. VI).

## III.2. Design of Prestressed Concrete Structures

For any group of limit states, prestressed concrete structures are designed to carry design loads together with the prestress.

The amount of prestress in reinforcing steel and concrete is very important for the subsequent behaviour of members under load. Among other things, the crack resistance of concrete subjected to tension in service increases with increasing prestress. Also, shrinkage and creep of the concrete and the relaxation of stress in the steel tend to diminish the prestressing effect. Therefore, if the amount of prestress was chosen low, its effect might be gone with time completely.

Experience shows that the prestress in concrete should be equal to about 40% to 50% of its strength at the time of prestress transfer from the steel to the concrete (the transfer strength $R_0$). With this amount of prestress, the loss of prestress by reinforced concrete structures due to creep will not practically exceed a maximum safe value.

Before the advent of cracking, loaded reinforced concrete members behave as elastic structures free from discontinuities, so that the external loads are shared by the concrete and the steel.

Because the strain in the concrete is equal to that in the metal, $\varepsilon_c = \varepsilon_s$, the stresses in the concrete $\sigma_c$ and the steel $\sigma_s$ in a member subjected to axial tension (before the onset of cracks) are related as

$$\sigma_c/E_c = \sigma_s/E_s \qquad \text{(III.1)}$$

where $E_c$ and $E_s$ are the moduli of elasticity of the concrete and steel, respectively.

The sum of forces acting in the concrete and steel is equal to the external force $N$

$$N = \sigma_c F_c + \sigma_s F_s \qquad \text{(III.2)}$$

where $F_c$ and $F_s$ are the cross-sectional areas of the concrete and steel in the reinforced concrete element.

Solving these equations simultaneously gives

$$\sigma_c = N/F_{tr} \qquad \text{(III.3)}$$

where

$$F_{tr} = F_c + nF_s$$

and

$$n = E_s/E_c$$

so that

$$\sigma_s = n\sigma_c \qquad \text{(III.4)}$$

Here, $F_{tr}$ is the area of the transformed section where the steel area is replaced by its equivalent "concrete area".

In the absence of cracks in the tension area, the moments of inertia and resistance of the transformed section are found in a similar manner.

As a rule, reinforced concrete structures can be prestressed mechanically or by electric heating.

In the case of pretensioning, the resultant tensile force in all the prestressed tendons is balanced by the reactive force taken up by the abutments. When the member is released from the abutments, the resultant force acts in the opposite direction and produces the prestressing force $N_0$ which is regarded as an external force. It is found as the product of the cross-sectional area $F_{pr}$ of the prestressed reinforcement and the prestress $\sigma_0$ in the reinforcing steel prior to its transfer to the concrete:

$$N_0 = F_{pr}\sigma_0 \qquad \text{(III.5)}$$

In an axially loaded member, the stress in the concrete subjected to the force $N_0$ is

$$\sigma_c = N_0/F_{tr} \qquad \text{(III.6)}$$

If the force acts with an eccentricity $e_0$ about the centroid of the transformed section, the stress is given by

$$\sigma_c = N_0/F_{tr} \pm (N_0 e_0/J_{tr})\, y \qquad \text{(III.7)}$$

where $J_{tr}$ is the moment of inertia of the transformed section, and $y$ is the distance from the centroid of the transformed section to the fibre in question.

The amount of prestress is taken as follows:

—for reinforced bars,

—for wire reinforcement,

$$0.2R_{sII} + p \leqslant \sigma_0 \leqslant 0.8R_{sII} - p \qquad \text{(III.9)}$$

where $R_{sII}$ is the design tensile strength applicable to the second group of limit states, and $p$ is the permissible deviation from the specified prestress. In mechanical tensioning, $p = 0.05\sigma_0$, and in tensioning by electric heating,

$$p \text{ (kgf cm}^{-2}) = 300 + 3\,600/l \qquad \text{(III.10)}$$

where $l$ is the length of the tendon being tensioned in m.

The initial prestress in the steel decreases with time for several reasons. Losses in prestress are customarily divided into the *early losses* which take place during the manufacture of a member and transfer of the prestress to the concrete, and the *late losses* which occur after the prestress has been transferred to the concrete.

In the case of pretensioning, the early losses $\sigma_{loss\,1}^{(pre)}$ are made up of the losses due to the relaxation of stress in the steel $\sigma_1$, the losses due to the difference in temperature $\sigma_2$, the losses due to the deformation of anchorage near the tensioning devices $\sigma_3^{(pre)}$, the losses due to steel friction $\sigma_4^{(pre)}$, the losses due to the deformation of steel forms $\sigma_5$, and the losses due to instantaneous creep $\sigma_6$:

$$\sigma_{loss\,1}^{(pre)} = \sigma_1 + \sigma_2 + \sigma_3^{(pre)} + \sigma_4^{(pre)} + \sigma_5 + \sigma_6 \qquad \text{(III.11)}$$

The late losses in pretensioning $\sigma_{loss\,2}^{(pre)}$ may be regarded as the sum of the losses due to shrinkage, $\sigma_8^{(pre)}$, and the losses due to creep, $\sigma_9^{(pre)}$:

$$\sigma_{loss\,2}^{(pre)} = \sigma_8^{(pre)} + \sigma_9^{(pre)} \qquad \text{(III.12)}$$

In posttensioning, the early losses, $\sigma_{loss\,1}^{(post)}$, are the sum of the losses due to the deformation of anchorages $\sigma_3^{(post)}$, and the losses due to steel friction $\sigma_4^{(post)}$:

$$\sigma_{loss\,1}^{(post)} = \sigma_3^{(post)} + \sigma_4^{(post)} \qquad \text{(III.13)}$$

whereas the late losses comprise the losses due to the relaxation of stress in the steel, $\sigma_7$, the losses due to shrinkage $\sigma_8^{(post)}$, the losses due to creep $\sigma_9^{(post)}$, the losses caused by the crushing of the concrete under the turns of spiral or ring reinforcement $\sigma_{10}$ (in structures up to 3 m in diameter),

and the losses due to deformed joints between the precast blocks $\sigma_{11}$:

$$\sigma_{loss\ 2}^{(post)} = \sigma_7 + \sigma_8^{(post)} + \sigma_9^{(post)} + \sigma_{10} + \sigma_{11} \qquad \text{(III.14)}$$

**Early Losses.** 1. The losses due to the relaxation of stress in the steel are as follows:
in mechanical tensioning (kgf cm$^{-2}$):
—high-strength wire and wire strands

$$\sigma_1 = (0.27\ \sigma_0/R_{sII} - 0.1)\ \sigma_0 \geqslant 0 \qquad \text{(III.15)}$$

—bars

$$\sigma_1 = 0.1\ \sigma_0 - 200 \geqslant 0 \qquad \text{(III.16)}$$

in tensioning by electric heating (kgf cm$^{-2}$):
—high-strength wire and wire strands

$$\sigma_1 = 0.05\ \sigma_0$$

—bars

$$\sigma_1 = 0.03\ \sigma_0$$

2. The losses due to the difference in temperature between the steel and abutments during the moist or heat curing of the concrete (kgf cm$^{-2}$):

$$\sigma_2 = 12.5\ \Delta t$$

where $\Delta t$ is the difference in temperature between the steel and the abutments in °C. If no data are available, $\Delta t$ should be taken as 65°C.

3. The losses due to the deformation of anchorages:
—in the case of pretensioning

$$\sigma_3^{(pre)} = \lambda E_s/l \qquad \text{(III.17)}$$

where $l$ is the length of the bar being tensioned, $\lambda = 2$ mm for upset shims or crushed button or rivet heads, and $\lambda = 1.25 + 0.15d$ for displaced bars in reusable grips ($d$ is the bar diameter in millimetres);
—in the case of posttensioning

$$\sigma_3^{(post)} = (\lambda_1 + \lambda_2)\ E_s/l \qquad \text{(III.18)}$$

where $\lambda_1 = 1$ mm for upset shims and $\lambda_2 = 1$ mm for deformed anchorages.

4. The losses due to steel friction:
—in the case of pretensioning

$$\left.\begin{aligned}&\sigma_4^{(pre)} = \sigma_0\,[1-\exp(-0.25\theta)]\\&\text{—in the case of posttensioning}\\&\sigma_4^{(post)} = \sigma_0\,[1-\exp(-kx-\mu\theta)]\end{aligned}\right\} \qquad \text{(III.19)}$$

where $\theta$ is the total angular change of the tendon along the curved portion in radians (Fig. III.1), $x$ is the length of the tendon from the jack to the section in question in

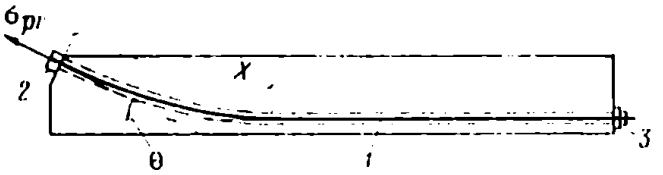



Fig. III.1. To determining frictional losses in raceways
*1*—tendon; *2*—jacking end; *3*—anchorage

metres, $\mu$ is the coefficient of friction between the tendon and the surrounding material, and $k$ is the wobble coefficient. The values of $\mu$ and $k$ are given in Table III.1.

**Table III.1. Friction and Wobble Coefficients**

| Duct | $k$ | $\mu$ | |
|---|---|---|---|
| | | cables and strands | deformed bars |
| With metal sheathing | 0.003 | 0.35 | 0.4 |
| With concrete surface: | | | |
| —made by rigid duct former | 0 | 0.55 | 0.65 |
| —made by flexible duct former | 0.0015 | 0.55 | 0.65 |

5. The losses due to the deformation of steel forms in tensioning by jacks

$$\sigma_5 = [(t-1)/2t]\,(\Delta l/l)\,E_s \qquad \text{(III.20)}$$

where $t$ is the number of tendon groups tensioned simultaneously, $l$ is the out-to-out distance of the bearing plates, and $\Delta l$ is the yield of the abutments along the resultant prestressing force, determined by the form design.

If no data on the form design are available, $\sigma_5$ is taken as 300 kgf cm$^{-2}$ (30 MPa).

6. The losses due to high-rate (instantaneous) creep. These losses occur due to an additional decrease in the length of the member during the first two or three hours after the transfer of the prestress from the steel to the concrete. This kind of loss is important in pretensioning. Creep in the concrete and, as a consequence, the respective losses increase with increasing prestress. When the member is allowed to harden fully in air, the loss of prestress is

$$\left.\begin{aligned} \sigma_6 &= 500\sigma_{pr}/R_0 \quad \text{for } \sigma_{pr}/R_0 \leqslant a \\ \sigma_6 &= 500a + 1\,000b\,(\sigma_{pr}/R_0 - a) \quad \text{for } \sigma_{pr}/R_0 > a \end{aligned}\right\} \quad \text{(III.21)}$$

where $\sigma_{pr}$ is the prestress in the concrete at the centroid of the prestressed steel taken with allowance for all the above losses, $a = 0.6$ and $b = 1.5$ for M-300 concrete (and higher brands), and $a = 0.5$ and $b = 3$ for M-200 concrete.

For heat-cured concrete, the losses defined by (III.21) are multiplied by 0.85.

**Late Losses.** 1. The losses due to the relaxation of stress in posttensioning are determined using Eqs. (III.15) and (III.16), that is, $\sigma_7 = \sigma_1$.

2. The losses due to shrinkage and contraction of a member are given in Table III.2.

**Table III.2. Loss of Prestress due to Shrinkage in kgf cm$^{-2}$ (MPa)**

| Concrete brand number | Pretensioning, $\sigma_8^{(pre)}$ | | Posttensioning, $\sigma_8^{(post)}$ |
|---|---|---|---|
| | air cured | heat-cured at atmospheric pressure | irrespective of hardening conditions |
| M-400 and lower | 400 (40) | 350 (35) | 300 (30) |
| M-500 | 500 (50) | 400 (40) | 350 (35) |
| M-600 and higher | 600 (60) | 500 (50) | 400 (40) |

3. The losses due to creep after the prestress has been transferred from the steel to the concrete (irrespective of the type of prestressing). When the concrete is allowed to harden fully in air, the losses are given by

$$\left.\begin{aligned}&\sigma_9^{(pre)} = \sigma_0^{(post)} = 2\,000\sigma_{pr}/R_0 \quad \text{for} \quad \sigma_{pr}/R_0 \leqslant 0.6\\&\sigma_9^{(pre)} = \sigma_9^{(post)}\\&\qquad = 4\,000\,(\sigma_{pr}/R_0 - 0.3) \quad \text{for} \quad \sigma_{pr}/R_0 > 0.6\end{aligned}\right\} \qquad \text{(III.22)}$$

where $\sigma_{pr}$ is determined as for the loss due to high-rate (instantaneous) creep.

When concrete is cured by heat at atmospheric pressure, the above losses are multiplied by 0.85.

4. The losses due to the crushing of the concrete under the turns of spiral or ring reinforcement (for a pipe or tank diameter of up to 3 m), equal to $\sigma_{10} = 300$ kgf cm$^{-2}$ (30 MPa).

5. The losses due to the deformation of joints between precast blocks, defined as

$$\sigma_{11} = n_j \lambda E_s / l \qquad \text{(III.23)}$$

where $n_j$ is the number of joints and $\lambda$ is the amount of deformation taken as 0.3 mm for grouted joints, and 0.5 mm for dry joints.

Any likely deviation from the specified prestress is accounted for by applying the tensioning accuracy factor

$$m_{ac} = 1 \pm \Delta m_{ac} \qquad \text{(III.24)}$$

The plus sign is taken for unfavourable conditions (the deviation increases), and the minus sign is taken for favourable conditions (the deviation decreases).

In mechanical tensioning, $\Delta m_{ac} = 0.1$; in tensioning by electric heating,

$$\Delta m_{ac} = 0.5\,(p/\sigma_0)\,(1 + 1/\sqrt{n}) \qquad \text{(III.25)}$$

but no less than 0.1. Here, $n$ is the number of prestressed bars in the section, and $p$ and $\sigma_0$ are the same as in Eqs. (III.8) and (III.9).

In the calculations of prestress loss, crack width, and sag, $\Delta m_{ac}$ may be taken as zero.

The total loss

$$\sigma_{loss} = \sigma_{loss\ 1} + \sigma_{loss\ 2}$$

may be as high as 2 000 to 2 500 kgf cm$^{-2}$ (200 to 250 MPa) or even higher. In design calculations, the total loss is usually taken to be not less than 1 000 kgf cm$^{-2}$ (100 MPa).

## III.3. Practical Hints for Design and Engineering

The brand number of concrete for a particular reinforced concrete structure is generally chosen on the basis of appropriate technical and economic estimates. Experience shows that for compressive members the best concrete brands are M-200 to M-400, for bending members M-150 to M-300, and for prestressed members M-300 to M-500.

The cross-sectional area of longitudinal reinforcement is usually 0.3% to 3% of that of the concrete. As compared with plain concrete members, the reinforced concrete products having this amount of reinforcement are free from brittle failure.

If the amount of reinforcement is low, reinforced concrete members are prone to brittle failure much as plain-concrete members are. To avoid this, relevant standards establish the minimum amount of longitudinal reinforcement to be installed (in more detail this point is discussed in subsequent chapters).

To ensure the composite action of the steel and the concrete at all stages of manufacture, erection, and service, and to protect the steel against corrosion and overheating, load-bearing reinforcement is installed in members so as to leave enough room for a protective concrete cover.

Nonprestressed reinforced concrete elements should preferably be reinforced with class A-III hot-rolled bars and class Bp-I or class B-I reinforcing wire 3 to 5 mm in diameter assembled into fabric and mats. Use may also be made of class A-II and class A-I hot-rolled steel, although it is customarily confined to transverse reinforcement. For nonprestressed structures intended to meet water tightness requirements, it is recommended to use class A-II and A-I reinforcing steel. Also, such structures may sometimes be

reinforced with class A-III hot-rolled bars and class Bp-I and B-I ordinary reinforcing wire.

Prestressed concrete structures are reinforced with class A-IV (and higher grades) hot-rolled bars, reinforcing strands, and class B-II and Bp-II high-strength reinforcing wire.

Some types of reinforcing steel and their welded joints tend to fail owing to cold brittleness. Because of this, reinforced concrete structures intended for service at subzero temperatures from —30° to —40°C may not be reinforced with class A-IV (grade 80C), class A-II (grade ВСт5пс2), and class A-I (grades Ст3кп2 and ВСт3Гпс2) hot-rolled bars.

Nonprestressed bars should be extended beyond the section where the steel is supposed to develop its full design strength for a length sufficient for proper anchorage:

$$l_{an} = (m_{an}R_s/R_{pr} + \Delta\lambda_{an})\, d \qquad \text{(III.26)}$$

but not less than $l_{an} = \lambda_{an}d$ or the minimum safe value $l_{an.\,min}$ whichever is the greater. The values of the quantities in Eq. (III.26) are given in Table III.3 for deformed bars. The figures in parentheses apply to plain bars.

For anchorage at the outer free supports of bending members, the longitudinal bars must be carried past the

**Table III.3. Anchorage Length of Nonprestressed Bars**

| Anchorage conditions | $m_{an}$ | $\Delta\lambda_{an}$ | $\lambda_{an}$ | $l_{an.\,min}$ |
|---|---|---|---|---|
| Tensile steel in tension concrete | 0.7 (1.2) | 11 | 20 | 250 |
| Tensile or compressive steel in compression concrete | 0.5 (0.8) | 8 | 12 (15) | 200 |

internal face of each support for not less than $10d$ (see Fig. III.2); if no transverse reinforcement is required by design calculations, the anchorage length may be reduced to not less than $5d$. If $l_{an}$ has to be reduced as compared with the required anchorage length, the bars must be anchor-

ed with the aid of short bars and plates, or by welding them to embedded parts.

Individual bars, welded-wire fabric and bar mats in reinforced concrete structures should usually be joined by welding. The types of welded joints are shown in Fig. II.12.

In both welded and tied-wire fabric and bar mats, the tensile nonprestressed bars up to 36 mm in diameter may be lap-spliced without welding. In this case, the lap, $l_l$,

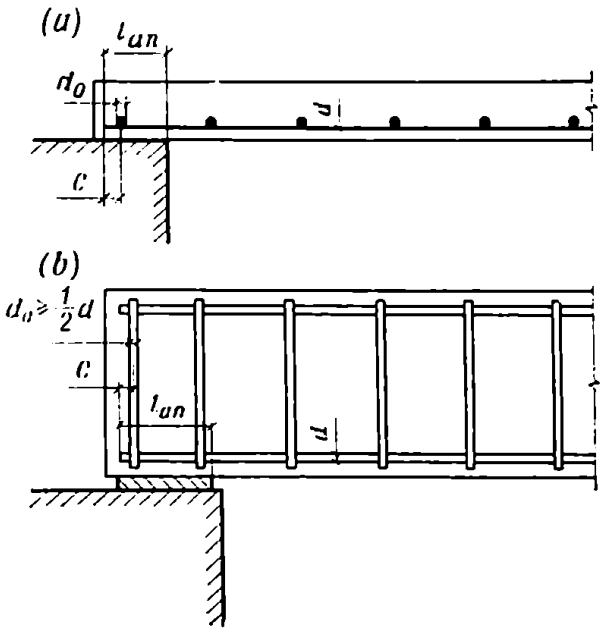



Fig. III.2. Anchorage of welded-wire fabric and bar mats at the free supports of bending members
(a) in slab; (b) in beam

should be not less than $l_{an}$ as defined by Eq. (III.26). The number of tensile bars that may be spliced at one joint should not exceed 25% of the total number in the case of plain bars and 50% in the case of deformed bars. It must be remembered that the joints should be located outside the sections where the tensile steel is supposed to act at its full strength. Reinforcing bars subjected to axial tension or to tension with small eccentricities are not allowed to be lap-spliced without welding.

Welded-wire fabric and welded-bar mats with the load-bearing bars located on one side of the transverse bars are lap-spliced as shown in Fig. III.3. Here, each lap should span at least two transverse bars.

During concreting, the steel is held in its design position by pins, suspensions, or additional pieces of fabric and mats.

The clear space between bars is chosen so as to facilitate placement and consolidation of the concrete mix. For

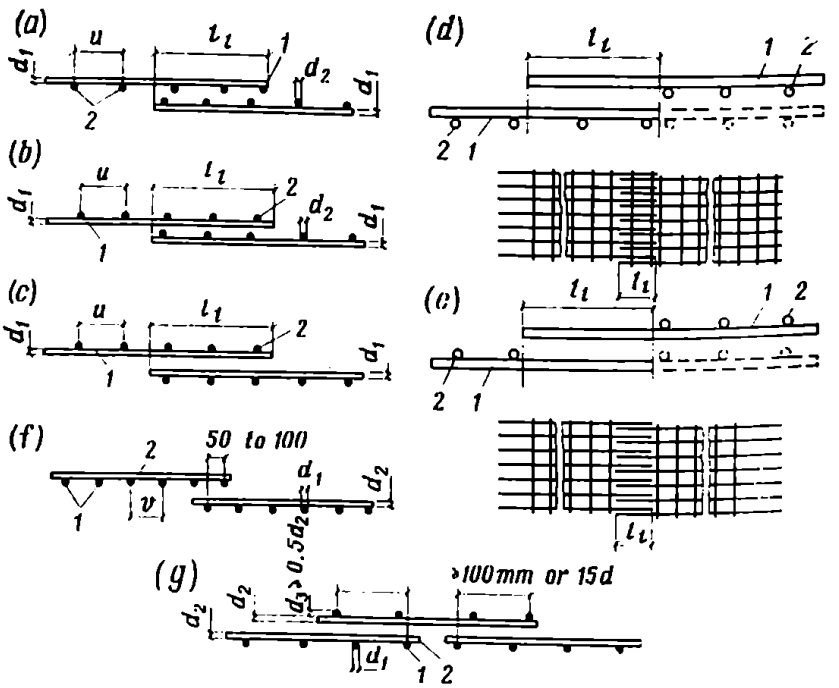



Fig. III.3. Splicing of welded-wire fabric and tied-bar mats

(*a*) lap splicing in load-bearing direction, plain wires, transverse wires in the same plane; (*b*) and (*c*) plain wires, transverse wires in different planes; (*d*) deformed bars, no transverse bars within lap in one of pieces to be spliced; (*e*) deformed bars, no transverse bars within lap in both pieces to be spliced; (*f*) lap splicing in direction of erection (distribution) bars; (*g*) with additional splice fabric; *1*—load-bearing wires; *2*—erection wires

horizontal bars, the clear space should be not less than the bar diameter, nor less than 25 mm (whichever is the greater) for the lower reinforcement, and not less than 30 mm for the upper reinforcement. For vertical bars, the clear space should be not less than 50 mm.

## III.4. Engineering Hints for Prestressed Members

In the case of posttensioning, prestressed reinforcement (strands, cables, or bars) should always be anchored at the ends (Fig. III.4). The most common types of anchorage are the socket-type anchor and the Freyssinet anchor. In the former, a threaded stud is inserted into a cable and a mild steel fitting is put on the cable. When the anchor

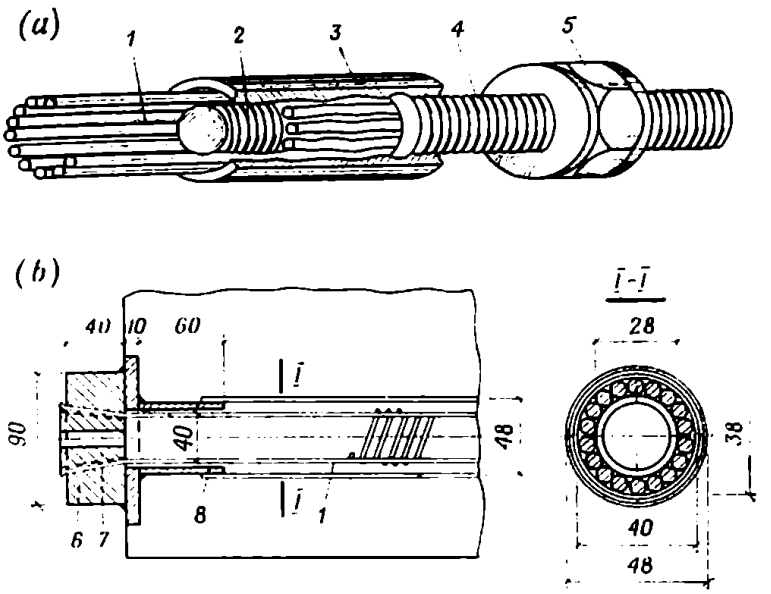



Fig. III.4. Anchorage of tendons (posttensioning)
(*a*) socket-type anchorage; (*b*) Freyssinet anchorage; *1*—wires; *2*—stud with circular grooves; *3*—socket; *4*—threaded stud; *5*—nut; *6*—female anchor cone; *7*—cone plug; *8*—sleeve

is drawn through an upsetting ring, the fitting yields and compresses the cable wires. The Freyssinet system uses a wedging principle. Here, a double-acting hydraulic jack is placed in position and grips the wires by means of wedges. The main piston is pumped to the required pressure and, while maintaining that pressure, the inner piston is pumped to drive home the plug. Deformed bars pretensioned by electric heating are usually anchored by upset bar ends and embedded shims with slots.

To ensure a proper bond between the concrete and the tendons, the raceways are pressure-filled with cement or sand-cement grout after the tendons have been tensioned and anchored.

In the case of pretensioning, deformed bars, deformed wire, and wire strands require no special anchoring devices, if the structure is made of a sufficiently strong concrete, (see Table III.4) to ensure an ample bond. In all other cases,

**Table III.4. Minimum Concrete Brand Numbers for Prestressed Members**

| Type of steel | Class of steel | Concrete brand number |
|---|---|---|
| Bars without anchors, 10 to 18 mm in diameter | A-IV, Ат-IV | M-200 |
| | A-V, Ат-V | M-250 |
| | Ат-VI | M-350 |
| Bars without anchors, 20 mm and more in diameter | A-IV, Ат-IV | M-250 |
| | A-V, Ат-V | M-350 |
| | Ат-VI | M-400 |
| Wire with anchors | B-II | M-250 |
| Wire without anchors, 5 mm and more in diameter | Bp-II | M-250 |
| Wire without anchors, 6 mm and more in diameter | Bp-II | M-400 |
| Wire strands | K-7 | M-350 |

prestressed reinforcement must appropriately be anchored by suitable devices, because no self-anchoring will take place.

Plain prestressed wire is always anchored at the ends (Fig. III.5).

Anchors impose a concentrated force on the concrete, thereby giving rise to an overstress. Because of this, the end portions of members near anchors should additionally be reinforced with welded-wire fabric and stirrups, and the concrete cover at such sections should generally be made thicker (Fig. III.6).

If reinforcing steel is installed without any anchoring devices, the prestress is transferred to the concrete within

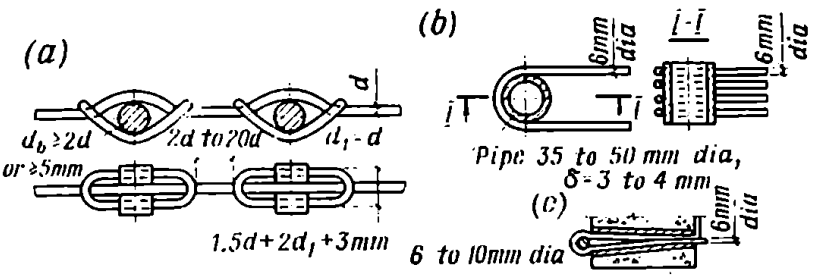



Fig. III.5. Anchorage of plain high-strength wire with
(*a*) short bars; (*b*) pipes; and (*c*) pins

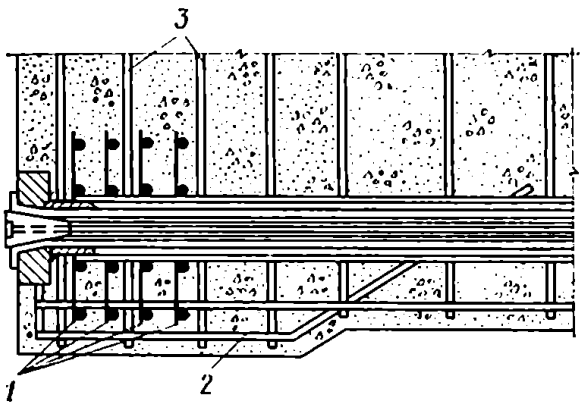



Fig. III.6. Strengthening of prestressed members within the anchorage zone
*1*—welded-wire fabric; *2*—longitudinal reinforcement placed for structural reasons; *3*—additional transverse bars

the self-anchoring length at the ends. This length may range from 10 to 15 diameters in the case of bars, 30 to 70 diameters in the case of seven-wire strands, and 45 to 100 diameters in the case of deformed wire. Within the self-anchoring length, the concrete should additionally be reinforced with fabric, spirals, etc.

The transmission length for prestressed steel without anchoring devices (the length where the stress is transferred from the steel to the concrete) is given by

**Table III.5. Transmission Length for Prestressed Steel Without Anchors**

| Type and class of reinforcing steel | $m_{tr}$ | $\Delta\lambda_{tr}$ |
|---|---|---|
| Deformed bars of any class and diameter | 0.3 | 10 |
| Class Bp-II high-strength wire: | | |
| 5 mm in diameter | 1.8 | 40 |
| 4 mm in diameter | 1.8 | 50 |
| 3 mm in diameter | 1.8 | 60 |
| Class K-7 wire strands: | | |
| 15 mm in diameter | 1.25 | 25 |
| 12 mm in diameter | 1.4 | 25 |
| 9 mm in diameter | 1.6 | 30 |
| 7.5, 6 and 4.5 mm in diameter | 1.8 | 40 |

where $m_{tr}$ and $\Delta\lambda_{tr}$ are given in Table III.5, and $\sigma_{tr}$ is taken to be equal to $R_s$ or $\sigma_0$, whichever is the greater.

For deformed reinforcing bars of any class, $l_{tr}$ is taken as not less than $15d$.

CHAPTER FOUR

# MEMBERS IN AXIAL COMPRESSION

## IV.1. Constructional Features

For design purposes, it is often assumed that intermediate columns in buildings and other structures, the top chords of trusses loaded at their joints, the uprising and vertical web members of trusses (Fig. IV.1), and so on are in pure axial compression.

In reality, however, various factors cause a departure from pure axial compression, so that what may be called eccentrical compression exists, with accidental eccentricities. The values of accidental eccentricities are limited by relevant standards. In design calculations, accidental eccentricities should not exceed 1/600 of the length of a member (as measured between the supports), 1/30 of the depth of a member, or 1 cm, whichever is the smaller.

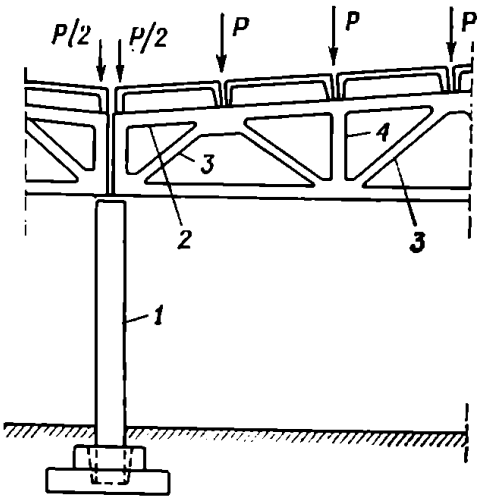



Fig. IV.1. Members in axial compression

*1*—intermediate column (equally loaded from either side); *2*—top chord of truss (with load applied at joints); *3*—uprising diagonals; *4*—verticals

As a rule, columns are made square or rectangular in cross-section. The cross-sectional dimensions of columns are found by calculation; to permit the use of standard formwork and reinforcing cages, they are chosen to be multiples of 50 mm. For better concreting, in-situ columns must measure not more than 25 cm on the side.

Columns are made with M-200, M-300, and M-400 concretes. They are reinforced with longitudinal bars 12 to 40 mm in diameter (load-bearing reinforcement), mostly of class A-II and A-III hot-rolled steel, and transverse bars of class A-I hot-rolled steel or cold-drawn low-carbon wire (Fig. IV.2). The longitudinal and transverse reinforcement is combined into bar mats or cages which may be either welded or tied (Fig. IV.3).

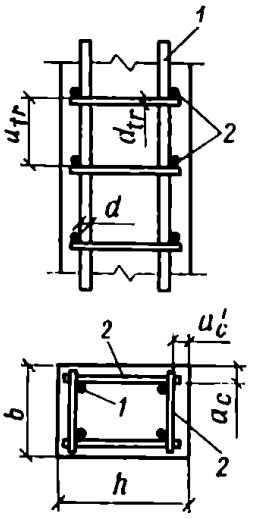



Fig. IV.2. Reinforcement of members in axial compression
*1*—longitudinal bars; *2*—transverse bars; $a_c$—concrete cover for longitudinal reinforcement; $a'_c$—concrete cover for transverse reinforcement

The amount of longitudinal steel in the cross section of members in compression is expressed in terms of the reinforcement ratio $\mu$ or the percentage of reinforcement

$$\mu\% = \mu \times 100 = (F_s/F) \times 100$$

where $F_s$ is the total longitudinal steel area and $F$ is the cross-sectional area of the member.

In practice, the percentage of reinforcement usually ranges between 0.4% and 3%.

In a column, the load-bearing bars should be installed as close to the surface as practicable, but leaving a sufficient concrete cover $a_c$ which, according to specifications, should be not less than the bar diameter or 20 mm, whichever is the greater.

Columns up to 40 × 40 cm in cross section may be reinforced with four longitudinal bars, which gives a clear space between the load-bearing bars close to the maximum allowed value. The minimum spacing between the bars depends on the concreting conditions (see Sec. III.3).

Transverse bars are installed without design calculations, but subject to appropriate rules. Among other things, the spacing between transverse bars should be such that the longitudinal bars can not buckle when loaded. This spacing, $u_{tr}$, (see Fig. IV.2) is $20d$ for welded reinforcing cages and

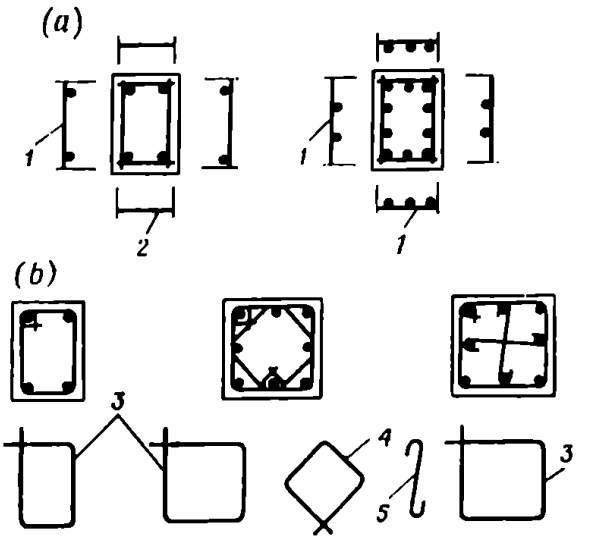



Fig. IV.3. Reinforcement of members in axial compression
(*a*) welded-bar mats; (*b*) tied mats; *1*—welded-bar mats; *2*—erection bars; *3*—stirrups; *4*—additional stirrups; *5*—hooks

$15d$ for tied cages, but not more than 500 mm (here, $d$ is the least diameter of the longitudinal compressive bars). The value of $u_{tr}$ is rounded to a multiple of 50 mm.

The transverse bar diameter $d_{tr}$ in welded reinforcing cages is chosen so as to meet the requirements for weldability (see p. 33). The stirrup diameter for tied cages should be not less than 5 mm nor less than $0.25d$, where $d$ is the maximum longitudinal bar diameter.

The concrete cover for transverse steel, $a'_c$, should be at least 15 mm.

It is not recommended to use spliced longitudinal bars within the length of a member.

In spliced reinforcing mats, the spacing between the bars within a splice should not exceed $10d$ (where $d$ is the diameter of the bars being spliced).

## IV.2. Design Under Accidental Eccentricity

Experiments (Fig. IV.4) have shown that the strength of short axially compressed members (often called stub columns) is contributed by the concrete and longitudinal reinforcement. In design calculations, the concrete is assum-

ed to work at its ultimate strength and the steel, at its yield point, because a heavily stressed concrete experiences relatively high plastic strains.

Obviously, if members are designed by the limit-state method, the resultant longitudinal force due to design loads should not exceed the sum of internal design forces in the concrete and reinforcement:

$$N \leqslant R_{pr}F + R_{s,\ com}F_s \quad \text{(IV.1)}$$

where $N$ is the design force, $R_{pr}$ is the design axial compressive strength of the concrete (prism crushing strength), and $R_{s,\ com}$ is the design compressive strength of the reinforcement.

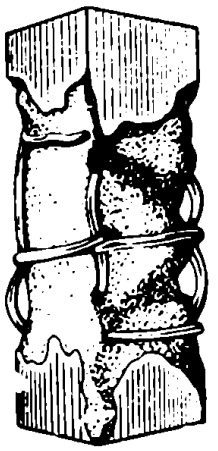


Fig. IV.4. Short specimen after loading by an axial compressive force

Appropriate specifications require that long (slender) members axially compressed with accidental eccentricities should be analyzed as eccentrically compressed members (see Sec. IV.1 and Chap. VII). At $l_0 \leqslant 20h$ (where $l_0$ is the effective length of a member which depends on how its ends are fixed, and $h$ is the depth of the member), some rectangular members symmetrically reinforced with class A-I, A-II, or A-III bars may be designed in terms of load-bearing capacity, assuming them to be axially loaded and subject to the following condition

$$N \leqslant m\varphi\,(R_{pr}F + R_{s,\ com}F_s) \quad \text{(IV.2)}$$

Here, $F = hb$ ($b$ is the width of the cross section), $F_s$ is the cross-sectional area of all longitudinal bars, $m$ is the service factor equal to 0.9 for $h \leqslant 200$ mm and to unity for $h > 200$ mm, $\varphi$ is the coefficient found experimentally according to the duration of loading, slenderness, and type of member reinforcement

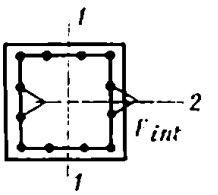
$$\varphi = \varphi_c + 2\,(\varphi_{sl} - \varphi_c)\,R_{s,\ com}F_s/R_{pr}F \quad \text{(IV.3)}$$

but taken as not more than $\varphi_{sl}$. The values of $\varphi_c$ and $\varphi_{sl}$ are looked up in Table IV.1, where $N_l$ is the longitudinal

force due to the dead and long-time live loads, and $F_{int}$ is the cross-sectional area of the intermediate bars (see the figure in the table).

Given all the data on the cross-sectional geometry, reinforcement, materials, and load, the load-bearing capacity of

**Table IV.1. Values of $\varphi_c$ and $\varphi_{st}$**




| $N_l/N$ | $l_0/h$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | under 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| | | | | $\varphi_c$ | | | | |
| 0 | 0.93 | 0.92 | 0.91 | 0.9 | 0.89 | 0.86 | 0.83 | 0.8 |
| 0.5 | 0.92 | 0.91 | 0.9 | 0.88 | 0.85 | 0.8 | 0.73 | 0.65 |
| 1 | 0.92 | 0.91 | 0.89 | 0.86 | 0.81 | 0.74 | 0.63 | 0.55 |
| | | | | $\varphi_{st}$ | | | | |
| A. The cross-sectional area of the intermediate bars at the faces parallel to the selected plane, $F_{int}$, is less than $F_s/3$ | | | | | | | | |
| 0 | 0.93 | 0.92 | 0.91 | 0.9 | 0.89 | 0.87 | 0.84 | 0.81 |
| 0.5 | 0.92 | 0.92 | 0.91 | 0.9 | 0.87 | 0.84 | 0.8 | 0.75 |
| 1 | 0.92 | 0.91 | 0.9 | 0.88 | 0.86 | 0.83 | 0.77 | 0.7 |
| B. $F_{int}$ is not less than $F_s/3$ | | | | | | | | |
| 0 | 0.92 | 0.92 | 0.91 | 0.89 | 0.87 | 0.84 | 0.8 | 0.75 |
| 0.5 | 0.92 | 0.91 | 0.9 | 0.87 | 0.83 | 0.79 | 0.72 | 0.65 |
| 1 | 0.92 | 0.91 | 0.89 | 0.86 | 0.8 | 0.74 | 0.66 | 0.58 |

*1 – 1* – selected plane
*2* – intermediate bars
$F_s$ – cross-sectional area of all longitudinal bars

a member in axial compression can be found from Eq. (IV.2), using the value of $\varphi$ as found from Eq. (IV.3) and Table IV.1.

If the cross-sectional dimensions of a member have already been assigned and it is only necessary to find the steel area,

use should be made of Eq. (IV.2) from which

$$F_s = N/m\varphi R_{s,\,com} - FR_{pr}/R_{s,\,com} \qquad \text{(IV.4)}$$

where $\varphi$ is determined by successive approximations.

When the load, effective length, and materials are specified in advance, the cross-sectional dimensions of a member and the steel area are determined on assuming $\varphi = m = 1$ and $F_s = 0.01F$. From Eq. (IV.2), we find

$$F = N/m\varphi\,(R_{pr} + \mu R_{s,\,com}) \qquad \text{(IV.5)}$$

and select the nearest standard cross-sectional dimensions. Then we calculate $l_0/h$ and select $F_s$ in the above manner. If it turns out that the percentage of reinforcement of the designed cross-section fails to meet the condition

$$\mu_{min}\% \leqslant \mu\% \leqslant \mu_{max}\%$$

the dimensions should be altered and $\varphi$ and $F_s$ calculated again. The cross-sectional dimensions may be considered to have been selected correctly, if $\mu = 1\%$ to $2\%$.

**Example IV.1.** Given: A column in axial compression. Its effective length is $l_0 = 6.4$ m, the design longitudinal force is $N = 150$ tf $= 1\,500$ kN, the force induced by the dead and long-time live loads is $N_l = 100$ tf $= 1\,000$ kN. The concrete is M-200 heavy concrete ($m_{cl} = 0.85$); the reinforcing steel is class A-II hot-rolled deformed bars.

To find: The cross-sectional dimensions and the steel area.

*Solution.* According to Appendices II and IV,

$$R_{pr} = 90 \text{ kgf cm}^{-2} \text{ (9 MPa)}$$

and

$$R_{s,\,com} = 2\,700 \text{ kgf cm}^{-2} \text{ (270 MPa)}$$

For $\varphi = m = 1$ and $\mu = 0.01$, Eq. (IV.5) yields

$$F = N/(m_{cl} R_{pr}) + (\mu R_{s,\,com})$$

$$= 150\,000/(0.85 \times 90 + 0.01 \times 2700)$$

$$= 1\,450 \text{ cm}^2$$

We choose a 40 × 40 cm column for which $F = 1\,600$ cm$^2$. The necessary ratios are

$$l_0/b = 640/40 \quad 16$$

$$N_l/N = 100/150 = 0.667$$

The service factor $m = 1$, because $h = 40$ cm, which is greater than 20 cm.

Interpolating from Table IV.1 gives

$$\varphi_c = 0.78$$

and

$$\varphi_{st} = 0.837$$

(assuming that $F_{int} < F_s/3$). From Eq. (IV.3), we get

$$\varphi = \varphi_c + 2\,(\varphi_{st} - \varphi_c)\,R_{s,\,com}\mu/m_{c1}R_{pr}$$

$$= 0.78 + 2\,(0.837 - 0.78) \times 2\,700 \times 0.01/(0.85 \times 90)$$

$$= 0.78 + 0.04 = 0.82$$

Since $\varphi$ ought not to exceed $\varphi_{st}$, we take $\varphi = 0.82$. Equation (IV.4) gives

$$F_s = N/m\varphi R_{s,\,com} - Fm_{c1}R_{pr}/R_{s,\,com}$$

$$= 150\,000/(1 \times 0.82 \times 2\,700) - (1\,600 \times 0.85 \times 90)/2700$$

$$= 22.4 \text{ cm}^2$$

For the above cross-sectional dimensions, the percentage of reinforcement is

$$\mu\% = (22.4 \times 100)/(40 \times 40) = 1.4$$

Using $\mu = 0.014$ thus found and Eq. (IV.3), we obtain a refined value of $\varphi = 0.83$. The re-calculation gives $F_s = 21.6$ cm$^2$. The obtained percentage of reinforcement ($\mu = 1.4\%$) lies within the prescribed limits. So, the final choice is a 40 × 40 cm column reinforced by four class A-II longitudinal bars 20 mm in diameter and four class A-II longitudinal bars 18 mm in diameter ($F_s = 22.74$ cm$^2$).

On distributing the bars over the cross section, we can see that the condition $F_{int} < F_s/3$ is met. So, no further calculation is necessary.

The diameter of the transverse bars in welded bar mats, $d_{tr}$, is taken as 8 mm (see Table II.2), the spacing is $u_{tr} = 35$ cm, because $u_{tr} \leqslant 20d = 20 \times 1.8 = 36$ cm.

CHAPTER FIVE

# REINFORCED CONCRETE MEMBERS IN AXIAL TENSION

## V.1. Constructional Features

For design purposes, it is assumed that axial tension exists in arch strings, the bottom chords and descending diagonals of trusses, and the walls of round tanks for liquids (Fig. V.1).

The basic engineering principles discussed in Chapter III also apply to members in axial tension. Main nonprestressed reinforcing bars are generally joined by welding; unwelded lap splicing is only allowed in slabs and walls.

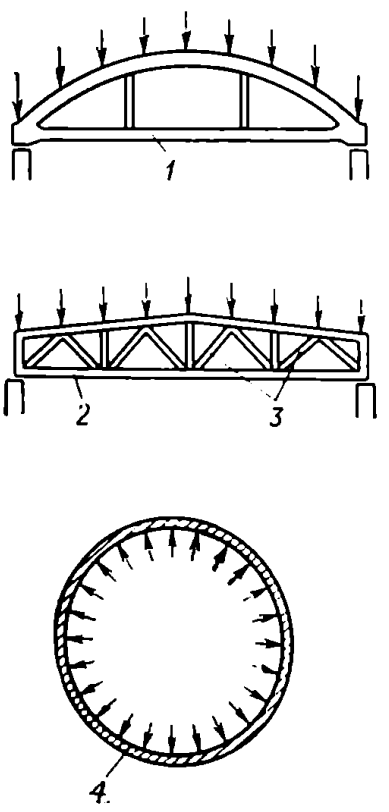



Fig. V.1. Members in axial tension

*1*—arch string; *2*—bottom chord of truss; *3*—descending diagonal of truss; *4*—wall of round tank

No joints are allowed in tensile prestressed reinforcement (bars, wire strands, and cables) used in linear members (such as arch strings and bottom truss chords). The steel should be installed symmetrically over the cross-section of a member (Fig. V.2) in order to avoid, as far as possible, an eccentrical prestress in the member while the prestressing force is transferred to the concrete (which may be done by releasing all bars at the same time or in groups).

In the case of posttensioning, the prestressed steel enclosed in ducts is unable to contribute to the strength of the members during the trans-

fer of the prestress. Therefore, it is advisable to provide prestressed members with a small amount of nonprestressed reinforcement (Fig. V.2*b*). It should be installed closer to the outer surface, so as to protect the member against the likely eccentrical effects of the prestress.

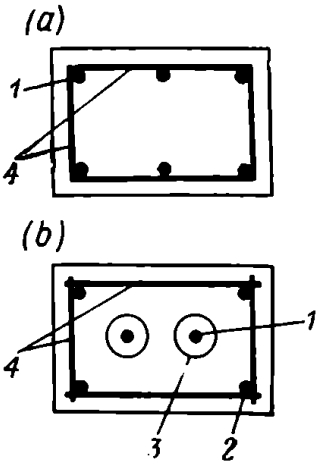



Fig. V.2. Distribution of reinforcing steel in prestressed linear members subjected to axial tension

(*a*) with pretensioning; (*b*) with posttensioning; *1*—prestressed steel (bars, wire strands, and cables); *2*—nonprestressed steel; *3*—duct for prestressed steel; *4*—transverse bars

## V.2. State of Stress and Design of Nonprestressed Members

As the external load is gradually raised, three characteristic stages can be traced in the stress-strain behaviour of a nonprestressed member subjected to axial tension. Stage I lasts until cracking begins in the concrete; stage II begins after cracking has occurred in the concrete; stage III is one of breakdown.

**Stage I.** Under the action of a tensile force $N$, the concrete and the steel of a member are deformed with an equal elongation, $\varepsilon_s = \varepsilon_{c,\,ten}$. Hence, the stresses in the steel and the concrete, $\sigma_s$ and $\sigma_{c,\,ten}$, can be related as follows:

$$\sigma_s/E_s = \sigma_{c,\,ten}/E_{c,\,ten} \text{ or } \sigma_s = \sigma_{c,\,ten}E_s/E_{c,\,ten} \qquad (V.1)$$

Experiments have shown that immediately before cracking the tensile modulus of elasticity of concrete, $E_{c,\,ten}$, is approximately equal to half the initial tangent modulus of elasticity of concrete in compression, $E_c$, that is, $E_{c,\,ten} \approx \approx 0.5\,E_c$. In the circumstances, the stress in the concrete is equal to its ultimate tensile strength $\bar{R}_{ten}$. Therefore,

$$\sigma_s = 2\,(E_s/E_c)\,\bar{R}_{ten} = 2n\bar{R}_{ten} \qquad (V.2)$$

where $n = E_s/E_c$.

Immediately before the advent of cracking, when the stress in the concrete is $\sigma_{c,\,ten} = \bar{R}_{ten}$, the external tensile force $N_{cr}$, defined as the sum of the forces in the concrete and the steel, is

$$N_{cr} = \sigma_{c,\,ten}F_c + \sigma_s F_s = \bar{R}_{ten}\,(F_c + 2nF_s) \qquad (V.3)$$

where $F_c$ and $F_s$ are the cross-sectional areas of the concrete and the steel, respectively.

In nonprestressed members, the tensile strength of the concrete is low, so the crack-inducing force is small and of minor importance.

In a tensile reinforced concrete member, cracking begins just as the stress in the concrete reaches its ultimate tensile strength. At a cracked section, the entire tensile force applied to the member is taken up by the steel alone, so that the stress in the steel increases (see $\sigma_{s,\,cr}$ in Fig. V.3*a*). Here, the stress in the concrete, $\sigma_{c,\,ten}$, is zero, but on moving away from the crack it rises to $\bar{R}_{ten}$. In regions where the stress reaches the ultimate tensile strength, adjacent cracks may develop (see Fig. V.3*a*). Consequently, the crack spacing, $l_{cr}$, depends on how soon the stress reaches the ultimate tensile strength. Within this length, tangential stresses $\tau_b$ develop due to the bond between the steel and the concrete. The respective stress curves and the force distribution over the distance $l_{cr}$ and other parts of the member are shown in Fig. V.3.

In the concrete portion of $l_{cr}$, the resultant of the tangential stresses $\tau_b$ is balanced out by the resultant of the normal stresses $\bar{R}_{ten}$. Therefore, we may write

$$\bar{R}_{ten}F_c = \tau_b S \omega l_{cr} \qquad (V.4)$$

where $\tau_b$ is the maximum tangential stress within the length $l_{cr}$, $S$ is the perimeter of the steel cross-section, and $\omega$ is the factor taking care of the shape of the tangential stress curve.

On designating

$$\begin{gathered} \bar{R}_{ten}/\omega\tau_b = \eta \\ F_c/S = F_s/\mu_1 S = u/\mu_1 \\ \mu_1 = F_s/F_c \\ u = F_s/S \end{gathered} \qquad (V.5)$$

we use Eq. (V.4) to obtain the crack spacing

$$l_{cr} = \eta F_c/S = \eta u/\mu_1 \qquad (V.6)$$

**Stage II.** After the onset of cracking, when the external force $N$ exceeds $N_{cr}$, the reinforced concrete member is divided by the cracks into blocks whose length is equal to $l_{cr}$, these blocks being held together by the reinforcing steel (see Fig. V.3*b*). In the cracked sections, the entire force $N$ is taken up by the steel alone. The respective stress is

$$\sigma_s = N/F_s \qquad (V.7)$$

Owing to the bond between the steel and the concrete, the steel stretched by the applied force makes the concrete work in tension within each block. Between the cracks, the stress in the steel $\sigma_{s1}$ is lower than it is in a cracked section, $\sigma_s$. In the concrete, the stress rises from zero at cracks to a maximum stress $\sigma_{c,ten}$ which, obviously, cannot exceed $\bar{R}_{ten}$ (see Fig. V.3*b*).

The average stress in the steel, $\sigma_{s,av}$, determined with allowance for the contribution from the con-

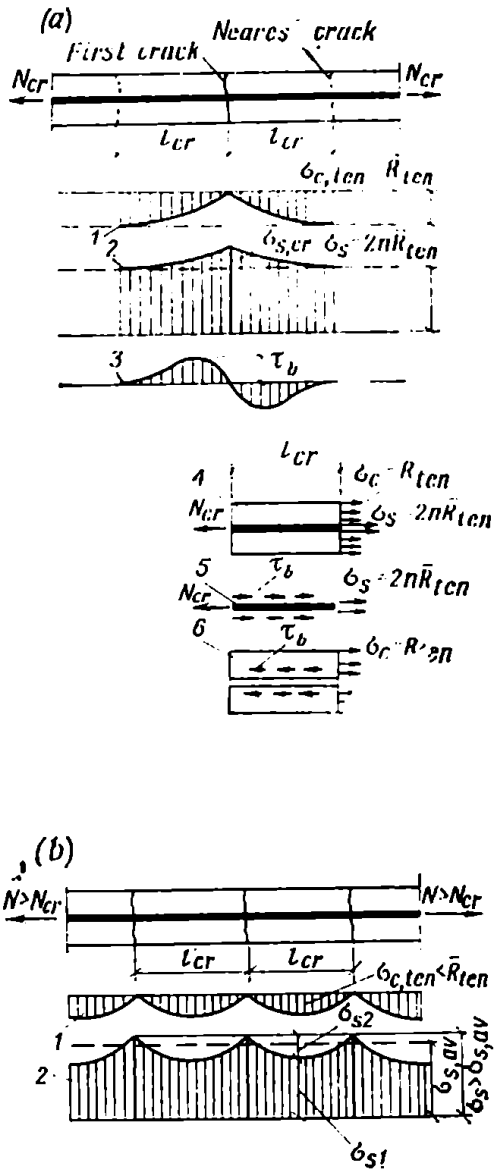



Fig. V.3. Stress-strain state of a member in axial tension

(*a*) during cracking; (*b*) after onset of cracks; *1*—normal stress diagram for concrete; *2*—stress diagram for steel; *3*—tangential stress diagram for concrete; *4*—loading system within $l_{cr}$; *5*—loading system for steel within $l_{cr}$; *6*—loading system for concrete within $l_{cr}$

crete between the cracks, is smaller than $\sigma_s$, and the average elongation $\varepsilon_{s,\,av}$ is smaller than the elongation $\varepsilon_s$ found without taking the concrete into consideration. In design, the contribution by the concrete between the cracks is expressed in terms of the following coefficient

$$\psi_s = \sigma_{s,\,av}/\sigma_s = \varepsilon_{s,\,av}/\varepsilon_s \qquad (V.8)$$

Graphically, the coefficient $\psi_s$ may be defined as the ratio between the area under the steel stress curve over the length $l_{cr}$ and the total area under the steel stress curve with the ordinate $\sigma_s$ (see Fig. V.3*b*):

$$\begin{aligned}\psi_s &= (\sigma_s l_{cr} - \omega_{ten}\sigma_{s2}l_{cr})/\sigma_s l_{cr} \\ &= 1 - \omega_{ten}\sigma_{s2}/\sigma_s \qquad (V.9)\end{aligned}$$

where $\omega_{ten}$ is the factor accounting for the shape of the steel stress curve over the length $l_{cr}$.

The crack width $a_{cr}$ may approximately be defined as the elongation in the steel over the spacing $l_{cr}$ between cracks (the elongation in the concrete may be neglected). Recalling Eq. (V.8), we may write

$$a_{cr} = \varepsilon_{s,\,av}l_{cr} = \psi_s\varepsilon_s l_{cr} = \psi_s\sigma_s l_{cr}/E_s \qquad (V.10)$$

The relations derived for Stage II hold until the stress in the steel exceeds the limit of proportionality.

**Stage III.** The member begins to break down when the stress in the steel exceeds the limit of proportionality and the reinforcement experiences a nonelastic deformation. It is assumed that the member fails completely when the stress in the steel reaches the physical yield point $\sigma_y$ (if the steel has one) or the proof yield strength $\sigma_{0.2}$ (if the steel has no definite yield point). The force at failure depends on the ultimate strength of the steel alone (because now the concrete does not work in tension).

Because the design method is based on the first group of limit states, the strength of a reinforced concrete member is checked for compliance with the condition

$$N \leqslant R_s F_s \qquad (V.11)$$

where $N$ is the design force found from design loads, and $R_s$ is the design tensile strength of the steel.

As previously mentioned, the cracking resistance of nonprestressed members is of minor significance. As a first approximation, the incipient-cracking force may be defined as

$$N_{c,\,cr} = 0.8R_{ten}^{ch}F_c \qquad (V.12)$$

where the factor 0.8 accounts for the reduction in the characteristic strength of the concrete due to shrinkage.

The crack width is determined for Stage II in the state of stress of a member. As already stated, Eq. (V.10) gives an approximate value of $a_{cr}$. According to relevant standards, the crack width should be calculated by the following empirical formula

$$a_{cr}\,(\text{mm}) = kc_d\eta\sigma_s 20\,(3.5 - 100\mu)\sqrt[3]{d\,(\text{mm})}/E_s \qquad (V.13)$$

where $k = 1.2$, $c_d$ is the factor accounting for the duration of loading and assumed to be unity for short-time dead and live loads and 1.5 for multiply repeated loading and long-time dead and live loads (for heavy concrete cured in the air entire time), $\eta$ is taken as unity for deformed bars, 1.2 for deformed wire and wire strands, and 1.4 for plain wire, $\sigma_s$ is the stress in the steel found from Eq. (V.7), $\mu$ is the reinforcement ratio calculated for the cross section assumed to be rectangular (leaving out the compressed overhangs) of depth $h_0$ ($\mu \leqslant 0.02$), and $d$ is the steel diameter.

## V.3. State of Stress and Design of Prestressed Members

For our discussion, let $F_c$ and $F_{pr}$ denote respectively the cross-sectional areas of the concrete and the prestressed steel of an axially prestressed member which is to work in axial tension, and $\sigma_0$ denote the stress in the steel before transfer of the prestress or when the prestress in the concrete falls to zero due to the action of external forces (real in pretensioning and imaginary in posttensioning).

**State of Stress in Pretensioning.** As noted in the previous chapters, the reinforcing tendons are laid into forms and tensioned to $\sigma_0$. The steel remains fastened to the abutments until the member is concreted and the concrete attains the transfer strength $R_0$. During this period, the steel suffers

the early losses, $\sigma_{loss\ 1}$, which occur prior to the transfer of prestress. As a result, the stress in the steel becomes equal to $\sigma_0 - \sigma_{loss\ 1}$.

The release of the tendons from the abutments is equivalent to application of an external compressive force

$$(\sigma_0 - \sigma_{loss\ 1})\, F_{pr} \tag{V.14}$$

When this force is applied to the concrete, the concrete and the steel contract by an equal amount, $\varepsilon_c = \varepsilon_s$. Accordingly, we may write

$$\Delta\sigma_s = (E_s/E_c)\, \sigma_c = n\sigma_c \tag{V.15}$$

where $\sigma_c$ is the stress in the concrete and $\Delta\sigma_s$ is the reduction in the steel stress due to the transfer of prestress when the tendons are released from the abutments.

After the tendons have been released from the abutments, the stress in the steel becomes

$$\sigma_0 - \sigma_{loss\ 1} - n\sigma_c \tag{V.16}$$

Before the application of an external force, the tensile force in the steel is in equilibrium with the compressive force in the concrete

$$F_{pr}\,(\sigma_0 - \sigma_{loss\ 1} - n\sigma_c) = F_c\sigma_c \tag{V.17}$$

Hence,

$$\sigma_c = (\sigma_0 - \sigma_{loss\ 1})\, F_{pr}/F_{tr} \tag{V.18}$$

where

$$F_{tr} = F_c + nF_{pr} \tag{V.19}$$

After the prestress has been transferred to the concrete, the late losses $\sigma_{loss\ 2}$ occur. Now the member is subjected to so-called steady-state stresses in the reinforcement

$$\sigma_0 - (\sigma_{loss\ 1} + \sigma_{loss\ 2}) - n\sigma_{c1} \tag{V.20}$$

and in the concrete

$$\sigma_c = [\sigma_0 - (\sigma_{loss\ 1} + \sigma_{loss\ 2})]\, F_{pr}/F_{tr} \tag{V.21}$$

When an external axial tensile force $N$ is just applied to the member, the prestress in the concrete gradually decreases until it is cancelled out completely. Now the stress

in the concrete is $\sigma_c = 0$, and that in the steel is

$$\sigma_0 - (\sigma_{loss\ 1} + \sigma_{loss\ 2}) \qquad \text{(V.22)}$$

As the external tensile force builds up, the state of stress in the concrete passes through three stages (see Sec. V.3), namely Stage I (prior to cracking), Stage II (after cracking), and Stage III (breakdown).

At the end of Stage I, the stress in the concrete reaches the ultimate tensile strength. As the stress in the concrete rises from zero to the ultimate tensile strength, the stress in the steel increases by the amount defined by Eq. (V.2), equal, on the average, to 300 kgf $cm^{-2}$ (30 MPa). Consequently, recalling Eq. (V.22), the stress in the steel immediately before cracking is

$$[\sigma_0 - (\sigma_{loss\ 1} + \sigma_{loss\ 2})] + 2n\bar{R}_{ten} \qquad \text{(V.23)}$$

From this state of stress, the cracking force is calculated.

During Stage II (after the onset of cracking), the external force in sections containing cracks is taken up by the steel alone. Between the cracks, the tensile force is resisted by both the steel and the concrete.

In a cracked section, the stress in the steel is given by

$$\sigma_s = (N - N_0)/F_s \qquad \text{(V.24)}$$

where the prestressing force is

$$N_0 = F_{pr}[\sigma_0 - (\sigma_{loss\ 1} + \sigma_{loss\ 2})] \qquad \text{(V.25)}$$

Finally, Stage III (breakdown) occurs. From the breakdown stage, the load-carrying capacity of the member is calculated.

More often than not, the prestress has an insignificant effect on the load-bearing capacity of a member.

**State of Stress in Posttensioning.** In posttensioning, the load-bearing longitudinal prestressed reinforcement is laid in inner ducts or surface slots of the member and tensioned against its ends. Before the transfer of the prestress to the concrete, the steel stress suffers the early losses, $\sigma_{loss\ 1}$. After the prestress has been transferred to the concrete, the stress in the concrete reaches the value $\sigma_c$. The stress in the tendons is found by Eq. (V.18).

With time, the late losses, $\sigma_{loss\ 2}$, occur in the steel. As this takes place, the stress in the concrete decreases to $\sigma_{c1}$, and that in the steel is given by Eq. (V.20).

Now the member receives what is known as the steady-state prestress.

When an external load is applied to the member and gradually increased, the prestress in the concrete is cancelled out and the concrete passes through the three stages of the state of stress.

**Limit-State Design of Members.** Design in terms of the first group of limit states is carried out, subject to

$$N \leqslant R_{s,\,pr}F_{pr} + R_sF_s \qquad \text{(V.26)}$$

where $R_{s,\,pr}$ and $F_{pr}$ are the design strength and the cross-sectional area of the prestressed steel, respectively, whereas $R_s$ and $F_s$ are the design strength and the cross-sectional area of the non-prestressed steel, respectively.

As is indicated in Appendices VI and VII, the values of $R_s$ ($R_{s,\,pr}$) for high-strength reinforcing steel are multiplied by the service factor $m_{s4}$ for class A-IV, Ат-IV, A-V, Ат-V, B-II, Bp-II, K-7 and Ат-VI steel.

The tendency of prestressed members to crack when in axial tension is evaluated from the condition

$$N \leqslant N_{cr}$$

where

$$N_{cr} = R_{ten\ II}\,(F_c + 2E_sF_s/E_c) + N_0 \qquad \text{(V.27)}$$

Here, $N$ is the design force determined assuming characteristic loads and $n > 1$ or $n = 1$, depending on the category of cracking resistance required (see Sec. III.1), $N_{cr}$ is the normal crack-inducing force, and $N_0$ is the prestressing force determined with allowance for the losses and the tensioning accuracy factor applicable to a given stage in the state of stress.

The tendency towards crack opening is evaluated, using Eq. (V.13) where

$$\sigma_s = (N - N_0)/F_s \qquad \text{(V.28)}$$

The requirements for crack closure (see Sec. III.1) are taken to be satisfied, if the following two conditions are met:

(a) no irreversible strain occurs in the prestressed reinforcement due to dead, long-time and short-time live loads, that is

$$\sigma_0 + \sigma_s \leqslant kR_{sII} \qquad \text{(V.29)}$$

where $\sigma_s$ is the increment in the stress in the prestressed reinforcement due to external loads, $k = 0.65$ for reinforcing wire, and $k = 0.8$ for bars;

(b) the action of dead and long-time live loads leaves the prestress in the concrete equal to at least 10 kgf cm$^{-2}$ (1 MPa).

CHAPTER SIX

# REINFORCED CONCRETE MEMBERS IN BENDING

## VI.1. Constructional Features

The most common reinforced concrete members in bending are *slabs* and *beams*. Slabs are defined as flat members whose thickness $h_s$ is much smaller than their length $l_s$ and width $b_s$. Beams are defined as structural members whose length $l$ considerably exceeds their depth $h$ and width $b$. These members are used in many reinforced concrete structures, most often in precast, in-situ and precast/in-situ floors and roofs (Fig. VI.1).

Slabs and beams can be single- and multi-span.

**Slabs.** The thickness of slabs cast in-situ ranges between 60 and 100 mm, whereas precast slabs should be as thin as permitted by relevant standards.

Figure VI.2*a* shows a one-way, single-span slab; Fig. VI.2*b* shows an in-situ one-way, multi-span slab carried by a number of parallel supports. Such slabs deform as beams under various loads, provided the loads do not change in a direction normal to the span.

Slab reinforcement consists of welded-wire fabric (see Sec. II.3). The fabric is installed with its load-bearing wires running along the span to resist tensile forces induced by

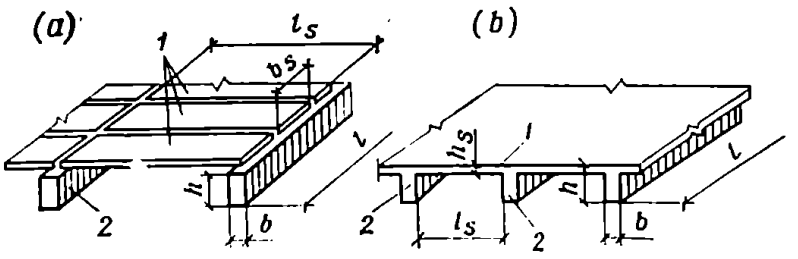



Fig. VI.1. Reinforced concrete floors
(*a*) precast; (*b*) in-situ; *1*—slabs; *2*—beams

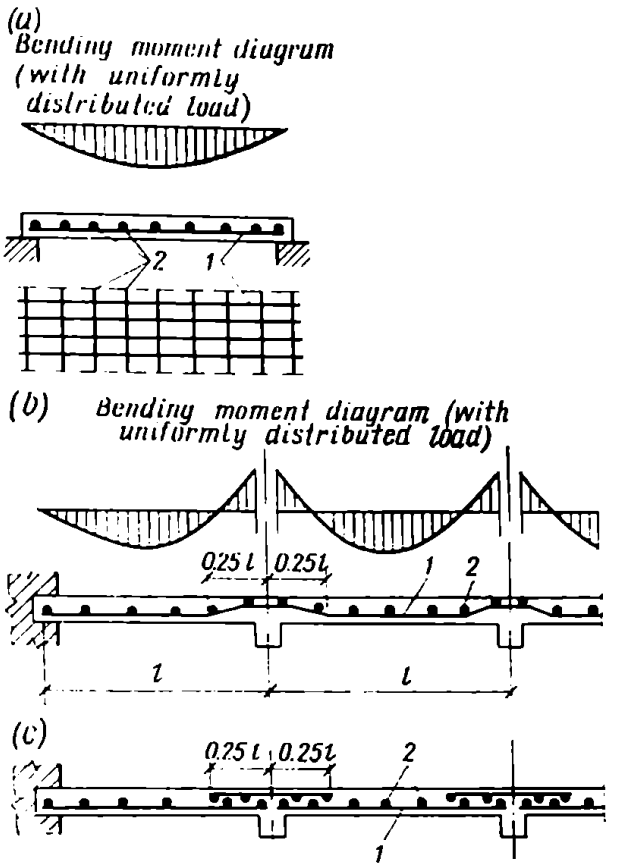



Fig. VI.2. Slab reinforcement

(*a*) single-span slab; (*b*) multi-span slab with continuous reinforcement; (*c*) multi-span slab with separate reinforcement; *1*—load-bearing bars; *2*—erection bars

bending. On the whole, its arrangement follows the bending moment curve (see Fig. VI.2). Thus, within the span, the fabric is placed in the lower part of the slab; in multi-span slabs, the fabric is additionally placed in the upper part, above the intermediate supports. The fabric reinforcement used in multi-span slabs may be continuous (see Fig. VI.2*b*) or sectionalized, that is, consisting of separate parts (Fig. VI.2*c*).

Load-bearing wires are from 3 to 10 mm in diameter, with a spacing of 100 to 200 mm.

The minimum concrete cover for load-bearing reinforcement is 10 mm; for super-thick slabs (in excess of 100 mm), it is 15 mm.

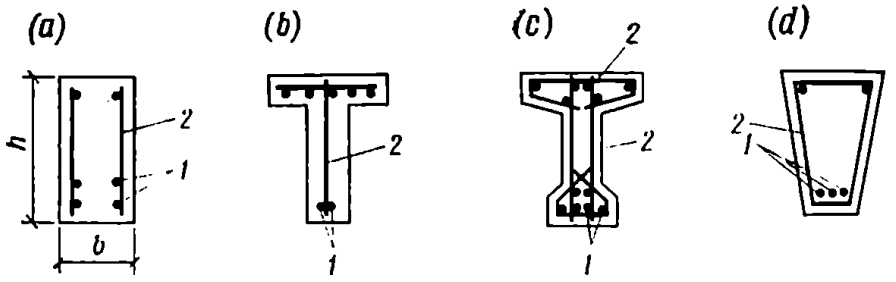



Fig. VI.3. Beam cross sections and types of reinforcement
(*a*) rectangular beam; (*b*) T-beam; (*c*) I-beam; (*d*) trapezoidal beam; *1*—longitudinal bars; *2*—transverse reinforcement

Transverse wires (distribution reinforcement) are installed to hold load-bearing wires in their design position, to reduce shrinkage and thermal strain in structures, and to distribute the stress due to concentrated loads over a larger area. They may be smaller in diameter than load-bearing wires, but the total transverse area should be not less than 10% of the load-bearing steel area at places where the bending moment is a maximum. Transverse wires are spaced 250 to 300 mm apart, but not wider than 350 mm.

At the outer free (infixed) supports, the load-bearing wires are anchored as shown in Fig. III.2*a*.

In some cases (slabs of a complex configuration in plan or having a large number of voids), where standard welded-wire fabric cannot be used, slabs are reinforced with separate wires bound together by manually applied tie wire.

**Reinforced Concrete Beams.** The most common types are rectangular beams, T-beams, I-beams, and trapezoidal beams (Fig. VI.3).

According to the load and type of structure, the beam depth $h$ may vary from 1/10 to 1/20 of the span. For unification, the beam depth is taken as a multiple of 50 mm if it does not exceed 600 mm or a multiple of 100 mm if it is greater.

The width of rectangular beams, $b$, ranges from $0.25h$ to $0.5h$. The actual sizes are 100, 120, 150, 200, 220, and 250 mm. Sizes in excess of 250 mm are made as multiples of 50 mm. To reduce the concrete consumption, the beam width should be made as small as possible. Over the cross section, the load-bearing reinforcement is installed in one

or two rows with a spacing sufficient for concreting without leaving voids or cavities. The necessary spacings and covers are given in Fig. VI.4.

As in slabs the arrangement of longitudinal load-bearing reinforcement in beams follows the shape of the bending moment curves in the tension zones where it is to resist the longitudinal tensile forces induced by bending loads. As a rule, it consists of deformed bars 12 to 32 mm in diameter; plain bars are used more rarely.

In addition to bending moments, reinforced concrete beams are subjected to shearing forces. To carry these forces, beams require transverse (or shear) reinforcement. The area of transverse reinforcement is found by calculation and from structural considerations.

Longitudinal and transverse reinforcement is welded into bar mats (see Sec. II.3); if no welding machines are available, they can be joined together with tie wire. Tied reinforcement is more labour-consuming, and its use is warranted only when it is impossible to fabricate welded-bar mats.

Welded-bar mats are combined into reinforcing cages in

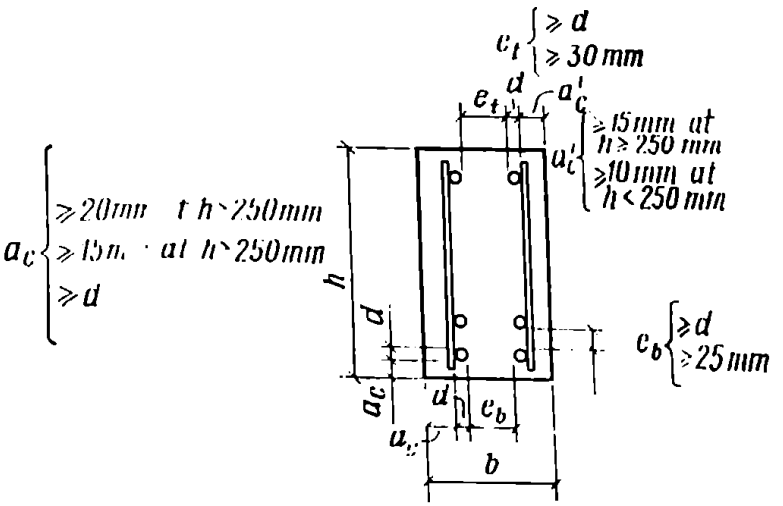



Fig. VI.4. Distribution of reinforcement over a beam cross section

$a_c$—concrete cover for load-bearing reinforcement; $a'_c$—concrete cover for erection reinforcement; $d$—largest diameter of load-bearing bars; $e_t$—clear space between bottom bars (when placing concrete); $e_t$—clear space between top bars

which the horizontal transverse bars are spaced 1 to 1.5 m apart.

The reinforcement of a single-span beam using welded-bar mats is shown in Fig. VI.5*a*. When use is made of tied-bar mats (Fig. VI.5*b*), stirrups in rectangular beams are made of closed design, whereas in T-beams where the rib is made fast to the solid flange on either side, the stirrups may be open at the top (they are then known as U-stirrups).

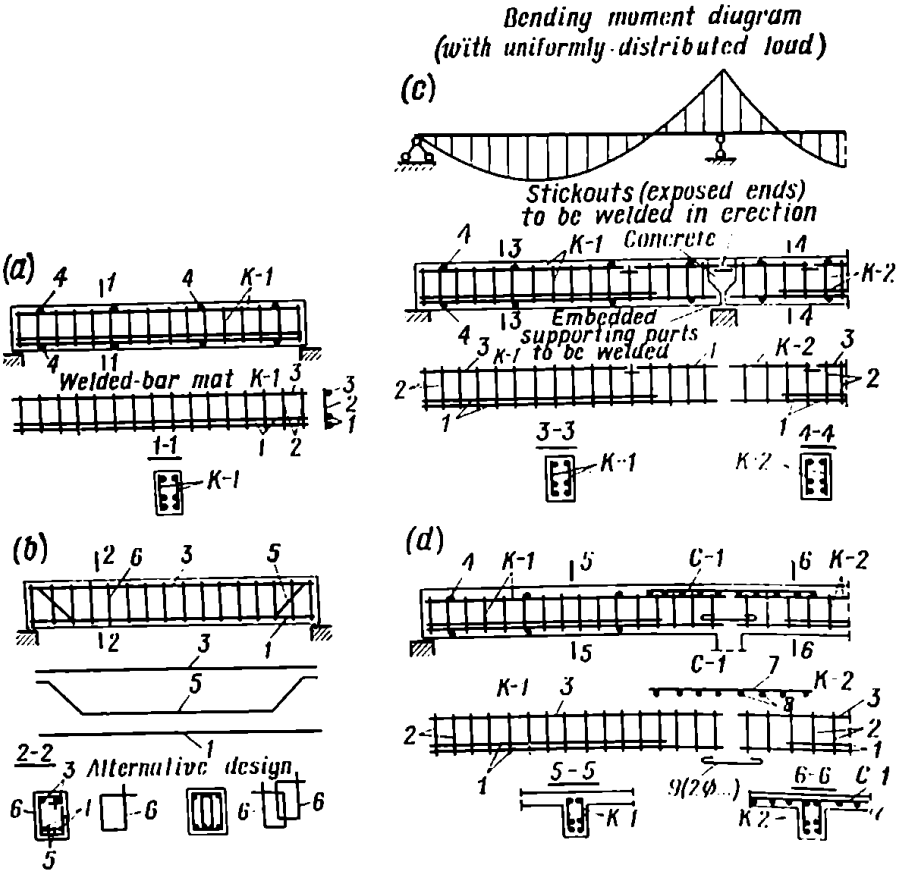



Fig. VI.5. Types of beam reinforcement

(*a*) single-span beam with welded-bar mats; (*b*) single-span beam with tied reinforcement; (*c*) precast multi-span rectangular beam; (*d*) in-situ multi-span T-beam; *1*—longitudinal load-bearing bars; *2*—transverse bars in mats; *3*—longitudinal erection bars; *4*—transverse erection bars; *5*—bent-up load-bearing bars; *6*—stirrups in tied mats; *7*—load-bearing bars in support fabric; *8*—spacer bars in support fabric; *9*—erection bars (two bars with diameter of not less than 10 mm, nor less than one half of bottom erection bar diameter)

Beams wider than 35 cm use multiple stirrups. In tied-bar mats, the stirrups should be at least 6 mm in diameter for beams up to 800 mm deep, and not less than 8 mm in diameter for deeper beams.

To meet design and erection requirements, the spacing between the transverse bars (or stirrups) in the longitudinal direction should be not more than $h/2$, or 150 mm for beams up to 450 mm in depth, and not more than $h/3$, or 500 mm for beams deeper than 450 mm, whichever is the smaller. In beams subjected to a uniformly distributed load, this requirement holds for 1/4 of the span starting from the support; in beams subjected to concentrated loads, this portion extends father out to the nearest load. In the remaining part of the span, the spacing between transverse bars (or stirrups) may be greater, but not more than $3/4h$ or 500 mm, whichever is the smaller.

Transverse bars (stirrups) in beams more than 150 mm in depth are installed even if they are not required by analysis.

In beams more than 700 mm deep, the side faces are additionally reinforced with longitudinal bars spaced not more than 400 mm apart in the vertical direction. These bars together with transverse reinforcement do not permit inclined cracks to open on the side faces.

The top beam faces are reinforced with longitudinal distribution bars 10 to 12 mm in diameter, installed during erection; they are necessary to combine all reinforcing elements into a cage that would be stable during concreting. In precast beams, the distribution bars may serve as load-bearing (or main) bars in transit and erection.

Instead of, or in addition to, transverse bars, use may be made of inclined bars. They are more effective than the former because they are better aligned with the direction of the principal tensile stresses in a beam. Transverse bars, however, are preferable because they are more convenient to install.

Inclined bars are usually installed at 45° to longitudinal bars. In beams deeper than 800 mm, they may be installed at up to 60°, whereas in beams with a smaller depth and beams subjected to concentrated loads, inclined bars are positioned at 30°.

When beams are reinforced with tied cages (made of class A-I and A-II steel), it is good practice to bend up some of the longitudinal load-bearing bars so as to reduce the steel consumption and to improve the construction of the cage (Fig. VI.5*b*). The straight portions of bends in plain bars should have hooks at their ends.

Precast multi-span beams are fabricated from individual single-span members containing welded reinforcing cages (Fig. VI.5*c*). The position and length of load-bearing reinforcement in a cage are chosen from bending moment diagrams plotted as for a multi-span beam. At the joints above the intermediate supports, the exposed ends (stick-outs) of the top load-bearing bars are field-welded together, using temporary back-up plates to contain a pool of molten metal; the bottom bars are welded to permanent back-up plates with the help of attachment fittings embedded into the precast members. After welding, the joints are concreted.

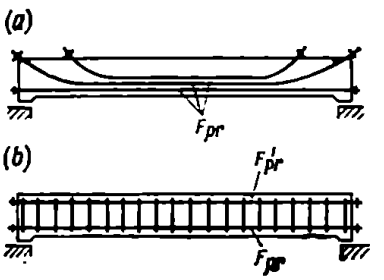



Fig. VI.6. Types of prestressed beam reinforcement
(*a*) curved prestressed tendons; (*b*) straight prestressed tendons

In-situ cast multi-span T-beams reinforced in the spans with welded bar mats (Fig. VI.5*d*) have welded fabric installed above the intermediate supports. The load-bearing bars of the fabric run along the span; they are intended to carry the forces existing in the tension zone above the supports.

In prestressed members subjected to bending, the arrangement of reinforcement follows the shape of curves plotted for bending moments and shearing forces caused by the load. Curved prestressed tendons (Fig. VI.6*a*) are most efficient because they follow the path of the principal tensile stresses best of all, but such reinforcement is more complex than the one using straight tendons (Fig. VI.6*b*). In addition to the steel of area $F_{pr}$ in the tension zone, beams reinforced with straight prestressed tendons also use the steel of area $F'_{pr}$ at the opposite face, ranging between $0.15F_{pr}$ and $0.25F_{pr}$. This reinforcement is useful in deep beams

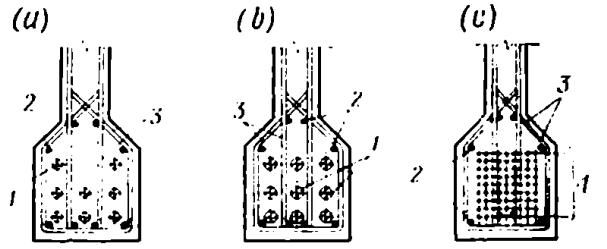



Fig. VI.7. Distribution of reinforcement in the tension zone of prestressed members

(a) deformed bars; (b) strands or cables in ducts; (c) high-strength wire; 1—prestressed steel; 2—longitudinal nonprestressed steel; 3—transverse reinforcement

where the prestressing force applied outside the kern causes at the top face a tension which may result in cracking during manufacture. The steel, $F'_{pr}$, is seldom used in slabs.

The best choice for beams in bending is I-beams (see Fig. VI.3*c*) and T-beams with sufficiently wide ribs (see Fig. VI.3*b*). The flanges in compression are proportioned to suit the compressive resultant of the internal couple induced by the bending moment under load. The tension zone should be sufficiently wide to accommodate the necessary reinforcement and resist the prestress in prestressed members.

Prestressed steel is installed in tension zones as is shown in Fig. VI.7. Here, the concrete cover and the spacing between the pretensioned bars and strands are chosen in accord with Fig. VI.4. In posttensioning, the concrete cover should be not less than 40 mm nor less than the duct width or half the duct height.

For prestressed members, special emphasis should be placed on the design of the reinforcement of beam ends near supports. If the prestressed reinforcement has no bent-up portions near supports it is necessary either to prestress the transverse reinforcement or to enlarge the sectional area within the transmission length at the ends of a prestressed member. In the latter case, additional nonprestressed transverse steel should be installed. Its cross-sectional area should be large enough to resist the acting force equal to at least 20% of the

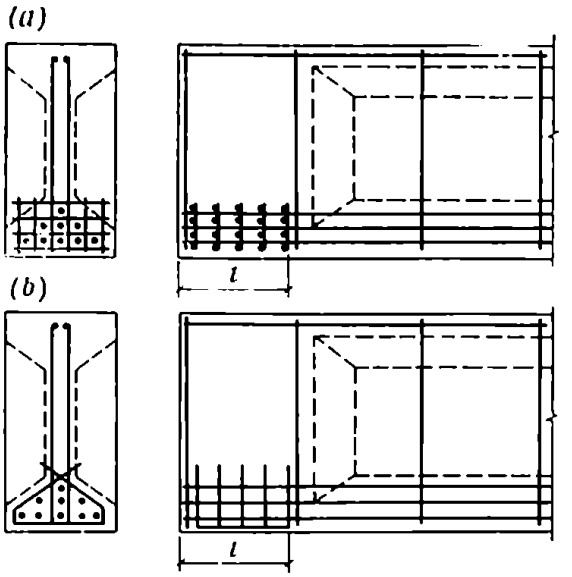



Fig. VI.8. Local reinforcement of prestressed beam ends
(a) transverse welded fabric; (b) stirrups or embracing welded fabric

ment (in the bottom zone at the support), as found by strength analysis (that is, $R_s F_{sl} \geqslant 0.2 R_s F_{p,r}$). The ends of non-prestressed transverse bars should be welded to embedded fittings.

The concrete at the ends of prestressed members containing steel with and without anchors, that is, where it has to resist considerable concentrated forces which may cause local overstresses and destroy the concrete, should additionally be reinforced with fabric or stirrups spaced 5 to 10 cm apart (Fig. VI.8). The length of the zone in question, $l$, is taken equal to twice the anchorage length, or, in the absence of anchors, it should be not less than $0.6 l_{tr}$, nor less than 20 cm.

## VI.2. State of Stress in Bending Members

Let us examine a single-span simply supported reinforced concrete beam loaded by two symmetric concentrated forces. The beam area between the loads is under pure bending, that is, it is subjected only to the bending moment $M$, the

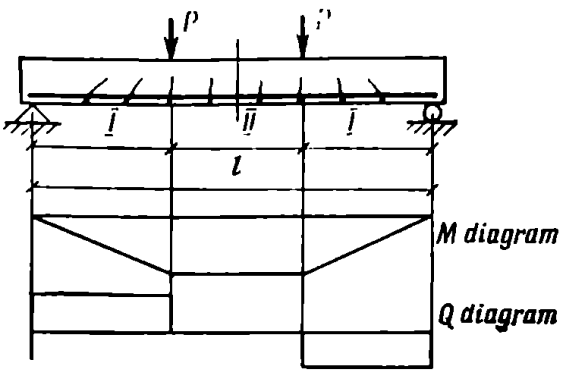



Fig. VI.9. Reinforced concrete beam in bending
*I*—portion subjected to both *M* and *Q*; *II*—portion subjected to *M* only

shearing force being zero (Fig. VI.9). At a certain amount of loading, normal cracks (those at right angles to the longitudinal axis of the beam) begin to form in the tension zone of this area. The portions between the support and load are subjected to both the bending moment $M$ and the shearing force $Q$. It is here that inclined cracks occur.

According to the amount of reinforcement in the tension zone of the member, one of two modes of failure may occur.

**Mode 1.** The member fails when the stress in the tensile reinforcement reaches the physical or proof yield stress, and the stress in the compressed concrete reaches the ultimate compressive strength (nonbrittle failure).

**Mode 2.** The member fails when the stress in the concrete reaches the ultimate compressive strength, and the stress in the reinforcement is below the yield point (brittle failure, overreinforced section).

According to the Loleyt principle*, relevant Soviet standards do not generally recommend to use overreinforced members, because they fail suddenly and not all of the tensile steel strength is used to advantage.

Numerous experiments have shown that under a gradually increasing load, the state of stress at normal sections within the pure-bending portions of nonprestressed beams reinfor-

* The Russian scientist A. F. Loleyt (1868-1933) was one of the founders of the present-day theory of reinforced concrete.

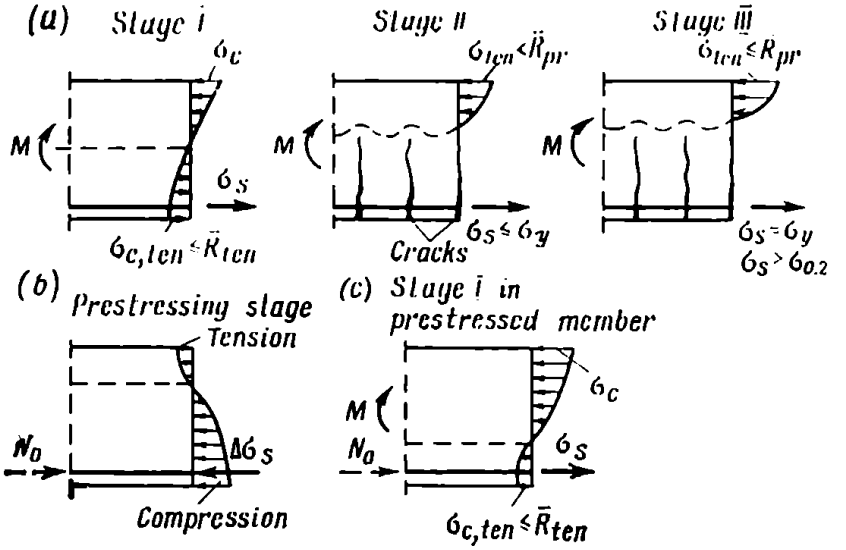



Fig. VI.10. Stages of the stress state at normal sections in the zone of pure bending

(*a*) nonprestressed member; (*b*) prestressing stage; (*c*) Stage *I* in a prestressed member

ced in compliance with the Loleyt principle passes through three successive stages (Fig. VI.10*a*).

**Stage I.** This refers to the initial stage of loading, before cracks occur in the tension zone of the concrete. During this stage, the steel and the concrete elongate together owing to the bond between them. By the end of Stage I, the stress in the tensile concrete, $\sigma_{c,\,ten}$, follows (in graphic interpretation) a curvilinear path owing to a nonlinear relationship between stress and strain. It attains a maximum value equal to the ultimate tensile strength of concrete ($\bar{R}_{ten}$ in Fig. VI.10*a*). In the compression zone, the $\sigma_c$ curve is nearly triangular in shape. At the end of Stage I, the member is on the verge of cracking. The cracking resistance of nonprestressed members is low, so the final phase of Stage I occurs in them at small loads.

**Stage II.** This applies to the state of stress after the concrete in the tension zone has developed cracks. In sections containing a crack, all of the tensile force is taken up by the reinforcing steel. Between the cracks, the concrete continues to work in tension owing to its bond with the steel, so the stress in the reinforcement is somewhat lower (see Fig. V.3).

As a result, the neutral axis assumes a varying position which shifts along the beam (see Fig. II.4). During Stage II, the stress $\sigma_c$ in the compression zone of the concrete is very high, and its plot becomes curvilinear owing to the inelastic properties of the concrete. At this stage, the stress in the compression zone is still below the ultimate compressive strength of the concrete, $\overline{R}_{pr}$ (see Fig. VI.10*a*).

Between cracks, each block of concrete is locally subjected to compressive forces induced above the cracks. Because of this, the stress in the concrete is distributed nonuniformly along the block length, being a little smaller in the middle portion.

At the end of Stage II, the reinforcing steel undergoes a marked plastic deformation which corresponds to the yield point.

**Stage III.** This stage takes place when the load imposed on the member exceeds the load-bearing capacity. As the load increases, the stress in the reinforcing steel $\sigma_s$ reaches either the physical yield stress $\sigma_y$ or the proof yield stress $\sigma_{0.2}$ (see Fig. VI.10*a*). With a further rise in the load, the stress in a steel having a definite yield point does not increase because at the yield point it undergoes a considerable deformation at a constant stress, whereas in a steel having no definite yield point it rises slowly. In the compression zone, the ordinates of the stress curve increase, the depth of the compression zone decreases and, as a consequence, the arm of the internal couple increases. The member fails because the concrete in the compression zone is crushed when the stress in the concrete, $\sigma_c$, reaches the ultimate compressive strength $\overline{R}_{pr}$ (failure mode 1). Overreinforced members tend to collapse due to the crushing of the concrete in the compression zone even when the stress in the tensile reinforcement is below the yield point (failure mode 2).

In prestressed members, the concrete in the zone which is expected to be in tension under service load is subjected to a prestressing force $N_0$ applied with an eccentricity so that a considerable amount of prestress is developed. In the circumstances, the zone likely to work in compression under service load usually contains an insignificant region of tensile stresses (Fig. VI.10*b*).

During prestress, the envelope of stress in the concrete is curved because the rate of stress is rather high.

At the initial stage of loading, the prestress is first gradually cancelled out (see Fig. VI.10*b*), then the member moves into the first stage discussed above (Fig. VI.10*c*).

Owing to a relatively high compressive prestress in the concrete, the member develops a considerable cracking resistance. Therefore, Stage I during which no cracks develop in the concrete lasts for a much longer time. At the end of Stage I, the stress curve for the concrete in prestressed members (see Fig. VI.10*c*) is similar in shape to that for nonprestressed members (see Fig. VI.10*a*), but the tension zone is less pronounced.

As compared with the prestress plot (see Fig. VI.10*b*), the stress diagram of a loaded member during Stage I (see Fig. VI.10*c*) is opposite in both the sign and size of the compression and tension zones.

Prestressed members deform almost elastically till the very end of Stage I (the onset of cracking).

During Stages II and III, prestressed members behave in the same manner as their nonprestressed counterparts. It should be noted that, according to experimental data, the prestress has generally little effect on the normal-section strength of a member in the region of pure bending.

The stages in the state of stress discussed above are useful in the analysis and design of reinforced concrete elements. Before cracking, members are considered to be working elastically. The end of Stage I serves as a basis for the incipient-cracking analysis of prestressed members. According to the category of cracking resistance requirements, Stages I and II are useful in the analysis of sags and crack closure for medium amounts of loading. The end of Stage III is useful in the analysis of members for load-bearing capacity and normal-section strength.

In the regions of a beam where a bending moment is accompanied by a shearing force (see Fig. VI.9), failure occurs along inclined sections. Here, two cases can be distinguished.

**Case I.** Under an increasing load in excess of the ultimate strength of the steel, the two portions of the beam rotate relative to each other about an axis located in the compres-

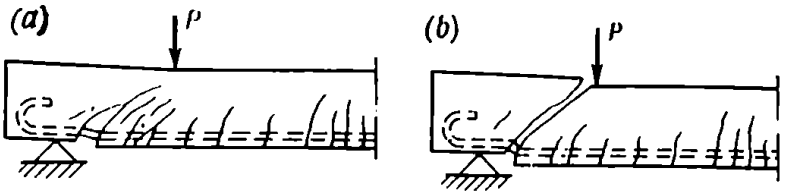



Fig. VI.11. Inclined-section failure of a beam
(*a*) Case *I*; (*b*) Case *II*

sion zone and lying on an extension of the inclined crack. As the crack opens up and grows longer, the compression zone shrinks progressively until it collapses (Fig. VI.11*a*). This mode of failure is similar to that along normal sections.

**Case II.** The compression zone fails owing to the joint action of the compressive stress and the shearing force (Fig. VI.11*b*). This mode of failure is characteristic of members using strong properly anchored longitudinal reinforcement which easily bends but does not come anywhere near the state of yield.

The nature of failure along an inclined section is taken into account in the analysis of members for load-bearing capacity in terms of inclined sections.

The stress-strain behaviour of reinforced concrete members discussed above is borne out by numerous experiments and theoretical research, and forms a basis for the present-day theory and design of reinforced concrete members. In the USSR, major contributions to this theory have been made by A. F. Loleyt and A. A. Gvozdev.

## VI.3. Normal-Section Strength Analysis of Members

**Members of an Arbitrary Symmetric Cross Section.** Let us consider a nonprestressed member subjected to bending. In the adopted loading system, it is assumed that the member is subjected to a bending moment $M$ computed for design loads, and forces in the steel and concrete, determined for stresses equal to their design strengths (Fig. VI.12). For simplicity, the curved stress distribution diagram for the compression zone is replaced by a rectangular diagram, but this affects the value of the moment insignificantly. The

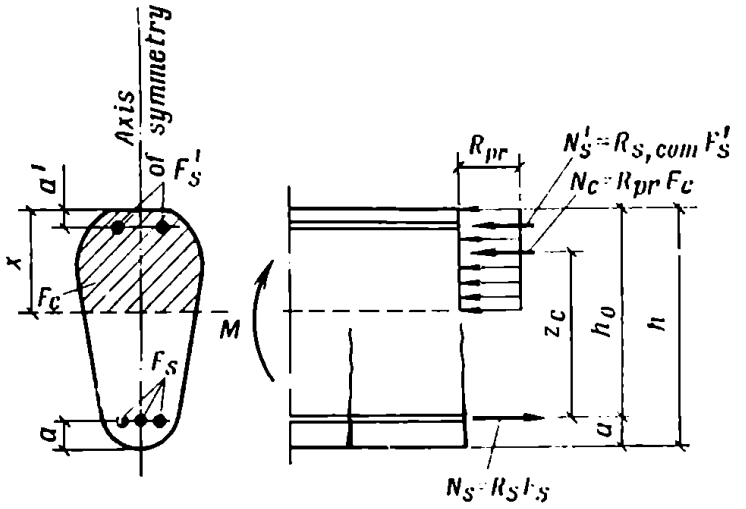



Fig. VI.12. Loading system for the normal-section strength analysis of members in bending

stress in the concrete is taken to be the same as for the entire compression zone and equal to $R_{pr}$, the design prism crushing strength.

The member may have a cross section of any geometry, provided it is symmetric about the plane of flexure. In the general case, the tension zone contains reinforcement of area $F_s$ and of design tensile strength $R_s$. The compression zone may also contain steel of area $F'_s$ and of design compressive strength $R_{s,com}$.

In Fig. VI.12, the letter $a$ designates the distance from the resultant of the forces in the steel of area $F_s$ to the tensile face, $a'$ is the distance from the resultant of the forces in the steel of area $F'_s$ to the compressive face, $h_0$ is the effective depth of the section, $F_c$ is the cross-sectional area of the compression zone in the concrete, and $z_c$ is the distance from the centroid of the compression zone to the resultant of the forces in the tensile steel.

The resultants of the forces acting in the steel and the concrete are

$$\left.\begin{aligned} N_s &= R_s F_s \\ N_c &= R_{pr} F_c \\ N'_s &= R_{s.com} F'_s \end{aligned}\right\} \qquad \text{(VI.1)}$$

From the equilibrium of the internal forces

$$R_s F_s - R_{pr} F_c - R_{s,\,com} F'_s = 0 \qquad \text{(VI.2)}$$

we may find the cross-sectional area $F_c$ of the compression zone in the concrete and its depth $x$ (see Fig. VI.12).

The member is considered to be sufficiently strong if the external design bending moment does not exceed the design bearing capacity of the member, expressed in terms of the opposing moment of the internal forces. In the case of moments about the axis normal to the plane of flexure and passing through the point where the resultant of the forces in the tensile steel of area $F_s$ is applied, the condition for strength is the following inequality

$$M \leqslant R_{pr} F_c z_c + R_{s,\,com} F'_s (h_0 - a') \qquad \text{(VI.3)}$$

For high-strength reinforcing steel, the values of $R_s$ in Eqs. (VI.1) through (VI.3) given in Appendices VI and VII, should be multiplied by the service factor

$$m_{s4} = \bar{m}_{s4} - (\bar{m}_{s4} - 1)\, \xi/\xi_R \qquad \text{(VI.4)}$$

where $\bar{m}_{s4} = 1.2$ for class A-IV and Ат-IV bars, $\bar{m}_{s4} = 1.15$ for class A-V, Ат-V, B-II, Bp-II, and K-7 reinforcement, and $\bar{m}_{s4} = 1.1$ for class Ат-VI steel, $\xi$ is the relative depth of the compression zone defined as $\xi = x/h_0$ (for $x$ calculated without $m_{s4}$ applied).

Members most effective in bending without brittle failure (Sec. VI.2) are those in which $x \leqslant \xi_R h_0$.

The limiting relative depth of the compression zone is given by

$$\xi_R = \xi_0/[1 + \sigma_A\, (1 - \xi_0/1.1)/4\,000] \qquad \text{(VI.5)}$$

where $\xi_0$ is the characteristic of the compression zone in the concrete; for heavy concrete, it is found from

$$\xi_0 = 0.85 - 0.000\,8\, R_{pr} \qquad \text{(VI.6)}$$

$\sigma_A$ is the proof stress in the steel given by

$$\sigma_A = R_s + 0.002\, E_s = R_s + 4\,000 \qquad \text{(VI.7)}$$

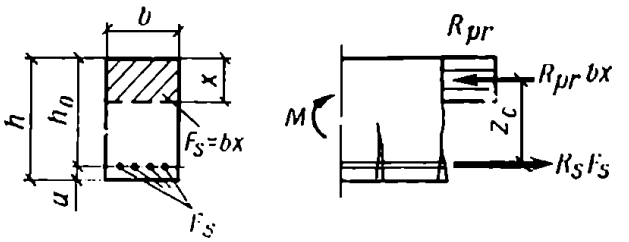



Fig. VI.13. Rectangular beam with tensile steel only and the loading system for the normal-section strength analysis

for steel without a definite yield point (class A-IV and higher), class B-II and Bp-II wire, and strands or by

$$\sigma_A = R_s$$

for steel with a definite yield point (classes A-I, A-II, and A-III), and wire (classes B-I and Bp-I), not generally used for prestressing. Here, $R_s$ is the design tensile strength of the steel, multiplied by all service factors $m_{sn}$, except $m_{s4}$.

For concrete with $m_{c1} = 0.85$ (see Sec. III.1), the factor "4 000" in Eq. (VI.5) must be replaced with "5 000".

If the depth of the compression zone as determined from Eq. (VI.2) exceeds $\xi_R h_0$, the member may be analyzed using Eq. (VI.3) for $x = \xi_R h_0$.

**Rectangular Beams with Tensile Steel Only.** A nonprestressed rectangular beam with tensile steel only has the following geometry (Fig. VI.13):

$$\left.\begin{aligned} F_c &= bx \\ z_c &= h_0 - 0.5x \end{aligned}\right\} \qquad \text{(VI.8)}$$

The depth of the concrete in compression, $x$, is determined on the basis of Eq. (VI.2) from the following expression

$$bxR_{pr} = R_sF_s \qquad \text{(VI.9)}$$

From Eq. (VI.3) it follows that the condition for strength has the form

$$M \leqslant R_{pr}bx\,(h_0 - 0.5x) \qquad \text{(VI.10)}$$

This may also be conveniently expressed in terms of the moments about the centroid of the compression zone

$$M \leqslant R_sF_s\,(h_0 - 0.5x) \qquad \text{(VI.11)}$$

Equations (VI.9) and (VI.10) or (VI.11) are used in combination, if

$$x \leqslant \xi_R h_0$$

where $\xi_R$ is given by Eq. (VI.5).

The reinforcement ratio $\mu = F_s/bh_0$ and the percentage of reinforcement ($\mu \times 100$) may, in view of Eq. (VI.9) and because $\xi = x/h_0$, be expressed as

$$\left.\begin{aligned} \mu &= \xi R_{pr}/R_s \\ \mu\,(\%) &= 100\xi R_{pr}/R_s \end{aligned}\right\} \qquad \text{(VI.12)}$$

Hence, we may find the maximum allowable steel area in a rectangular section from the limiting values of $\xi_R$, subject to the condition in Eq. (VI.5).

If $x > \xi_R/h_0$, the bending moment is found by Eq. (VI.10) or (VI.11) as for $x = \xi_R/h_0$.

From an analysis of Eqs. (VI.10) and (VI.11) it follows that a member may be sufficiently strong with various combinations of cross-sectional dimensions and steel area.

In practice, the cost of reinforced concrete members is close to optimal if

$$\mu = 1 \text{ to } 2\% \text{ and } \xi = 0.3 \text{ to } 0.4 \text{ for beams}$$

$$\mu = 0.3 \text{ to } 0.6\% \text{ and } \xi = 0.1 \text{ to } 0.15 \text{ for slabs}$$

A section with specified $b$, $x$, and $F_s$ (the materials and bending moment $M$ are assumed to be known) is tested for strength in the following sequence. First, we find the depth of the compression zone $x$ from Eq. (VI.9), then we test it to see if it satisfies the condition $x \leqslant \xi_R h_0$, and finally use Eq. (VI.10) or (VI.11).

A section is considered to be chosen correctly, if its load-bearing capacity expressed in terms of the moment is equal to the specified design moment or exceeds it by not more than 3 to 5%.

**Example VI.1.** Given: The design bending moment induced by dead, long- and short-time live loads is $M = 7.6$ tf m (76 kN m); the beam width is $b = 20$ cm, and the depth is $h = 40$ cm; the longitudinal steel consists of four class A-III deformed bars 16 mm in diameter; the concrete is M-200 (the service factor for the concrete is $m_{c1} = 0.85$),

Check the member for strength at a normal section.

*Solution.* From Appendices IV and VI we find

$$R_{pr} = 90 \text{ kgf cm}^{-2} \text{ (9 MPa)}$$
$$R_s = 3\,600 \text{ kgf cm}^{-2} \text{ (360 MPa)}$$
$$F_s = 8.04 \text{ cm}^2$$

The effective depth of the member for $a = 3.5$ cm is

$$h_0 = h - a = 40 - 3.5 = 36.5 \text{ cm}$$

From Eq. (VI.9), the depth of the concrete in compression is

$$x = R_s F_s / m_{c1} R_{pr} b$$
$$= (3\,600 \times 8.04)/(0.85 \times 90 \times 20)$$
$$= 18.9 \text{ cm}$$

From Eqs. (VI.6) and (VI.5), we find

$$\xi_0 = 0.85 - 0.000\,8\, R_{pr} = 0.85 - 0.000\,8 \times 0.85 \times 90$$
$$= 0.79$$
$$\xi_R = \xi_0/[1 + R_s (1 - \xi_0/1.1)/5\,000]$$
$$= 0.79/[1 + 3\,600\,(1 - 0.79/1.1)/5\,000] = 0.653$$

The condition $x \leqslant \xi h_0$ is met because

$$\xi = x/h_0 = 18.9/36.5 = 0.518 < \xi_R = 0.653$$

According to Eq. (VI.11), the load-bearing capacity of the section is

$$R_s F_s (h_0 - 0.5x) = 3\,600 \times 8.04\,(36.5 - 0.5 \times 18.9)$$
$$= 783\,000 \text{ kgf m}$$
$$= 7.83 \text{ tf m}$$
$$= 78.3 \text{ kN m}$$

which is 3% in excess of the design moment, $M = 7.6$ tf m (76 kN m). So, the member meets the condition for strength.

Sections are chosen for the specified moment, using Eqs. (VI.9) and (VI.10)) or (VI.11), with their left-hand sides set equal to the right-hand sides.

In practice, rectangular beams with tensile steel only are designed with reference to an auxiliary table, such as Table VI.1 and Eqs. (VI.9) and (VI.11) are re-written as follows

$$M = A_0 b h_0^2 R_{pr} \tag{VI.13}$$

$$F_s = M/\eta h_0 R_s \tag{VI.14}$$

where

$$A_0 = x\,(1 - 0.5x/h_0)/h_0 = \xi\,(1 - 0.5\xi) \tag{VI.15}$$

$$\eta = z_c/h_0 = 1 - 0.5\ x/h_0 = 1 - 0.5\xi \tag{VI.16}$$

The effective depth of the section is found from Eq. (VI.13):

$$h_0 = \sqrt{M/A_0 b R_{pr}} \tag{VI.17}$$

The coefficients $A_0$ and $\eta$, as found from Eqs. (VI.15) and (VI.16), are given in Table VI.1 for ease of computation.

The dimensions $b$ and $h$ should be chosen as follows. Assume the section width $b$ and look up the coefficient $A_0$ in Table VI.1 for the recommended $\xi$. Then using Eq. (VI.16), determine the effective depth $h_0$ and the overall depth $h = h_0 + a$, and select the nearest standard size. If the values of $b$ and $h$ thus found do not meet the required conditions of design or manufacture, refine them by going through the above sequence of steps once more.

The necessary steel area $F_s$ should be found as follows. First, deduce $A_0$ from Eq. (VI.13). Then get $\eta$ and $\xi$ from Table VI.1 for $A_0$ thus found. Determine $F_s$ by Eq. (VI.14). Finally, check to see if the condition of Eq. (VI.4) is satisfied.

Table VI.1 may also be used to check a member for strength. In this case, calculate $\mu = F_s/bh_0$, using the known data on the section, and also $\xi$ from Eq. (VI.12), checking it for compliance with the condition $x \leqslant \xi_R h_0$. Then, using $\xi$, find $A_0$ from Table VI.1, and calculate the maximum bearable bending moment by Eq. (VI.13).

**Example VI.2.** Given: A rectangular beam subjected to a bending moment $M = 8.5$ tf m (85 kN m), induced by dead, long- and short-time live loads. Concrete: M-200 ($m_{c1} = 0.85$). Reinforcement: class A-II deformed bars.

To find: The values of $b$, $h$, and $F_s$.

**Table VI.1. Coefficients $A_0$ and $\eta$ for Design of Bending Rectangular Beams with Tensile Steel Only**

| $\xi = x/h_0$ | $\eta = z_c/h_0$ | $A_0$ | $\xi = x/h_0$ | $\eta = z_c/h_0$ | $A_0$ |
|---|---|---|---|---|---|
| 0.01 | 0.995 | 0.01 | 0.37 | 0.815 | 0.301 |
| 0.02 | 0.99 | 0.02 | 0.38 | 0.81 | 0.309 |
| 0.03 | 0.985 | 0.03 | 0.39 | 0.805 | 0.314 |
| 0.04 | 0.98 | 0.039 | 0.4 | 0.8 | 0.32 |
| 0.05 | 0.975 | 0.048 | 0.41 | 0.795 | 0.326 |
| 0.06 | 0.97 | 0.058 | 0.42 | 0.79 | 0.332 |
| 0.07 | 0.965 | 0.067 | 0.43 | 0.785 | 0.337 |
| 0.08 | 0.96 | 0.077 | 0.44 | 0.78 | 0.343 |
| 0.09 | 0.955 | 0.085 | 0.45 | 0.775 | 0.349 |
| 0.1 | 0.95 | 0.095 | 0.46 | 0.77 | 0.354 |
| 0.11 | 0.945 | 0.104 | 0.47 | 0.765 | 0.359 |
| 0.12 | 0.94 | 0.113 | 0.48 | 0.76 | 0.365 |
| 0.13 | 0.935 | 0.121 | 0.49 | 0.755 | 0.37 |
| 0.14 | 0.93 | 0.13 | 0.5 | 0.75 | 0.375 |
| 0.15 | 0.925 | 0.139 | 0.51 | 0.745 | 0.38 |
| 0.16 | 0.92 | 0.147 | 0.52 | 0.74 | 0.385 |
| 0.17 | 0.915 | 0.155 | 0.53 | 0.735 | 0.39 |
| 0.18 | 0.91 | 0.164 | 0.54 | 0.73 | 0.394 |
| 0.19 | 0.905 | 0.172 | 0.55 | 0.725 | 0.399 |
| 0.2 | 0.9 | 0.18 | 0.56 | 0.72 | 0.403 |
| 0.21 | 0.895 | 0.188 | 0.57 | 0.715 | 0.408 |
| 0.22 | 0.89 | 0.196 | 0.58 | 0.71 | 0.412 |
| 0.23 | 0.885 | 0.203 | 0.59 | 0.705 | 0.416 |
| 0.24 | 0.88 | 0.211 | 0.6 | 0.7 | 0.42 |
| 0.25 | 0.875 | 0.219 | 0.61 | 0.695 | 0.424 |
| 0.26 | 0.87 | 0.226 | 0.62 | 0.69 | 0.428 |
| 0.27 | 0.865 | 0.236 | 0.63 | 0.685 | 0.432 |
| 0.28 | 0.86 | 0.241 | 0.64 | 0.68 | 0.435 |
| 0.29 | 0.855 | 0.248 | 0.65 | 0.675 | 0.439 |
| 0.3 | 0.85 | 0.255 | 0.66 | 0.672 | 0.442 |
| 0.31 | 0.845 | 0.262 | 0.67 | 0.665 | 0.446 |
| 0.32 | 0.84 | 0.269 | 0.68 | 0.66 | 0.449 |
| 0.33 | 0.835 | 0.275 | 0.69 | 0.655 | 0.452 |
| 0.34 | 0.83 | 0.282 | 0.7 | 0.65 | 0.455 |
| 0.35 | 0.825 | 0.289 | | | |
| 0.36 | 0.82 | 0.295 | | | |

*Solution.* From Appendices IV and VI, we find

$$R_{pr} = 90 \text{ kgf cm}^{-2} \text{ (9 MPa)}$$

and

$$R_s = 2\,700 \text{ kgf cm}^{-2} \text{ (270 MPa)}$$

Assume $b = 20$ cm and $\xi = 0.35$. In Table VI.1, $\xi = 0.35$ corresponds to $A_0 = 0.289$. From Eq. (VI.17), we obtain

$$h_0 = \sqrt{M/A_0 b R_{pr} m_{c1}} = \sqrt{850\,000/(0.289 \times 20 \times 90 \times 0.85)}$$
$$= 43.8 \text{ cm}$$

The overall depth is $h = h_0 + a = 43.8 + 3.5 = 47.3$ cm. We choose $h = 45$ cm (a multiple of 5 cm) and $h_0 = 45 - 3.5 = 41.5$ cm. From Eq. (VI.13), we get

$$A_0 = M/bh_0^2 R_{pr} m_{c1} = 850\,000/(20 \times 41.5^2 \times 90 \times 0.85)$$
$$= 0.323$$

In Table VI.1, this corresponds to $\eta = 0.797$ and $\xi = 0.405$. From Eq. (VI.14) we obtain

$$F_s = M/\eta h_0 R_s = 850\,000/(0.79 \times 41.5 \times 2\,700) = 9.55 \text{ cm}^2$$

Equations (VI.6) and (VI.5) yield

$$\xi_0 = 0.85 - 0.000\,8 R_{pr} = 0.85 - 0.000\,8 \times 0.85 \times 90$$
$$= 0.79$$

$$\xi_R = \xi_0/[1 + R_s (1 - \xi_0/1.1)/5\,000]$$
$$= 0.79/[1 + 2\,700 (1 - 0.79/1.1)/5\,000] = 0.687$$

The condition $x \leqslant \xi_R h_0$ is met, because $\xi = 0.405 < \xi_R = 0.687$. So, we may take four class A-II bars 18 mm in diameter with an area $F_s = 10.18 \text{ cm}^2$ (see Appendix VIII).

**Example VI.3.** Given: A slab subjected to a design moment of 380 kgf m (3 800 N m) per metre of its cross-sectional length (due to dead, long- and short-time live loads). The thickness is $h = 8$ cm. Reinforcement: standard welded-wire fabric 150/250/6/4, made of class B-I plain wire. Concrete: M-150 heavy concrete ($m_{c1} = 1$).

Check the slab for strength.

*Solution.* From Appendices IV, VII, and IX,

$$R_{pr} = 70 \text{ kgf cm}^{-2} \text{ (7 MPa)}$$

$$R_s = 3\,150 \text{ kgf cm}^{-2} \text{ (315 MPa)}$$

The design cross-sectional area of the load-bearing (longitudinal) bars per metre run of the fabric is

$$F_s = 2.07 \text{ cm}^2$$

The effective depth of the slab is

$$h = h_0 - a = 8 - 1.5 = 6.5 \text{ cm}$$

The percentage of reinforcement is

$$\mu = 100F_s/bh_0 = (100 \times 2.07)/(100 \times 6.5) = 0.32\%$$

From Eq. (VI.12), we get

$$\xi = \mu R_s/100R_{pr} = (0.32 \times 3\,150)/(100 \times 70) = 0.144$$

which is obviously smaller than $\xi_R$.

In Table VI.1, $\xi = 0.144$ corresponds to $A_0 = 0.134$. From Eq. (VI.13), we obtain

$$A_0bh_0^2R_{pr} = 0.134 \times 100 \times 6.5^2 \times 70$$

$$= 39\,600 \text{ kgf cm}$$

As is seen, the load-bearing capacity of the section is greater than it is required for the design moment $M = 380$ kgf m (3 800 N m).

**T-Section Members.** These are used rather frequently as separate reinforced concrete T-beams (Fig. VI.14*a* and *b*) and as parts of in-situ and precast panel floors (Fig. VI.14*c* and *d*). A T-section consists of a flange and a rib.

T-section members are more advantageous as compared with rectangular members (the dashed line in Fig. VI.14*a*), because for the same load-bearing capacity (which does not depend on the cross-sectional area of the tension zone in a reinforced concrete member) they require less concrete owing to a reduced tension zone. For the same reason, T-section members with the flange in the compression zone (see Fig. VI.14*a*) are preferable because the flange located in the tension zone (see Fig. VI.14*b*) does not contribute to the

bearing capacity of the member. As a rule, T-members have tensile steel only.

With wide flanges, the overhang areas more distant from the rib are stressed less. So, design calculations use the equivalent overhang $b_{ov}$ (see Fig. VI.14*c* and *d*). It is taken equal to not more than half the clear space between the ribs, $c$, nor more than 1/6 of the span of the member on either side of the rib. In members with flanges having $h'_f \leqslant 0.1h$ without transverse ribs or with ribs spaced wider apart than the longitudinal ribs, $b_{ov}$ should not exceed $6h'_f$. For separate T-beams with overhanging flanges (see Fig. VI.14*a*), $b_{ov}$ should be

$$\max 6h'_f \text{ for } h'_f \geqslant 0.1h$$

$$\max 3h'_f \text{ for } 0.05h \leqslant h'_f \leqslant 0.1h$$

For $h'_f < 0.05h$, flange overhangs are not taken into account in design calculations.

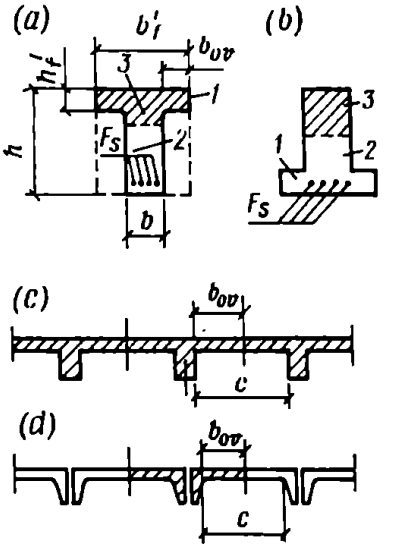



Fig. VI.14. T-sections

(*a*) beam with the flange in compression; (*b*) beam with the flange in tension; (*c*) T-section in an in-situ floor; (*d*) T-section in a precast floor; *1*—flange; *2*— rib; *3*—compression zone

In T-section members, the neutral axis may run either within the flange (Fig. VI. 15*a*) or below the flange (Fig. VI.15*b*).

The former case, where $x \leqslant h'_f$, occurs in T-sections with large overhangs. Here, the T-section is designed as a rectangular section with the dimensions $b'_f$ and $h_0$ (see Fig. VI.15*a*), because the concrete area in the tension zone does not affect the load-bearing capacity of the member.

The design formulas for nonprestressed members are

$$R_{pr}b'_f x = R_s F_s \tag{VI.18}$$

$$M \leqslant R_{pr}b'_f (h_0 - 0.5x) \tag{VI.19}$$

or

$$M \leqslant A_0 R_{pr} b'_f h_0^2 \tag{VI.20}$$

where $A_0$ is taken from Table VI.1.

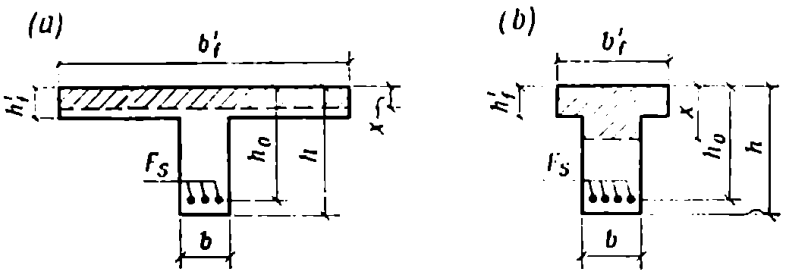



Fig. VI.15. Two design cases for T-sections
(a) neutral axis within the flange; (b) neutral axis below the flange

The latter case, where $x > h'_f$, occurs in T-members with narrow overhangs. Here, the compression zone of the section includes the compression zone of the rib and the overhangs.

The depth of the compression zone is found from the following equation

$$R_s F_s = R_{pr} bx + R_{pr} (b'_f - b) h'_f \qquad \text{(VI.21)}$$

With moments taken about the axis normal to the plane of flexure and passing through the point of application of the resultant force in the tensile steel, the condition for strength is as follows:

$$M \leqslant R_{pr} bx (h_0 - 0.5x) + R_{pr} (b'_f - b) h'_f (h_0 - 0.5h'_f) \qquad \text{(VI.22)}$$

For T-section members, $x$ should not exceed $\xi_R h_0$.

Experience shows that the overall depth of a T-beam may approximately be taken in the range

$$h = 15 \sqrt[3]{M} \text{ to } 20 \sqrt[3]{M} \qquad \text{(VI.23)}$$

where $h$ is in cm and $M$ is in tf m.

The rib width is usually taken as

$$b = 0.4h \text{ to } 0.5h \qquad \text{(VI.24)}$$

The dimensions $b'_f$ and $h'_f$ are most frequently known from the layout of the structure. The steel area $F_s$ necessary to accommodate the design moment is found according to the position of the neutral axis. If the neutral axis lies within the flange, $F_s$ is determined from Table VI.1, assuming

that the section is rectangular, with the width $b_f'$ and the depth $h_0$, and has tensile steel only.

Which of the two cases is involved may be found as follows.

1. All of the section parameters, including $F_s$, are known. Then, if

$$R_s F_s \leqslant R_{pr} b_f' h_f' \qquad \text{(VI.25)}$$

the neutral axis lies within the flange. Otherwise, it is located in the rib.

2. The dimensions $b_f'$, $h_f'$, $b$ and $h$ and the design bending moment are specified, but $F_s$ is unknown. Then, if

$$M \leqslant R_{pr} b_f' h_f' (h_0 - 0.5h_f') \qquad \text{(VI.26)}$$

the neutral axis lies within the flange. Otherwise, it crosses the rib.

For the neutral axis lying below the flange, Eqs. (VI.21) and (VI.22) may be re-written, recalling that $x = \xi h_0$ and taking into consideration Eq. (VI.15), thus

$$R_s F_s = \xi R_{pr} b h_0 + R_{pr} (b_f' - b) h_f' \qquad \text{(VI.27)}$$

$$M \leqslant A_0 R_{pr} b h_0^2 + R_{pr} (b_f' - b) h_f' (h_0 - 0.5h_f') \qquad \text{(VI.28)}$$

where the coefficients $\xi$ and $A_0$ are taken from Table VI.1.

The above expressions may be used to choose the section geometry. If it is necessary to determine $F_s$, calculate from Eq. (VI.28)

$$A_0 = [M - R_{pr} (b_f' - b) h_f' (h_0 - 0.5h_f')]/R_{pr} b h_0^2 \qquad \text{(VI.29)}$$

Then find $\xi$ corresponding to the computed $A_0$ from Table VI.1, and determine the steel area from Eq. (VI.27)

$$F_s = [\xi b h_0 + (b_f' - b) h_f'] R_{pr}/R_s \qquad \text{(VI.30)}$$

If it is necessary to check a section for strength, when all data are known, which of the two cases applies can better be determined from Eq. (VI.26). Then (if the neutral axis happens to run below the flange), the depth of the compression zone is found from Eq. (VI.21). Further calculations are carried out by Eq. (VI.22).

**Example VI.4.** Given: A T-beam subjected to a design bending moment of 45 tf m (450 kN m) due to dead, long- and short-time live loads; $h = 70$ cm ($h_0 = 66$ cm), $b =$

= 25 cm. Design flange width is $b_f' = 60$ cm, $h_f' = 8$ cm. Concrete: M-200 heavy concrete ($m_{c1} = 0.85$). Reinforcement: class A-III deformed bars.

To find: $F_s$.

*Solution.* From Appendices IV and VI, we find that $R_{pr} = 90$ kgf cm$^{-2}$ (9 MPa) and $R_s = 3\,600$ kgf cm$^{-2}$ (360 MPa) for bars more than 10 mm in diameter.

Which of the two cases applies can be determined, using Eq. (VI.26):

$$m_{c1}R_{pr}b_f'h_f'(h_0 - 0.5h_f') = 0.85 \times 90 \times 60 \times 8\,(66 - 0.5 \times 8) = 2\,280\,000 \text{ kgf cm}$$

This is less than $M = 45$ tf m (450 kN m). So, the neutral axis crosses the rib. Equation (VI.29) gives

$$A_0 = [M - m_{c1}R_{pr}(b_f' - b)h_f'(h_0 - 0.5h_f')]/m_{c1}R_{pr}bh_0^2$$
$$= [4\,500\,000 - 0.85 \times 90\,(60 - 25) \times 8 \times (66 - 0.5 \times 8)]/(0.85 \times 90 \times 25 \times 66^2)$$
$$= 0.381$$

In Table VI.1, this corresponds to $\xi = 0.512$. According to Eq. (VI.30), the necessary steel area is

$$F_s = [\xi bh_0 + (b_f' - b)h_f']\,m_{c1}R_{pr}/R_s$$
$$= [0.512 \times 25 \times 66 + (60-25) \times 8]\,0.85 \times 90/3\,600$$
$$= 23.9 \text{ cm}$$

We may take (see Appendix VIII) two class A-III bars 18 mm in diameter and with $F_s = 24.72$ cm$^2$.

The coefficient $\xi = 0.512$ is smaller than $\xi_R = 0.653$ (see Example VI.1), so the condition for the use of the design formulas is met.

## VI.4. Inclined-Section Shear Strength Analysis

**Basic Design Formulas.** Members in bending fail at inclined sections owing to the simultaneous action of bending moments and shearing forces at a given section. Consider the internal forces developing in the steel and the concrete in the compression zone at an inclined crack. Figure VI.16

shows a near-support part of a reinforced concrete member containing longitudinal, transverse, and diagonal reinforcement. This part is separated from the rest of the member by a section coinciding with an inclined crack.

To have an ample shear strength, a member should satisfy two conditions, because both $M$ and $Q$ exist in the portion in question (see Fig. VI.9, portion *1*). In the design force diagram (Fig. VI.16), it is assumed that the member is subjected to the moment and the shearing force that would exist under design loads, and that the stresses in the steel and the concrete are equal to their design strengths. Then, the conditions for strength are

$$M_D \leqslant R_s F_s z + \Sigma R_s F_b z_b + \Sigma R_s F_{tr} z_{tr} \qquad \text{(VI.31)}$$

$$Q_D \leqslant \Sigma R_{s,tr} F_b \sin \alpha + \Sigma R_{s,tr} F_{tr} + Q_c \qquad \text{(VI.32)}$$

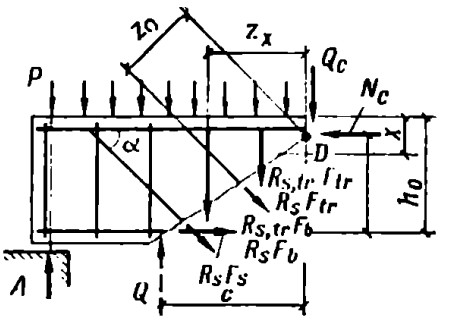



Fig. VI.16. Design loading diagram at an inclined section (quantities in the brackets give the forces in the transverse and diagonal bars, taken for the analysis of the inclined sections for resistance to the applied bending moment)

Here, $F_s$, $F_{tr}$, and $F_b$ are the areas of the longitudinal reinforcement, transverse reinforcement (stirrups), and diagonal reinforcement (bent-up bars), respectively; $Q_c$ is the shearing force resisted by the concrete in the compression zone at the inclined section; $\alpha$ is the angle between the bent-up bars and the longitudinal axis of the member; $z$, $z_b$ and $z_{tr}$ are the lever arms for the forces in the longitudinal, diagonal, and transverse bars; $M_D$ and $Q_D$ are the design bending moment and shearing force taken about point $D$.

Equation (VI.31) gives the condition for strength with regard to the bending moment. The shear strength of a member is considered adequate if the bending moment $M_D$ due to all the forces applied to the part in question does not exceed the sum of the moments due to the internal design forces in the longitudinal, transverse, and diagonal reinforcement, taken about the same fulcrum.

Equation (VI.32) gives the condition for the shear strength of a member. A member is considered to have an ample shear strength, if the shearing force $Q_D$ due to all the forces applied to the selected part does not exceed the sum of the projections of the internal design forces on the axis normal to the longitudinal axis of the member, and the shearing force carried by the concrete of the compression zone.

Some of the transverse and diagonal bars in an inclined crack lying closer to the compression zone cannot act at their full strength because the crack is narrow there. Accordingly, Eq. (VI.32) uses a design strength $R_{s.tr}$ which is somewhat lower than $R_s$ (see Appendix VII). In Eq. (VI.31), the design strength is not reduced because the moments of the forces in the transverse and diagonal bars located close to the fulcrum are small and do not significantly affect the final results.

The design shearing forces carried by the concrete in the compression zone is given by the following empirical formula

$$Q_c = k_2 R_{ten} b h_0^2 / c = B/c \qquad \text{(VI.33)}$$

where $B = k_2 R_{ten} b h_0^2$, $k_2$ is an empirical coefficient taken as 2 for heavy concrete, $h_0$ is the effective depth of the member, $b$ is the width of a rectangular member, or the rib or web width of a T- or I-beam, and $c$ is the projection of the inclined section on the axis of the member.

The condition defined by Eq. (VI.31) can usually be met without calculation, by adequate proportioning which will be discussed later. The condition defined by Eq. (VI.32) generally requires an appropriate computation.

Practical recommendations require that the maximum shearing force for rectangular, T-, and other similar sections should be

$$Q \leqslant 0.35 R_p b h_0 \qquad \text{(VI.34)}$$

If this condition is met, the web concrete between inclined cracks will stand well the inclined compressive stress.

If design calculations predict no inclined cracks in a member, shear strength analysis need not be carried out. The required strength is given by the following empirical

formula

$$Q \leqslant k_1 R_{t\,en} b h_0 \qquad \text{(VI.35)}$$

where $k_1$ is equal to 0.6 for heavy concrete.

**Transverse Bar Design.** Let us examine a member in bending, which contains transverse reinforcement without bent-up bars. This is the most commonly used type of reinforcement.

Of all inclined sections likely to originate at point $B$ (Fig. VI.17), we shall choose for design purposes the section having the least load-bearing capacity. As will be recalled

$$\left.\begin{aligned} Q_D &= Q - pc \\ \Sigma R_{s,\,tr} F_{tr} &= q_{tr} c \end{aligned}\right\} \qquad \text{(VI.36)}$$

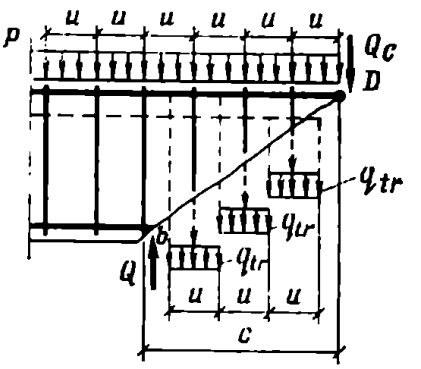



Fig. VI.17. Forces in transverse bars, taken for the inclined-section analysis of a beam

where $Q$ is the shearing force at the beginning of the inclined section (see Fig. VI.17), and $q_{tr}$ is the force carried by the transverse bars per unit length of the member.

Substituting Eqs. (VI.33) and (VI.36) into Eq. (VI.32) gives

$$Q \leqslant (q_{tr} + p)\, c + B/c \qquad \text{(VI.37)}$$

The least load-bearing capacity of the inclined section will obviously be given by

$$dQ/dc = (q_{tr} + p) - B/c^2 = 0$$

Hence, the projection of the chosen ("design") inclined section is

$$c = \sqrt{B/(q_{tr} + p)} = \sqrt{k_2 R_{t\,en} b h_0^2/(q_{tr} + p)} \qquad \text{(VI.38)}$$

Substituting Eq. (VI.38) into Eq. (VI.37) gives the condition for the shear strength in terms of the least load-bearing capacity of the inclined section

$$Q \leqslant 2\sqrt{B\,(q_{tr} + p)}$$

Substituting for $B$ from Eq. (VI.33) into the above expression gives the shearing force $Q_{tr,c}$ carried by the stirrups and the concrete at the design inclined section

$$Q_{tr,c}=2\sqrt{k_2R_{ten}bh_0^2(q_{tr}+p)} \qquad \text{(VI.39)}$$

For design purposes, the load $p$ is frequently taken to be uniformly distributed. Actually, however, it is concentrated at few points, so it may so happen that it does not exist at all over the entire length of an inclined section. Therefore, the load should be taken into account only when it is uniformly distributed (such as earth or water pressure).

Putting $p = 0$ in Eqs. (VI.38) and (VI.39), we find that the shearing force that the concrete of the compression zone and the stirrups can carry is

$$Q_{tr,c}=2\sqrt{k_2bh_0^2R_{ten}q_{tr}} \qquad \text{(VI.40)}$$

The projection of the design inclined section is

$$c_0=\sqrt{k_2R_{ten}bh_0^2/q_{tr}} \qquad \text{(VI.41)}$$

Referring to the diagram of Fig. VI.18, we may write

$$q_{tr}u = R_{s,tr}f_{tr}n \text{ or}$$

$$q_{tr}u = R_{s,tr}F_{tr},\ F_{tr} = f_{tr}n \qquad \text{(VI.42)}$$

where $u$ is the spacing between the transverse bars (stirrups), $f_{tr}$ is the cross-sectional area of one transverse bar (stirrup), and $n$ is the number of transverse bars within the section of the member.

In design calculation, the diameter of transverse bars and their number within the cross-section of the member are usually specified in advance, so $f_{tr}n = F_{tr}$ is regarded as a term known beforehand.

An idea about the required amount of reinforcement may be formed from Eq. (VI.40)

$$q_{tr} = Q^2/4k_2bh_0^2R_{ten} \text{ or } q_{tr} \geqslant R_{ten}b/2 \qquad \text{(VI.43)}$$

whichever is the greater. This value of $q_{tr}$ should be matched by the force in stirrups per unit length of the member

$$q_{tr} = R_{s,tr}F_{tr}/u \qquad \text{(VI.44)}$$

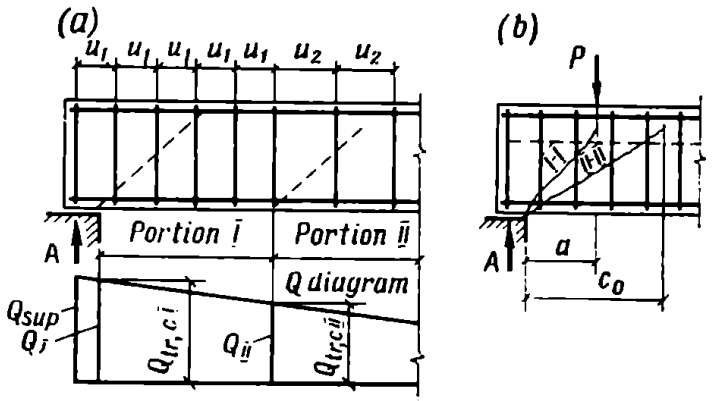



Fig. VI.18. Design inclined sections in different beam portions
(*a*) with different spacings between transverse bars; (*b*) between support and concentrated load, provided $a < c_0$

It should be kept in mind that the spacing between the transverse bars ought not to exceed a value such that a likely crack would lie anywhere between two adjacent bars where the member owes its strength solely to the concrete in the compression zone. In this case, the condition $Q \leqslant Q_c$ holds. Using Eq. (VI.33) and multiplying the result by the coefficient 0.75 which accounts for likely departure of cracks from design direction due to the nonuniformity of the concrete, and also for the likely inaccuracy in the location of the stirrups, we find that the spacing between the transverse bars should not exceed

$$u_{max} = 0.75 k_2 R_{ten} b h_0^2 / Q \qquad \text{(VI.45)}$$

Apart from design conditions, the spacing between the transverse bars should also be satisfactory from certain engineering considerations (see Sec. VI.1).

The spacing and diameter of transverse bars may vary from part to part of a beam. Therefore, the origin of the design diagonal sections is chosen at the support face and where $Q = Q_{tr,c\,II}$ (Fig. VI.18*a*), and the shearing force i assigned the design value applicable at these points. The member portion with $q_{tr\,I}$ runs from the support to the point where $Q = Q_{tr,cII}$ (see Fig. VI.18*a*).

If a member is subjected to a concentrated load $P$ applied at a distance $a < c_0$ from the support, an inclined crack

may occur between the support and the point where the load is applied (Fig. VI.18*b*, section *I-I*). The condition for strength in this case is derived from Eq. (VI.37) in which $c$ is put equal to $a$:

$$Q_{\mathrm{I}} \leqslant q_{tr}a + B/a$$

or, recalling Eq. (VI.33),

$$Q_{\mathrm{I}} \leqslant q_{tr}a + k_2 R_{ten} b h_0^2/a \qquad \text{(VI.46)}$$

Hence, the force per unit length of the beam, carried by the transverse reinforcement in section *I-I*, is

$$q_{tr}^{\mathrm{I-I}} = Q_{\mathrm{I}}/a - k_2 R_{ten} b h_0^2/a^2 \qquad \text{(VI.47)}$$

According to Eq. (VI.43), the force in the stirrups for section *II-II* past the point of application of load $P$ may be determined from the following formula

$$q_{tr}^{\mathrm{II-II}} = (Q_{\mathrm{I}} - P)^2/4k_2 b h_0^2 R_{ten} \qquad \text{(VI.48)}$$

The amount of transverse reinforcement is assigned according to which of the two terms, $q_{tr}^{\mathrm{I-I}}$ or $q_{tr}^{\mathrm{II-II}}$, is the larger.

**Example VI.5.** Given: A beam with $b = 20$ cm and $h = 40$ cm ($h_0 = 36.5$ cm), subjected to a shearing force of 8 tf (80 kN). Concrete: M-200 heavy concrete ($m_{c1} = 1$). Transverse reinforcement: class A-II bars.

To find: The values of $F_{tr}$ and $u_{max}$.

*Solution.* From Appendices IV and VI, we find that $R_{pr} = 90$ kgf cm$^{-2}$ (9 MPa), $R_{ten} = 7.5$ kgf cm$^{-2}$ (0.75 MPa), and $R_{s,tr} = 2\,150$ kgf cm$^{-2}$ (215 MPa).

A check to see if Eqs. (VI.34) and (VI.35) are satisfied gives

$$k_1 R_{ten} b h_0 \leqslant Q \leqslant 0.35 R_{pr} b h_0$$

On substituting the numerical values, we obtain

$$0.6 \times 7.5 \times 20 \times 36.5 = 3\,300 \text{ kgf (33 kN)} < Q$$
$$= 8\,000 \text{ kgf (80 kN)} < 0.35 \times 90 \times 20 \times 36.5$$
$$= 23\,000 \text{ kgf (23 kN)}$$

As is seen, transverse reinforcement is necessary, and the chosen cross-sectional dimensions of the beam are acceptable.

We take two transverse bars 6 mm in diameter ($f_{tr}$ = $= 0.283$ cm$^2$, $F_{tr} = 0.283 \times 2 = 0.566$ cm$^2$).

According to Eq. (VI.43), the design force per unit length of the beam, carried by the transverse bars is

$$q_{tr} = Q^2/4k_2bh_0^2R_{ten} = 8\,000^2/(4 \times 2 \times 20 \times 36.5^2 \times 7.5)$$
$$= 40 \text{ kgf cm}^{-1} \text{ (400 N cm}^{-1}\text{)}$$

which is less than

$$R_{ten}b/2 = 7.5 \times 20/2 = 75 \text{ kgf cm}^{-1} \text{ (750 N cm}^{-1}\text{)}$$

Equation (VI.44) gives the spacing between the transverse bars

$$u = R_{s,tr}F_{tr}/q_{tr} = 2\,150 \times 0.566/40 = 30 \text{ cm}$$

According to the condition in Eq. (VI.45),

$$u_{max} = 0.75k_2R_{ten}bh_0^2/Q$$
$$= 0.75 \times 2 \times 7.5 \times 20 \times 36.5^2/8\,000 = 37 \text{ cm}$$

From engineering considerations (see Sec. VI.1),

$$u \leqslant h/2 = 40/2 = 20 \text{ cm}$$

and $u \leqslant 15$ cm. We take the smaller of these values, that is, $u = 15$ cm.

**Bent-up Bars.** Nowadays, bent-up bars are used relatively seldom, and then to strengthen some parts of a beam subjected to large shearing forces. The locations for bends (the places where some of the longitudinal reinforcement is carried from the tension to the compression zone) are chosen on the basis of normal- and inclined-section analysis (see Fig. VI.16). Bends are provided where $Q > Q_{tr,c}$ [see Eq. (VI.40)].

According to Eq. (VI.32), the condition for shear strength with allowance for bends may be written as

$$Q \leqslant R_{s,tr}F_b \sin \alpha + Q_{tr,c} \qquad \text{(VI.49)}$$

Hence, the cross-sectional area of the bent-up bars is

**Constructional Features to Ensure Bending Moment Strength for Inclined Sections.** As regards the applied bending moment, the load-bearing capacity of an inclined section [see the right-hand side of (VI.31)] should not be less than that of the normal section passing through the same point $D$ (see Fig. VI.16). This requirement can be met by incorporating appropriate engineering features (to be discussed below), and the analysis of inclined sections for bending moment may be omitted.

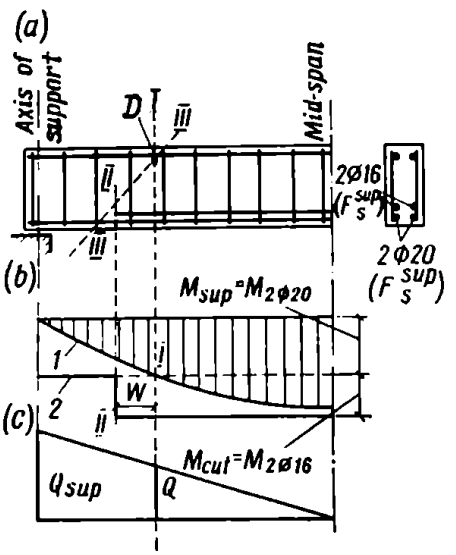



Fig. VI.19. Location of the cut-off point in the span

(a) reinforcement scheme; (b) moment diagram; (c) shearing force diagram; *I-I*—theoretical cutoff point for two bars 16 mm in diameter; *II-II*—actual cutoff point for these bars; *1*—distribution of moments due to load; *2*—distribution of moments resisted by normal sections of member (what is known as material diagram)

If the longitudinal reinforcement at a free support is anchored as advised in Sec. III.3, so that the longitudinal steel acts at full strength over the span, the member will have an ample bending strength at any inclined section beginning at the support face.

In practice, beams are most often reinforced without bent-up bars. If all of the tensile longitudinal steel is extended as far as the support and properly anchored, any inclined section will resist the applied bending moment well owing to the longitudinal reinforcement alone, provided it is designed for the normal section carrying at least the same bending moment. In this case, too, an analysis of the inclined section for bending moment is unnecessary.

To reduce steel consumption, some longitudinal reinforcement (not more than 50% of the design area) may be cut off in the span at a point beyond which it is no longer required by the normal-section strength analysis.

These bars should be extended beyond their theoretical

cutoff point found from the bending moment diagram (section *I-I* in Fig. VI.19) for a length $w$ where the transverse steel will make up for the missing longitudinal bars at inclined sections (section *III-III* in Fig. VI.19*a*) and guarantee the required resistance to bending moments. From the above considerations and for proper anchorage of cutoff bars, the length $w$ must be taken equal to the greater of the two following values

$$\left.\begin{aligned} w &= (Q - Q_b)/2q_{tr,\,w} + 5d \\ \text{or} \qquad w &= 20d \end{aligned}\right\} \qquad \text{(VI.51)}$$

Here, $Q$ is the design shearing force at the theoretical cutoff point (section *I-I* in Fig. VI.19), corresponding to the load for which this point is determined; $Q_b$ is the shearing force carried by the diagonal bars at the theoretical cutoff point, if the member contains bent-up bars in addition to transverse steel; $q_{tr,\,w}$ is the force per unit beam length, carried by the transverse bars, as found from the bending moment at an inclined section (section *III-III* in Fig. VI.19*a*); and $d$ is the diameter of the cutoff bar.

The values of $Q_b$ and $q_{tr,\,w}$ are determined from the following expressions

$$Q_b = R_s F_b \sin \alpha \qquad \text{(VI.52)}$$

$$q_{tr,\,w} = R_s F_{tr}/u \qquad \text{(VI.53)}$$

If there are no bent-up bars in the zone where the longitudinal steel is cut off, $Q_b = 0$ in the first line of Eqs. (VI.51).

Figure VI.19 illustrates how the location of cutoff points in the span is found. To begin with, we lay off on the moment diagram the ordinate of the moment resisted by the normal section of the reinforced concrete member containing the steel which is extended intact as far as the support (in Fig. VI.19, $F_s^{sup}$ is for two bars 20 mm in diameter, $M_{sup} = M_{2\varnothing 20}$). This ordinate is given by

$$M_{sup} = R_s F_s^{sup} z_c \qquad \text{(VI.54)}$$

The intersections between the ordinate $M_{sup}$ and the design moment curve locate the theoretical cutoff points,

*I-I*. The actual cutoff points, *II-II*, must lie within the distance $w$ of the theoretical cutoff points. The ordinate $Q$ which is used in Eq. (VI.51) to determine $w$ is marked on the shearing force diagram.

## VI.5. Incipient-Cracking Resistance of Prestressed Members

In normal-section analysis, the limit state attained by a bending prestressed member is taken to be the final state of stress during Stage I (see Fig. VI.10*c*). Figure VI.20*b* shows the force and stress scheme for the incipient-cracking analysis of a rectangular member. In this scheme, the curves representing the distribution of stress in the concrete are approximated for simplicity by a rectangle in the tension zone, and a triangle in the compression zone (for $\sigma_c \leqslant \leqslant 0.7R_{prII}$).

Immediately before cracking, the stress in the concrete is assumed to be equal to $R_{tenII}$ which is the design tensile strength of concrete adopted in the design in terms of the second group of limit states (see Appendix IV).

The member is subjected to the bending moment $M$ induced by the design loads multiplied by the overload factor which is chosen to be $n > 1$ or $n = 1$, according to the category of crack resistance requirements the structure is

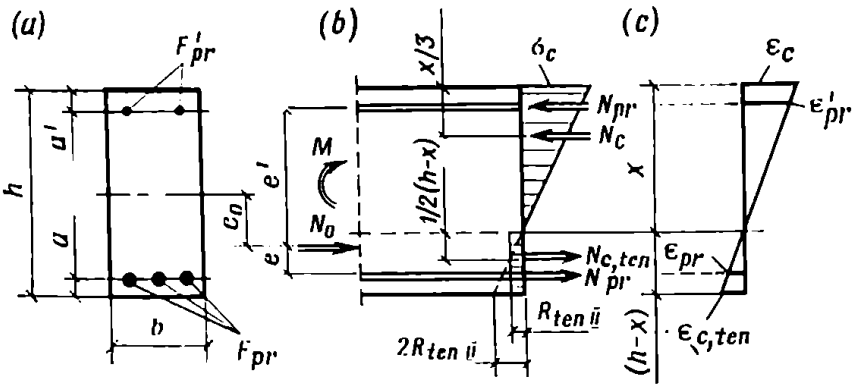



Fig. VI.20. To the incipient-cracking analysis of prestressed members in bending

(*a*) cross section; (*b*) stress diagram and design forces in concrete; (c) strain diagram

expected to meet (see Sec. III.1), and the resultant of the prestressing forces $N_0$ applied to the member by the prestressed steel of area $F_{pr}$ and $F'_{pr}$. In calculations, the prestressing force $N_0$ is multiplied by the tensioning accuracy factor $m_{ac}$ (see Sec. III.2).

Over the depth of the member, the strain is assumed to be varying linearly (Fig. VI.20*c*), so

$$\left.\begin{aligned} \varepsilon_c &= \varepsilon_{c,\,ten} x/(h-x) \\ \varepsilon_{pr} &= \varepsilon_{c,\,ten}\,(h-x-a)/(h-x) \\ \varepsilon'_{pr} &= \varepsilon_{c,\,ten}\,(x-a')/(h-x) \end{aligned}\right\} \qquad \text{(VI.55)}$$

Present evidence shows that when the stress in concrete reaches a maximum, its modulus of elasticity is about half the initial tangent modulus of elasticity, $E_c$. Therefore, the marginal strain in the tensile concrete may be written as

$$\varepsilon_{c,\,ten} = R_{ten\,II}/0.5E_c = 2R_{ten\,II}/E_c \qquad \text{(VI.56)}$$

In view of the foregoing, the marginal stresses in the concrete and the steel, induced by the moment $M$ and the prestressing force $N_0$ before the advent of cracks, are given by

$$\sigma_c = E_c\varepsilon_c = 2R_{ten\,II}\,x/(h-x) \qquad \text{(VI.57)}$$

$$\begin{aligned} \sigma_{pr} &= E_s\varepsilon_{pr} = E_s\varepsilon_{c,\,ten}\,(h-x-a)/(h-x) \\ &= 2nR_{ten\,II}\,(h-x-a)/(h-x) \end{aligned} \qquad \text{(VI.58)}$$

$$\begin{aligned} \sigma'_{pr} &= E_s\varepsilon'_{pr} = E_s\varepsilon_{c,\,ten}x/(h-x-a') \\ &= 2nR_{ten\,II}\,(x-a')/(h-x) \end{aligned} \qquad \text{(VI.59)}$$

where $n = E_s/E_c$.

The stresses in the steel of area $F_{pr}$ and $F'_{pr}$ are added to the prestresses $\sigma_0$ and $\sigma'_0$.

The forces developed in the steel and concrete by the moment $M$ and the prestressing force $N_0$ may be found as follows:

$$\left.\begin{aligned} N_{pr} &= F_{pr}\sigma_{pr} \\ N'_{pr} &= F'_{pr}\sigma'_{pr} \\ N_c &= 0.5\sigma_c xb \\ N_{c,\,ten} &= (h-x)\,bR_{ten\,II} \end{aligned}\right\} \qquad \text{(VI.60)}$$

The depth of the compression zone is found from the equilibrium of the sum of the projections of all normal forces acting in the member on its longitudinal axis

$$N_0 + N_{pr} - N'_{pr} + N_{c,\,ten} - N_c = 0 \quad \text{(VI.61)}$$

Hence, on re-arranging

$$x = [bh^2 + 2nF_{pr}(h-a) + 2nF'_{pr}a' + N_0 h/R_{ten\,II}]/[2bh + 2n(F_{pr} + F'_{pr}) + N_0/R_{ten\,II}] \quad \text{(VI.62)}$$

No cracking will occur in a member, if the bending moment $M$ due to external loads does not exceed the moment due to the internal forces at the normal section which is on the verge of cracking. With the fulcrum taken for simplicity on the direction of the force $N_0$, this condition may be written as

$$M \leqslant N_{pr}e + N'_{pr}e' + N_c(h - e + a - x/3) + N_{c,\,ten}[e + a - 0.5(h - x)] \quad \text{(VI.63)}$$

For members of a more complex shape (such as T- or I-section, Fig. VI.21), incipient-cracking analysis may be done approximately using the kern moment method. According to this technique, the condition for cracking resistance is

$$M^k_{ext} \leqslant M_{cr} \quad \text{(VI.64)}$$

where $M^k_{ext}$ is the bending moment induced by external forces on one side of the section and taken about the axis which is normal to the plane of flexure and passes through the kern point the most distant from the tension zone (point $C$ in Fig. VI.21), and $M_{cr}$ is the moment induced by internal forces immediately before cracking and taken about the same axis.

The distance from the kern point to the centroid of the transformed section is

$$r_k = 0.8W_0/F_{tr} \quad \text{(VI.65)}$$

(for nonprestressed bending members, 0.8 in the above equation is replaced by unity).

The cracking moment is given by

$$M_{cr} = R_{ten\,II}\,W_{cr} \pm M^k_{pr} \quad \text{(VI.66)}$$

where $M^k_{pr}$ is the moment due to the prestressing force $N_0$ (with allowance for all losses), taken about the same

kern point, $W_0$ is the elastic moment of resistance of the tension zone in the transformed section, $W_{cr}$ is the moment of resistance of the tension zone in the transformed section as adjusted for the inelastic deformation in the concrete of the tension zone (the rectangular stress diagram in Fig. VI.20*b*). Without considerably affecting the value of $M_{cr}$, the value of $W_{cr}$ may approximately be taken as

$$W_{cr} \approx \gamma W_0 \quad \text{(VI.67)}$$

Here, $\gamma = 1.75$ for rectangular sections and T-sections with their flanges in the compression zone; for I-sections (see Fig. VI.21) it should be looked up in Table VI.2.

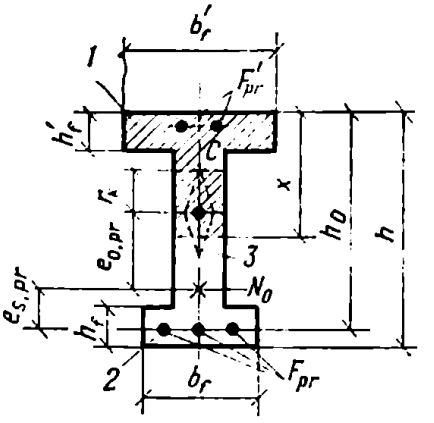



Fig. VI.21. To cracking-resistance analysis by the kern moment method

*1*—compression zone; *2*—tension zone; *3*—kern

**Table VI.2. Coefficient γ for Nonsymmetric I-sections (see Fig. VI.21)**

| Section characteristic | | | γ |
|---|---|---|---|
| $b'_f/b$ | $b_f/b$ | $h_f/h$ | |
| not over 3 | not over 2 | Any | 1.75 |
| | 2 to 6 | Any | 1.5 |
| | over 6 | over 0.1 | 1.5 |
| 3 to 8 | not over 4 | Any | 1.5 |
| | over 4 | equal to or over 0.2 | 1.5 |
| | over 4 | under 0.2 | 1.25 |
| over 8 | Any | over 0.3 | 1.5 |
| | Any | not over 0.3 | 1.25 |

When the stress in the concrete of the compression zone, $\sigma_c$, is such that the maximum marginal ordinates on the stress diagram exceed $R_{pr\,II}$, it is necessary to allow for the inelastic strain of the concrete. In this case, the triangular stress diagram must be replaced by a trapezoidal one. This, however, happens seldom and calls for no special discussion.

**Example VI.6.** Given: A solid-section slab subjected to bending. Cross-sectional dimensions: $b = 100$ cm, $h = 12$ cm, $a = 2.2$ cm. Concrete: M-400 heavy concrete. Reinforcement: five class K-7 strands made of wire 3 mm in diameter (see Appendix VIII). $F_{pr} = 2.545$ cm$^2$, $F'_{pr} = 0$. Prestressing force is $N_0 = 21$ tf (210 kN) with allowance for all losses; $M = 1.8$ tf m (18 kN m).

To find: The cracking resistance of the slab.

*Solution.* From Appendices III, V, VII, we find

$$R_{ten\,II} = 18 \text{ kgf cm}^{-2} \text{ (1.8 MPa)}$$

$$R_{pr\,II} = 225 \text{ kgf cm}^{-2} \text{ (22.5 MPa)}$$

$$E_c = 33 \times 10^4 \text{ kgf cm}^{-2} \text{ (33} \times 10^3 \text{ MPa)}$$

$$E_s = 18 \times 10^5 \text{ kgf cm}^{-2} \text{ (18} \times 10^4 \text{ MPa)}$$

$$n = E_s/E_c = 18 \times 10^5/(33 \times 10^4) = 5.45$$

Equation (VI.62) gives

$$x = [100 \times 12^2 + 2 \times 5.45 \times 2.545\,(12 - 2.2) + (21\,000/18) \times 12]/(2 \times 100 \times 12 + 2 \times 5.45 \times 2.545 + 21\,000/18) = 8 \text{ cm}$$

According to Eq. (VI.57), the maximum compressive stress in the concrete is

$$\sigma_c = 2R_{ten\,II}x/(h - x) = 2 \times 18 \times 8/(12 - 8) = 72 \text{ kgf cm}^{-2} \text{ (7.2 MPa)} < 0.7\,R_{pr\,II} = 0.7 \times 225 = 157 \text{ kgf cm}^{-2} \text{ (15.7 MPa)}$$

Equation (VI.60) gives the forces in the concrete

$$N_c = 0.5\ \sigma_c x b = 0.5 \times 72 \times 8 \times 100 = 28\,800 \text{ kgf (288 kN)}$$

$$N_{c,\,ten} = (h - x)\, bR_{ten\,II} = (12 - 8) \times 100 \times 18$$
$$= 7\,200 \text{ kgf (72 kN)}$$

Now we check to see if the cracking resistance of the member meets the condition in Eq. (VI.63), bearing in mind that $e = 0$ and $N'_{pr} = 0$:

$$M \leqslant N_c\,(h - a - x/3) + N_{c,\,ten}\,[a - 0.5\,(h - x)]$$

On substituting the numerical values, we get

180 000 kgf cm (18 kN cm) $<$ 28 800 (12 — 2.2 — 8/3)

+ 7 200 [2.2 — 0.5 (12 — 8)] = 206 900 kgf cm (20.69 kN cm)

As is seen, the slab has an adequate cracking resistance.

In bending members subjected to $M$ and $Q$, inclined cracks may occur at the centroid level or, in T-section

Table VI.3. Coefficients $m_1$ and $m_2$

| Concrete (heavy) | $m_1$ | $m_2$ |
|---|---|---|
| M-400 and lower | 0.5 | 2 |
| M-500 | 0.375 | 1.6 |
| M-600 | 0.25 | 1.33 |
| M-700 | 0.125 | 1.14 |

members, near the joint between the rib and the compressive flange. Inclined cracks owe their origin to the principal tensile and compressive stresses

$$\left.\begin{matrix}\sigma_{prin,\,com}\\ \sigma_{prin,\,ten}\end{matrix}\right\} = (\sigma_x + \sigma_y)/2 \pm \sqrt{[(\sigma_x - \sigma_y)/2]^2 + \tau_{xy}^2} \qquad \text{(VI.68)}$$

where $\sigma_x$ and $\sigma_y$ are the normal stresses along and across the member, respectively; $\tau_{xy}$ is the tangential stress induced by external loads and the prestressing force.

No inclined cracking will occur, if the following conditions are satisfied

$$\left.\begin{aligned} \sigma_{prin.\,com} &\leqslant m_1 R_{pr\,II} \\ \sigma_{prin.\,ten} &\leqslant R_{ten\,II} \end{aligned}\right\} \qquad \text{(VI.69)}$$

or

$$\left.\begin{aligned} \sigma_{prin.\,com} &> m_1 R_{prII} \\ \sigma_{prin.\,ten} &\leqslant m_2 R_{tenII}\,(1-\sigma_{prin.\,com}/R_{prII}) \end{aligned}\right\} \qquad \text{(VI.70)}$$

where $m_1$ and $m_2$ are taken from Table VI.3.

## VI.6. Sag Analysis

**Determination of Sag.** In the general case, the equation for the sag of a bending member may be written as

$$y(x) = \int\int M(x)\,(1/\rho)\,(x)\,dx^2 + C_1 x + C_2 \qquad \text{(VI.71)}$$

where $M(x)$ is the equation for bending moments and $(1/\rho)(x)$ is the term defining the curvature of the deflected axis of the member in bending.

In reinforced concrete members, the curvature of the deflected axis is a function of not only the axial variable $x$, but also of the stress-strain state of the member. For members with the bending moment curve entirely positive or entirely negative, the sag may be found, using equations usually given in a course in the strength of materials, recalling that

$$y = (1/B)\int\int M(x)\,dx^2 + C_1 x + C_2 \qquad \text{(VI.72)}$$

Here, the rigidity $EI$, that remains constant during loading, has been replaced by the flexural rigidity $B$ which varies according to a given stress-strain state.

**Curvature of the Deflected Axis in Members Without Normal Cracks in the Tension Zone.** The curvature of the deflected axis in bending members free from normal cracks in the tension zone is determined for prestressed members which are to meet the cracking resistance requirements of the first and second categories

The curvature of the deflected axis, $1/\rho$, is defined as

$$1/\rho = 1/\rho_s + 1/\rho_l - 1/\rho_h - 1/\rho_{creep} \qquad \text{(VI.73)}$$

Here, $1/\rho_s$ and $1/\rho_l$ are the curvatures of the deflected axis due to (1) design short-time live loads and (2) dead plus long-time live loads given by

$$1/\rho_i = \overline{M}c/0.85E_cJ_{tr} \qquad \text{(VI.74)}$$

where $\overline{M}$ is the bending moment due to the respective loads, and $c$ is the factor accounting for the increase in curvature due to sustained creep in the concrete, taken as unity for short-time live loads and as 2 for dead and long-time live loads (at a relative humidity of over 40%). The factor 0.85 accounts for the increase in curvature due to short-time creep in the concrete.

The curvature of the deflected axis, $1/\rho_h$, caused by the short-time action of the prestressing force $N_0$, is

$$1/\rho_h = N_0e_0/0.85E_0J_{tr} \qquad \text{(VI.75)}$$

The curvature due to shrinkage and creep in the prestressed concrete is

$$1/\rho_{creep} = (1/E_sh_0)\,(\sigma_{loss} - \sigma'_{loss}) \qquad \text{(VI.76)}$$

where $\sigma_{loss}$ and $\sigma'_{loss}$ are the stresses numerically equal to the sum of the losses in the prestress caused by shrinkage and creep in the concrete, that is, $\sigma_6 + \sigma_8 + \sigma_9$ (see Sec. III.2), calculated at the centroid of the tensile longitudinal steel and at the compressive face of the concrete, respectively.

If normal cracks in the tension zone remain closed under load the curvatures $1/\rho_s$, $1/\rho_h$, and $1/\rho_l$ are increased by 20%.

**Curvature of the Deflected Axis in Members Having Normal Cracks in the Tension Zone.** The curvature of the deflected axis in bending members having normal cracks in the tension zone is determined for nonprestressed members and prestressed members which are to meet the cracking resistance requirements of the third category. The stress-strain state of such members is illustrated in Fig. VI.22.

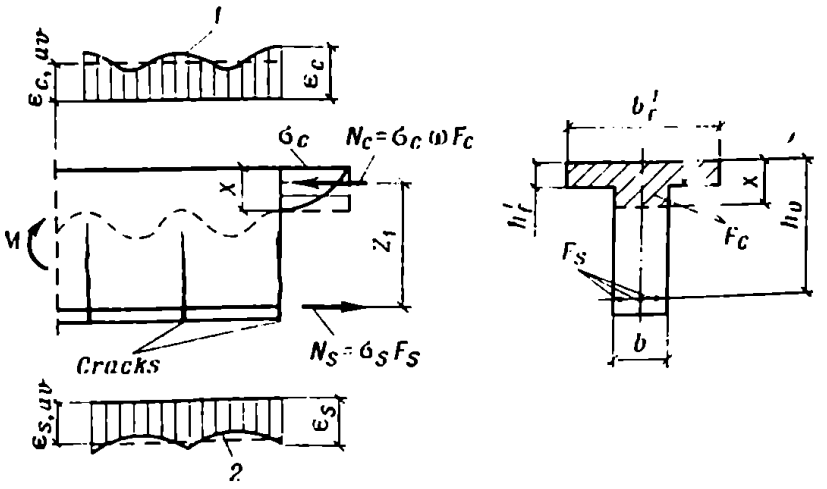



Fig. VI.22. Stress-strain state of a bending member having normal cracks in the tension zone
*1*—marginal strain in concrete of compression zone; *2*—strain in tensile steel

tensile steel and the compressive face of the concrete may be written as

$$\varepsilon_{s,\,av} = \psi_s \varepsilon_s = \psi_s \sigma_s / E_s = \psi_s M / E_s F_s z_1 \qquad \text{(VI.77)}$$

$$\varepsilon_{c,\,av} = \psi_c \varepsilon_c = \psi_c \sigma_c / \nu E_c = \psi_c M / \nu E_c F_c z_1 \qquad \text{(VI.78)}$$

where $\psi_s$ is the ratio of the average strain in the steel to the maximum strength at a crack, characterizing the behaviour of the tensile concrete between cracks; $\psi_c$ accounts for variations in the distribution of strain at the compressive face of the concrete between cracks (for heavy concrete, $\psi_c = 0.9$); $\nu$ characterizes the elasto-plastic state of the concrete in the compressive zone (for short-time live loads, $\nu = 0.45$, and for long-time live loads, $\nu = 0.15$).

The curvature of the deflected axis of the member shown in Fig. VI.23 can be deduced from the similarity of the triangles *OAB* and *CDE*:

$$1/\rho = (\varepsilon_{s,\,av} + \varepsilon_{c,\,av})/h_0 \qquad \text{(VI.79)}$$

Taking into consideration the prestress and recalling Eqs. (VI.77) and (VI.78), we may re-write the above expres-

sion as

$$1/\rho = (M_{eq}/h_0 z_1)\,[\psi_s/E_s F_s + \psi_c/(\gamma' + \xi)\,bh\nu E_c] - N_0\psi_s/h_0 E_s F_s \qquad \text{(VI.80)}$$

Here, $M_{eq}$ is the equivalent moment induced by all external forces (including the prestressing force $N_0$) on one side of the section, and taken about the axis which is normal to the plane of flexure and passes through the centroid of the tensile steel. The arm of the internal couple $z_1$ at the cracked section is given by

$$z_1 = h_0\,[1 - (\gamma' h_f'/h_0 + \xi^2)/2\,(\gamma' + \xi)] \qquad \text{(VI.81)}$$

$$\gamma' = [(b_f' - b)\,h_f' + F_s' n'/2\nu]/bh_0 \qquad \text{(VI.82)}$$

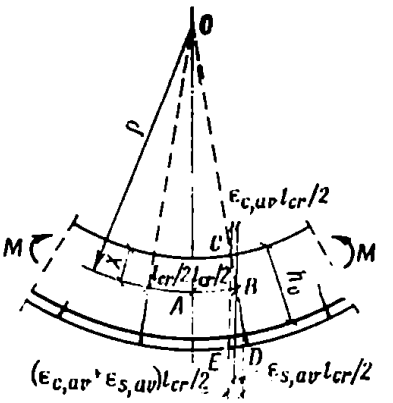



Fig. VI.23. Strain of a bending member after the advent of cracking

According to a relevant standard, the relative depth of the compression zone at a section containing a normal crack is found by an empirical formula

$$\xi = x/h_0 = 1/\{1.8 + [1 + 5\,(L + T)]/10\,\mu n\} \qquad \text{(VI.83)}$$

where

$$\left.\begin{aligned} L &= M_{eq}/bh_0^2 R_{pr\,II} \\ T &= \gamma'\,(1 - h_f'/2h_0) \end{aligned}\right\} \qquad \text{(VI.84)}$$

For heavy concrete structures,

$$\psi_s = 1.25 - sm - (1 - m^2)/[(3.5 - 1.8\,m)\,e_{s,\,av}/h_0] \leqslant 1 \qquad \text{(VI.85)}$$

Here, for bending members

$$m = R_{ten\,II} W_{cr}/(M_{cx}^k - M_{pr}^k) \leqslant 1 \qquad \text{(VI.86)}$$

where $M_{cx}^k = M$ (for bending moments);

$$M_{pr}^k = N_0\,(e_{0,\,pr} + r_k) \qquad \text{(VI.87)}$$

where $e_{0,\,pr}$ is as explained in Fig. VI.21.

The eccentricity $e_{s,\,av}$ of the prestressing force $N_0$ about the centroid of the tensile steel and the ratio $e_{s,\,av}/h_0$ are given by

$$\begin{aligned} e_{s,\,av} &= M_{cq}/N_0 \\ e_{s,\,av}/h_0 &= M_{cq}/N_0 h_0 \geqslant 1.2/s \end{aligned} \qquad \text{(VI.88)}$$

In Eqs. (VI.85) and (VI.88), $s = 1.1$ and 1 for short-time live loads in the case of deformed bars and wire respectively, and $s = 0.8$ for long-time live loads irrespective of the type of reinforcement.

Equation (VI.80) yields the flexural rigidity of a bending member

$$B = h_0 z_1 E_s F_s / [\psi_s\,(1 - z_1/e_{s,\,av}) + \psi_c r_{s,\,c}] \qquad \text{(VI.89)}$$

where

$$r_{s,\,c} = E_s F_s / (\gamma' + \xi)\, b h_0 E_c \nu \qquad \text{(VI.89a)}$$

**Limit of sag.** The sag of reinforced concrete members is found by taking loads multiplied by the overload factor $n = 1$. It should not exceed the limit established by relevant standards. The limit of sag for basic structures is as follows:

| Structure | Limit of sag |
|---|---|
| Flat-slab floors with spans: | |
| less than 6 m | $l/200$ |
| 6 to 7·5 m | 3 cm |
| over 7·5 m | $l/300$ |
| Beam-and-slab floors and stairs with spans: | |
| less than 5 m | $l/200$ |
| 5 to 10 m | 2.5 cm |
| over 10 m | $l/400$ |

## VI.7. Crack-Opening Analysis

The width of normal cracks, $a_{cr}$ (mm), is found from Eq. (V.13) where $k = 1$ and $\sigma_s$, the stress in the outer row of tensile reinforcement (or the increment in stress in the same reinforcement due to external loads, when prestressed members are considered), is

$$\sigma_s = [M - N_0\,(z_1 - e_{s,\,pr})]/F_s z_1 \qquad \text{(VI.90)}$$

Here, $z_1$ is given by Eq. (VI.78) and $e_{s,pr}$ is explained in Fig. VI.21.

The value of $\sigma_s$ should not exceed $R_{s\,II}$ for bars and $0.8\,R_{s\,II}$ for reinforcing wire.

Within portions subjected to both $M$ and $Q$, the width of inclined cracks, $a_{cr}$ (mm) (for members reinforced with transverse steel) should be calculated as

$$a_{cr} = c_d k\,(h_0 + 30\,d_{\max})\,\eta t^2/\mu_{tr}E_s^2 \qquad \text{(VI.91)}$$

where $c_d$ and $\eta$ are the same as in Eq. (V.13), $d_{\max}$ is the maximum diameter of stirrups and bent-up bars in mm,

$$k = (20 - 1\,200\,\mu_{tr}) \times 10^3 \geqslant 8 \times 10^3 \qquad \text{(VI.92)}$$

$$\mu_{tr} = \mu_{st} + \mu_b = F_{st}/bu + F_b/bu_b \qquad \text{(VI.93)}$$

$$t = Q/bh_0 - 0.25N_0/F \qquad \text{(VI.94)}$$

In Eq. (VI.94), $Q$ is the maximum shearing force within a given portion having a constant amount of transverse steel. This analysis should be carried out for sections within at least a distance $h_0$ of the support.

CHAPTER SEVEN

# REINFORCED CONCRETE MEMBERS IN ECCENTRICAL COMPRESSION AND TENSION

## VII.1. Constructional Features of Members in Eccentrical Compression

Members subjected to the simultaneous action of a longitudinal compressive force $N$ and a bending moment $M$ are said to be in eccentrical compression. Eccentrical compression is induced in the columns of one-storey industrial buildings subjected to crane loads (Fig. VII.1*a*), the top cords of trusses without diagonal web members (Fig. VII.1*b*), and the walls of underground tanks rectangular in plan, which carry the lateral pressure of earth and liquids and the vertical pressure of the roof (Fig. VII.1*c*).

The distance between the point of application of a direct load to a member and its longitudinal axis is termed the eccentricity, $e_0^*$. In the general case, the eccentricity at any point of a member of statically determinate structures is given by

$$e_0 = M/N + e_0^a \qquad \text{(VII.1)}$$

where $e_0^a$ is the accidental eccentricity (see Sec. IV.1).

For the members of statically indeterminate structures, it is found by assuming the structures to be statically determinate, but it should be not less than $e_0^a$.

It is advisable to proportion eccentrically loaded members so that their larger dimensions lie in the plane of the applied bending moment. They may be rectangular, $I$- or $T$-shaped in cross section.

Members in eccentrical compression are reinforced with longitudinal load-bearing bars (installed along the short sides of the cross section), for which the cross-sectional area is found by design calculations, and with transverse bars (or stirrups) installed from engineering considerations according to relevant specifications. The longitudinal and trans-

* Sometimes, this is called the arm.—Translator's note.

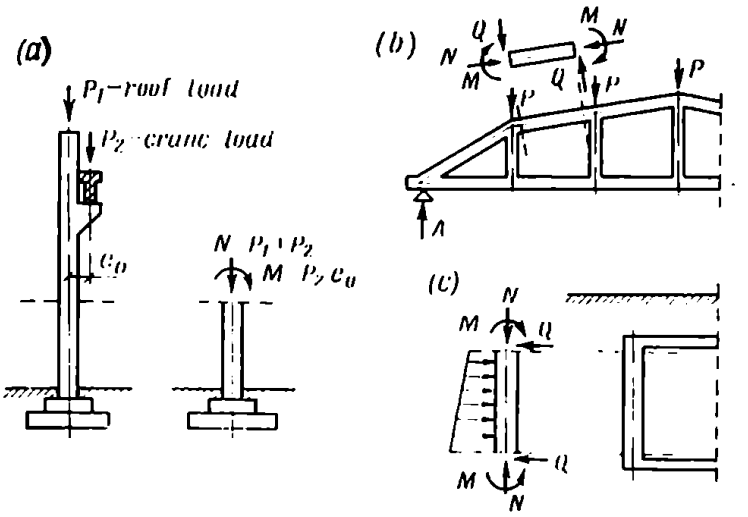



Fig. VII.1. Members in eccentrical compression
(*a*) column of industrial building; (*b*) top chord of truss without diagonals; (*c*) wall of buried tank

verse steel is combined into bar mats of reinforcing cages which may be either welded or (though seldom) tied.

The optimal steel area at either side of the cross section ranges between 0.5% and 1.2% of the cross-sectional area of the member.

According to appropriate standards, the minimum steel area at either side of the cross section for members in compression should be as follows:

0.05 for $l_0/r_g$ under 17
0.1 for $l_0/r_g$ ranging between 17 and 35
0.2 for $l_0/r_g$ ranging between 30 and 83
0.25 for $l_0/r_g$ over 83

Here $l_0/r_g$ is the slenderness ratio of a member, $r_g$ is the radius of gyration of the cross section in the plane of the longitudinal force eccentricity, and $l_0$ is the effective height of the compression member.

Members in eccentrical compression should be reinforced in compliance with general requirements for reinforced concrete structures (see Sec. III.3) and also the requirements for reinforced concrete structures subjected to axial com-

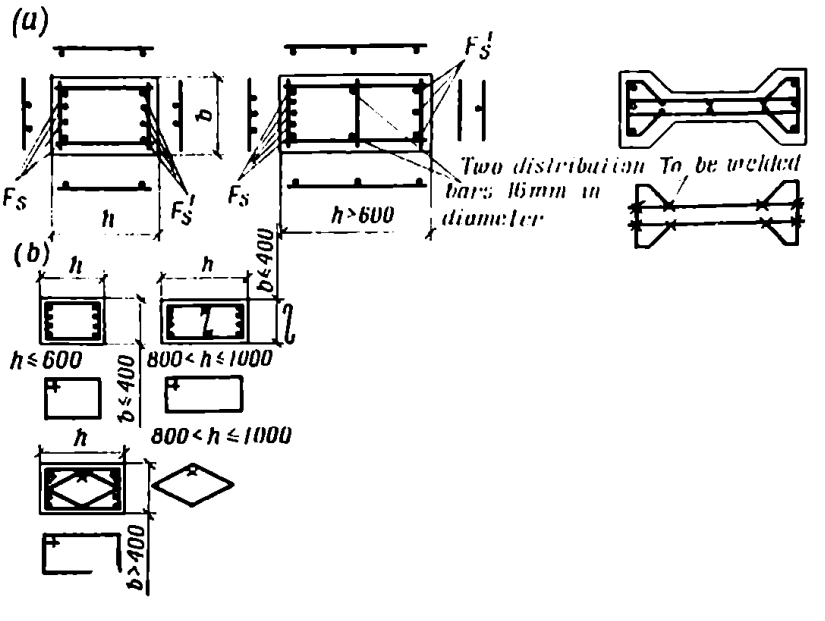



Fig. VII.2. Reinforcement of members in eccentrical compression (*a*) welded-bar mats; (*b*) separate bars and stirrups

pression (see Sec. IV.1) or bending members (see Sec. VI.1), depending on the amount of eccentricity.

The placement of reinforcement in eccentrically loaded members is shown in Fig. VII.2. The amount of load-bearing reinforcement at one side of the cross section may differ from that at the other, in which case we speak of nonsymmetric reinforcement. However, symmetric reinforcement is preferable, provided that it does not involve a prohibitive increase in the amount of steel used.

Welded reinforcing cages are assembled from flat bar mats. Tied reinforcing cages are made of separate longitudinal bars and closed stirrups.

If bending moments are too high and might cause a certain part of the cross section to work in tension, as may be the case with eccentrically loaded members subjected to forces which are applied with a considerable amount of eccentricity, or very slender members, resort is made to prestressing. It improves the cracking resistance and rigidity in service in the former case and the same characteristics during manufacture, transit and erection in the latter.

## VII.2. Design of Members in Eccentrical Compression

**Design Cases.** According to the state of stress and the manner of failure, two cases of eccentrical compression may be considered.

*Case 1.* This applies to eccentrically loaded members to which a longitudinal force is applied with a relatively large amount of eccentricity. The stress-strain state (as well as the failure) of the member is close in nature to that of members in bending (Fig. VII.3 and VII.4*a*). The part of the section most distant from the point at which the force is applied is in tension and contains cracks normal to the longitudinal axis of the member. The tensile force in this zone is carried by the reinforcing steel. The part of the section closer to the compressive force is in compression together with the steel. The member begins to fail as soon as the stress in the tensile steel reaches the yield point (or the proof yield stress). The failure is complete when the stresses in the concrete and the steel of the compression zone reach their ultimate strengths, whereas the stress in the tensile steel remains constant, if the latter has a definite yield point, or rises, if it has no definite yield point. The failure is gradual.

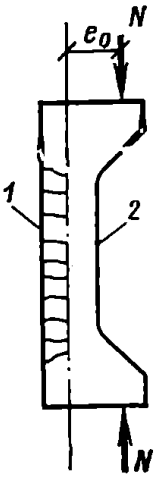



Fig. VII.3. Reinforced concrete specimen in eccentrical compression (the compressive force is applied with a large eccentricity)
*1*—tension zone; *2*—compression zone

*Case 2.* This refers to eccentrically loaded members to which a compressive force is applied with a relatively small amount of eccentricity. This case covers two types of stress-strain behaviour. In one, all of the cross-section is in compression (the dashed curve, *1*, in Fig. VII.4*b*); in the other, most of the section closer to the longitudinal force is in compression and the opposite part is exposed to a relatively weak tension (curve *2* in Fig. VII.4*b*). The member fails

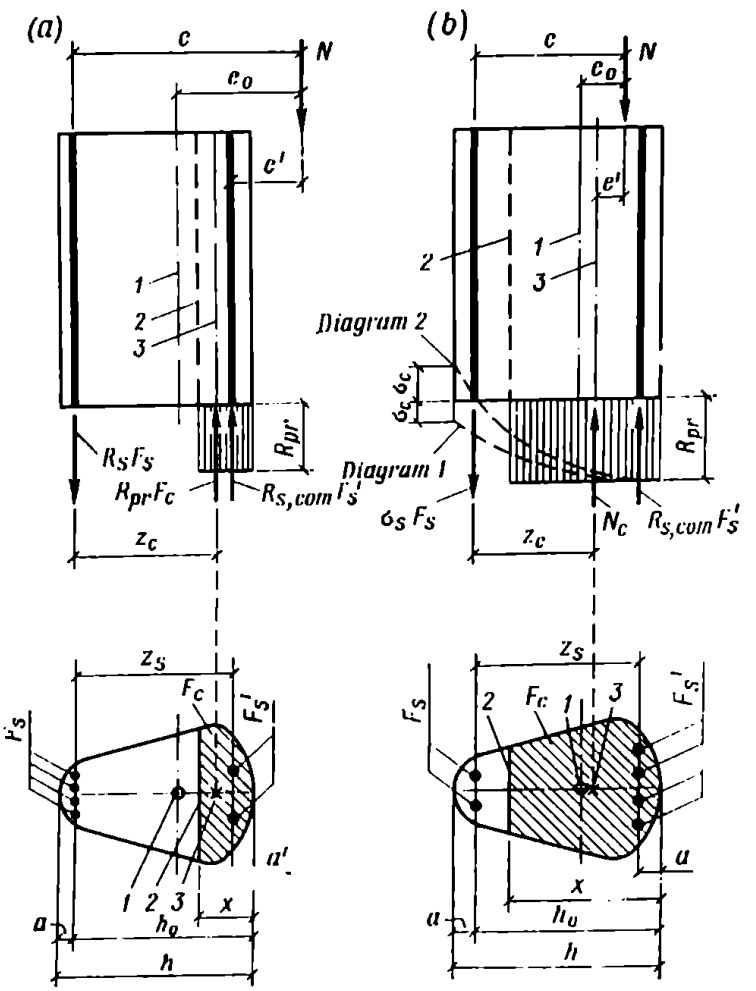



Fig. VII.4. Loading systems for members in eccentrical compression (*a*) Case *1*; (*b*) Case *2*; *1*—geometrical axis of member in design loading system; *2*—neutral axis; *3*—centroid of concrete in compression zone; $F_s$—steel more distant from compressive force; $F'_s$—steel closer to compressive force

as the stress in it exceeds the ultimate strength of the concrete and steel in the part of the section closer to the applied force. As this takes place, the stress (compressive or tensile) in the distant part of the section remains low, and the material does not act at its full strength.

**Members of Arbitrary Symmetric Cross Section.** Figure VII.4 shows the loading diagrams for the strength analysis

of members having an arbitrary cross section with an eccentricity in the plane of symmetry.

For members falling in Case 1, the design strength of the concrete in the compression zone is taken to be constant and equal to $R_{pr}$, and that in the tensile and compressive steel is taken equal to $R_s$ and $R_{s,\,com}$, respectively. For members falling in Case 2, the actual compressive stress curve shown dashed in Fig. VII.4*b* is replaced by a rectangular shape with the ordinate equal to $R_{pr}$ and the design strength of the compressive steel, $F'_s$, is taken to be $R_{s,\,com}$. The stress $\sigma_s$ in the steel, $F_s$, is below the design value.

The loading diagram in Fig. VII.4*a* is valid when $\xi = x/h_0 \leqslant \xi_R$ and that in Fig. VII.4*b*, when $\xi = x/h_0 > \xi_R$, where $\xi_R$ is the limiting relative depth of the compression zone, defined by Eq. (VI.5).

When $\xi = x/h_0 \leqslant \xi_R$ (see Fig. 4*a*), the neutral axis is located by equating the design longitudinal force due to the external design loads $N$, to the sum of the projections of all internal design forces in the steel and the concrete in compression on the longitudinal axis of the member

$$N = R_{pr}F_c + R_{s,\,com}F'_s - R_sF_s \qquad \text{(VII.2)}$$

The load-bearing capacity of a member is said to be adequate if the bending moment $M = Ne$ due to the external design loads does not exceed the sum of the moments caused by the above internal forces and taken about an axis which is normal to the plane of the bending moment and passes through the point where the tensile resultant is applied to the steel of area $F_s$

$$Ne \leqslant R_{pr}F_cz_c + R_{s,\,com}F'_sz_s \qquad \text{(VII.3)}$$

where

$$z_s = h_0 - a' \qquad \text{(VII.4)}$$

In Fig. VII.4*a*, $e$ and $e'$ are the distances between the longitudinal force $N$ and the centroids of the tensile steel $F_s$ and the compressive steel $F'_s$, respectively.

When $\xi = x/h_0 > \xi_R$ (Fig. VII.4*b*), the strength of members in eccentrical compression is calculated by Eq. (VII.3), and the depth of the compression zone is found from

$$N = R_{pr}F_c + R_{s,\,com}F'_s - \sigma_sF_s \qquad \text{(VII.5)}$$

Here, the stress in the less stressed reinforcing steel, $\sigma_s$, is specified according to the materials used. As an example, for members made of M-400 (or lower-brand) concrete and class A-I, A-II, or A-III nonprestressed steel, the value of $\sigma_s$ is given by

$$\sigma_s = [2\,(1 - x/h_0)/(1 - \xi_R) - 1]\,R_s \qquad \text{(VII.6)}$$

in which, as experiments have shown, the stress in the steel may be assumed to be linearly related to $\xi$ in the range from $\xi = x/h_0 = 1$ to $\xi = \xi_R$. For members made of M-400 (and higher brand) concrete and higher than class A-III reinforcement, $\sigma_s$ is found from the empirical expression

$$\sigma_s = 4\,000\,(\xi_0/\xi - 1)\,/(1 - \xi_0 h_0/h) \qquad \text{(VII.7)}$$

where $\xi_0$ is given by Eq. (VI.6).

The value of $\sigma_s$ given by Eqs. (VII.6) and (VII.7) is taken with the sign obtained; it ranges between $R_s$ and $-R_{s,\,com}$, both multiplied by an appropriate service factor, except $m_{s4}$ [see Eq. (VI.4)].

For high-strength reinforcing steel with no definite yield, Eq. (VII.7) holds up to $\sigma_s \approx 0.8\,R_s$. So, if $\sigma_s$ as calculated by Eq. (VII.7) exceeds $0.8R_s$, the value of $\sigma_s$ is found by linear interpolation between $0.8R_s$ and $R_s$, according to the following equation

$$\sigma_s = [0.8 + 0.2\,(\xi_{el} - \xi)/(\xi_{el} - \xi_R)]\,R_s \qquad \text{(VII.8)}$$

Here, $\xi_{el}$ is calculated by Eq. (VI.5) for $\sigma_s = 0.8R_s$. If $\sigma_s > R_s$, then $R_s$ is multiplied by $m_{s4}$ given by Eq. (VI.4). For members with $l_0/r > 35$, one neglects $m_{s4}$.

A bending moment causes a slender member to buckle, thereby increasing the initial eccentricity $e_0$ of the longitudinal force $N$ (Fig. VII.5). As a consequence, the bending moment increases, too, and the member fails at a lower longitudinal force than does a short (stub) column.

If the slenderness ratio is $l_0/r > 14$, slender members subjected to eccentrical compression may be designed by the above expressions, but with allowance for the increased eccentricity which is obtained by multiplying the initial eccentricity $e_0$ by $\eta > 1$. The latter is found from

$$\eta = 1/(1 - N/N_{cr}) \qquad \text{(VII.9)}$$

where

$$N_{cr} = (6.4E_c/l_0^2) \times \\ \times \{J\,[0.11/(0.1 + t/k_{pr}) + 0.1]/k_l + J_{s,tr}\} \quad \text{(VII.10)}$$

The above equation takes into account the special properties of reinforced concrete, namely the presence of reinforcing steel in the cross section, inelastic properties of concrete in compression, cracks in the tension zone, and the effect of long-time loading on the rigidity of a member in the limit state.

In Eq. (VII.10), $l_0$ is the effective length of the member, $J$ is the moment of inertia of the concrete section, $J_{s,\,tr}$ is the transformed moment of inertia of the steel about the centroid of the concrete section, $k_l$ is the coefficient allowing for the effect that long-time loading has on the buckling of the member in the limit state, and $k_{pr}$ accounts for the effect of the prestress in the steel on the rigidity of the member in a limit state (the steel is assumed to be uniformly prestressed). The values of $k_l$ and $k_{pr}$ are found from the following empirical relations

$$k_l = 1 + M_1^l/M_1 \quad \text{(VII.11)}$$

$$k_{pr} = 1 + 40\sigma_{c,\,pr}e_0/R_{pr\,II}h \quad \text{(VII.12)}$$

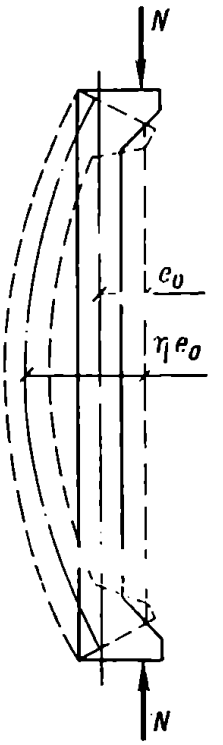



Fig. VII.5. Eccentricity due to buckling

In Eq. (VII.11), $M_1$ and $M_1^l$ are the moments taken about the most tensioned or least tensioned side of the cross section (the latter situation applies to when the cross section is entirely in compression). They are due to the combination of all loads and the dead and long-time live loads, respectively.

In Eq. (VII.12), $\sigma_{c,\,pr}$ is the prestress in the concrete found with allowance for all losses.

In Eq. (VII.10), $t$ is taken as

$$t = e_0/h \tag{VII.13}$$

but not smaller than that calculated from the empirical formula

$$t_{\min} = 0.5 - 0.01 l_0/h - 0.001\, R_{pr} \tag{VII.14}$$

where $R_{pr}$ is in kgf cm$^{-2}$.

For $l_0/r < 14$, $\eta$ is taken equal to unity. If $N$ turns out to exceed $N_{cr}$, the cross-sectional dimensions should be increased.

The transverse reinforcement in members subjected to eccentrical compression is designed to resist shearing forces, using formulas for members in bending (see Sec. VI.4).

**Rectangular Members.** For the rectangular section in Fig. VII.6,

$$\begin{aligned} F_c &= bx \\ N_c &= R_{pr} bx \\ z_c &= h_0 - x/2 \end{aligned} \tag{VII.15}$$

Taking into consideration the above expressions, we may re-write Eq. (VII.3) for the load-bearing capacity as follows

$$Ne \leqslant R_{pr} bx\,(h_0 - 0.5x) + R_{s,\,com} F'_s\,(h_0 - a') \tag{VII.16}$$

The depth of the compression zone is given by any one of the following equations

(a) for $\xi = x/h_0 \leqslant \xi_R$

$$N = R_{pr} bx + R_{s,\,com} F'_s - R_s F_s \tag{VII.17}$$

(b) for $\xi = x/h_0 > \xi_R$

$$N = R_{pr} bx + R_{s,\,com} F'_s - \sigma_s F_s \tag{VII.18}$$

where $\sigma_s$ is determined from Eq. (VII.6) or (VII.7), depending on the materials used.

*Load-bearing capacity check.* In checking a member for load-bearing capacity when its characteristics are known, the depth of the compression zone is found from Eq. (VII.17) on the assumption that $\xi = x/h_0 \leqslant \xi_R$

$$x = (N - R_{s,\,com} F'_s + R_s F_s)/R_{pr} b \tag{VII.19}$$

Then $\xi_R$ is found from Eq. (VI.5). After that, the condition $x \leqslant \xi_R h_0$ is checked; if it is met, the member is checked for load-bearing capacity by Eq. (VII.16) for the found value of $x$. If the condition $\xi = x/h_0 \leqslant \xi_R$ is not satisfied, $x$ should be determined from Eq. (VII.18) for $\xi = x/h_0 > \xi_R$

When $x > \xi_R h_0$ and use is made of concrete with a brand number of not higher than M-400 and class A-I, A-II, or A-III reinforcement, $\sigma_s$ as found by Eq. (VII.6) should be substituted into Eq. (VII.18), whence

$$x = (N + C - R_s F_s - R_{s,\,com} F'_s)/(R_{pr} b + C/h_0) \quad \text{(VII.20)}$$

where

$$C = 2R_s F_s/(1 - \xi_R) \quad \text{(VII.21)}$$

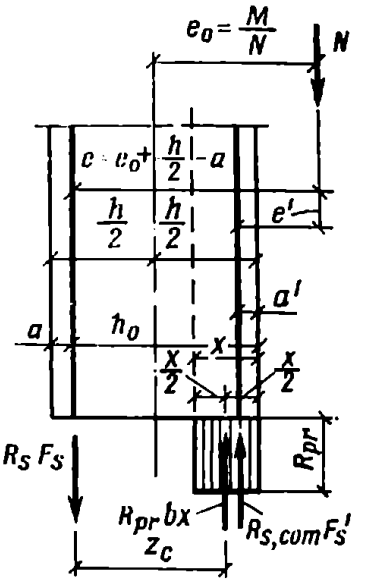
The value of $x$ as found from the above expression should be substituted into Eq. (VII.16) so as to check the member for load-bearing capacity.

If $x > \xi_R h_0$ and use is made of a concrete stronger than M-400 and class A-IV and higher reinforcing steel, $\sigma_s$ as found by Eq. (VII.7) should be substituted into Eq. (VII.18), whence the following quadratic equation results:

$$\xi^2 - (N - R_{s,\,com} F'_s - DF_s)\,\xi/R_{pr} b h_0 - DF_s \xi_0/R_{pr} b h_0 = 0 \quad \text{(VII.22)}$$

where

$$D = 4\,000/(1 - \xi_0 h_0/h) \quad \text{(VII.23)}$$



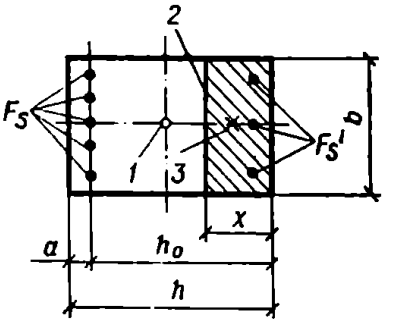




Fig. VII.6. To the design of rectangular members in eccentrical compression

*1*—geometrical axis of member; *2*—neutral axis; *3*—centroid of concrete in compression zone.

On finding $\xi$ from Eq. (VII.22), we can calculate $x = \xi h_0$, which is finally used in Eq. (VII.16) to check the member for load-bearing capacity.

*Steel area calculation.* To calculate the necessary steel area $F_s$ and $F'_s$ when $N$, $l_0$, $b$, and $h$ are assumed to be known and the condition

$$\xi = x/h_0 \leqslant \xi_R$$

is satisfied, the design equation must be transformed as follows.

It is obvious that the steel of area $F'_s$ will be required from design considerations only if the relative depth of the compression zone as calculated with only the tensile steel of area $F_s$ present exceeds the limiting value of $\xi_R$. Using this depth of the compression zone and the respective value of $A_R$ from Table VI.1, we obtain on the basis of Eqs. (VII.16) and (VII.17)

$$F'_s = (Ne - A_R R_{pr} b h_0^2)/R_{s,\,com} z_s \qquad \text{(VII.24)}$$

$$F_s = (\xi_R R_{pr} b h_0 - N)/R_s + F'_s R_{s,\,com}/R_s \qquad \text{(VII.25)}$$

The steel area $F'_s$ should be not less than the minimum value given in Sec. VII.1. When $F'_s$ is specified in advance (from constructional or design considerations), Eq. (VII.16) yields

$$x\,(h_0 - 0.5x) = [Ne - R_{s,\,com} F'_s\,(h_0 - a')]/R_{pr} b$$

All terms on the right-hand side of the above equation are known. On the other hand, recalling Eq. (VI.15), we find that

$$A_0 = \xi\,(1 - \xi/2) \qquad \text{where} \qquad \xi = x/h_0 \qquad \text{(VII.26)}$$

is known, so

$$A_0 = [Ne - R_{s,\,com} F'_s\,(h_0 - a')]/(R_{pr} b h_0^2) \qquad \text{(VII.27)}$$

Once $A_0$ is known, we can look up the value of $\xi$ in Table VI.1. Thus, knowing $x = \xi h_0$, we can deduce the necessary steel area from Eq. (VII.17) as

$$F_s = (\xi R_{pr} b h_0 - N)/R_s + F'_s R_{s,\,com}/R_s \qquad \text{(VII.28)}$$

In practice, members, especially those subjected to bending moments differing in sign but close in value, are often

reinforced symmetrically. If $F_s = F'_s$ and $R_{s,\,com} = R_s$, that is, $R_{s,\,com}F'_s = R_sF_s$, we may use Eq. (VII.17) to get

$$x = N/R_{pr}b \qquad \text{(VII.29)}$$

Then, using this value of $x$, we get from Eq. (VII.16)

$$F_s = F'_s = N\,[e - h_0 + N/(2R_{pr}b)]/R_{s,\,com}\,(h_0 - a') \qquad \text{(VII.30)}$$

To sum up, the steel area for rectangular members with asymmetrical reinforcement should be calculated in the following sequence.

(1) Write down the design values of $R_{pr}$, $R_s$, $R_{s,\,com}$, $E_s$, and $E_c$, and calculate $h_0$, $z_s$, $e_0 = M/N$, $e_0/h$, $l_0/h$, and $n$.

(2) Assume a trial reinforcement ratio

$$\mu = (F_s + F'_s)/bh_0$$

anywhere between 0.005 and 0.03 and determine $t$, $k_l$, and $N_{cr}$ from Eqs. (VII.11) through (VII.14).

If $N_{cr} < N$, the cross-sectional area of the member should be increased.

(3) Find the value of $\eta$ from Eq. (VII.9) and the distance from the force $N$ to the steel of area $F_s$

$$e = e_0\eta + h/2 - a \qquad \text{(VII.31)}$$

where $e_0$ is given by Eq. (VII.1).

(4) Assuming the expected value of $F_s/F'_s$, find the depth of the compression zone, $x$, from Eq. (VII.19), then $\xi = x/h_0$. Calculate $F_s$ and $F'_s$ by Eqs. (VII.24) through (VII.28), but take them as not less than the minimum values specified in Sec. VII.1.

(5) Using the steel areas thus found, calculate the reinforcement ratio $\mu$. If it differs from the initial value by not more than 0.005, the problem may be taken as solved. Otherwise, go through the above steps again, assuming an improved trial reinforcement ratio.

If the design calculations yield $\mu > 0.03$, the cross-sectional dimensions $b$ and $h$ should be changed, or use should be made of other types of concrete and reinforcing steel.

(6) Check the member for strength with allowance for the effect of buckling in a plane normal to the plane of flex-

ure, treating it as a member subjected to compression with accidental eccentricities.

(7) If necessary, check the member for load-bearing capacity, using Eqs. (VII.19) through (VII.26) and the condition defined in Eq. (VII.16).

The necessary steel area is found for $\xi = x/h_0 > \xi_R$ by successive approximations.

## VII.3. Members in Eccentrical Tension

Members subjected to eccentrical tension are the walls of rectangular tanks (bins) resisting the internal pressure of the contents (Fig. VII.7*a*), the bottom chords of trusses

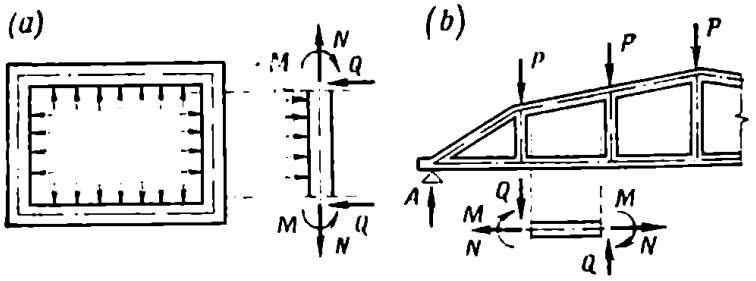



Fig. VII.7. Members in eccentrical tension
(*a*) wall of tank (bin); (*b*) bottom chord of truss without diagonals

without diagonals (Fig. VII.7*b*), and the like. Such members are subjected to both a longitudinal tensile force $N$ and a bending moment $M$, which is equivalent to the eccentrical tension that would be induced by the force $N$ if it were applied with an eccentricity $e_0 = M/N$ about the longitudinal axis of the member.

Depending on the value of $e_0$, two cases may be distinguished (Fig. VII.8).

**Case 1.** This refers to the external longitudinal tensile force $N$ applied within a distance $z_s$ (Fig. VII.8*a*). The concrete is traversed by 'through' cracks, and at a cracked section the external force is resisted by the longitudinal steel alone. The member fails when the stress in the longitudinal steel reaches its ultimate value.

**Case 2.** This refers to $N$ applied outside $z_s$. The part of the section more distant from $N$ is in compression and the opposite part in tension, as in the case of eccentrical compression with a large amount of eccentricity. After the onset of cracking in the concrete of the tension zone, all of the tensile

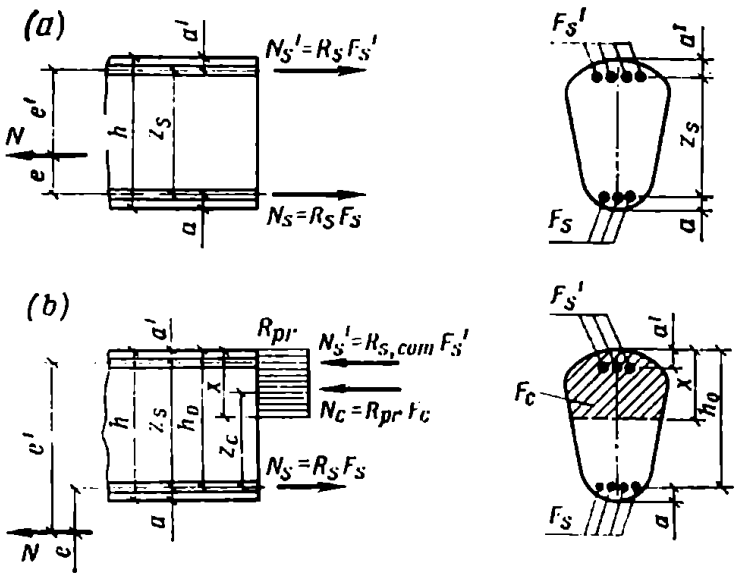



Fig. VII.8. Loading systems for members in eccentrical tension
(*a*) Case *1* (tensile force is applied within distance $z_s$); (*b*) Case *2* (tensile force is applied outside this range)

force at the cracked sections is taken up by the tensile steel alone. The member fails as soon as the stresses in the tensile steel and the compressive concrete reach their ultimate strengths. The steel in the compression zone may act at its ultimate strength if the depth of the compression zone is such that $z_c < z_s$.

Eccentrically loaded members falling under Case 2 are fabricated with longitudinal and lateral bars similarly to members in bending. Those falling under Case 1 are reinforced as members designed to carry axial tension.

It is advisable to use prestressing in eccentrically loaded members under the same conditions as for axially loaded members (Case 1) and bending members (Case 2).

The strength analysis of eccentrically loaded members of any symmetrical cross section is carried out, assuming the loading system given in Fig. VII.8.

In Case 1 (see Fig. VII.8*a*), the conditions for strength may be derived by equating the moments due to the internal and external forces about the axes normal to the plane of flexure and passing through the centroids of the steel of areas $F_s$ and $F'_s$, respectively:

$$\begin{aligned} Ne' &\leqslant R_s F_s z_s \\ Ne &\leqslant R_s F'_s z_s \end{aligned} \qquad \text{(VII.32)}$$

In Case 2 (see Fig. VII.8*b*), the condition for strength may be written in terms of the bending moment

$$Ne \leqslant R_{pr} F_c z_c + R_{s,\,com} F'_s z_s \qquad \text{(VII.33)}$$

Here, the depth of the compression zone can conveniently be found from the following expression

$$R_s F_s - R_{s,\,com} F'_s - N = R_{pr} bx \qquad \text{(VII.34)}$$

Additional data on the design of members in eccentrical tension (the effect of prestress, use of high-strength reinforcement, inclined-section strength, cracking resistance, and sag) can be found in appropriate standards and specifications.

# CHAPTER EIGHT

# MASONRY AND REINFORCED MASONRY STRUCTURES

## VIII.1. Masonry Materials and Strength

Masonry structures are assembled from natural or man-made stones bonded by mortar. According to the type of binding agent they use, mortars may be classed into cement, lime, and mixed (such as cement-lime and cement-clay mortars). Lime and clay enhance the plasticity and water-retaining capacity of mortar, thereby improving the quality of masonry. Mortars having a bulk unit weight (on a dry basis) of $\gamma \geqslant 1\,500$ kg $m^{-3}$ are called heavy mortars, and those with $\gamma < 1\,500$ kg $m^{-3}$ are known as light mortars.

In Soviet practice, the strength of stone and mortar is expressed in terms of a brand number which refers to their ultimate compressive strength in kgf $cm^{-2}$. Strength tests are carried out in compliance with relevant standards. The brand number of high-strength stone (such as natural heavy-rock stone and high-strength brick) ranges between 300 and 1 000, that of medium-strength stone (various types of brick, structural, clay tile, concrete blocks, and natural lightweight stone) ranges between 35 and 250, and that of low-strength stone (such as weak limestone and raw materials) is from 4 to 25.

For design purposes, the strength brand numbers for mortars are set at 4, 10, 25, 50, 75, 100, 150, and 200. Freshly placed mortar (or the thawed out mortar in a masonry structure erected by freezing) is assumed to have zero strength.

The frost-resistance brand number of stone ranges between Mpa 10 and MPa 300.

Appropriate standards set up minimum design brand numbers for masonry materials in terms of strength and frost resistance, according to the type of structure, exposure

conditions, and degree of durability*. For example, the stone to be used in the exterior walls of building which are to meet the requirements of the first degree of durability, and also in the protruding parts of walls (such as cornices, parapets, and the like) should have a frost-resistance of at least MPa 50, and the mortar should have a minimum strength brand number of 50.

The strength of masonry varies with age and depends on the size and strength of the stone, type and strength of the mortar, and quality of construction.

The average (expected) ultimate strength** $\overline{R}$, measured in kgf $cm^{-2}$ (MPa), for various types of masonry may be found empirically. The design strength is given by

$$R = \overline{R}/K_s$$

where $K_s$ is the safety factor taken as 2 for compression and as 2.25 for tension.

Design strength values for all types of masonry can be found in relevant specifications. This text includes only a concise table (Table VIII.1); the values are given in kgf $cm^{-2}$ (MPa).

**Table VIII.1. Design Compressive Strength of Brick Masonry Using Heavy Mortars**

| Brick brand number | Mortar brand number | | | | | Zero mortar strength |
|---|---|---|---|---|---|---|
| | 100 | 75 | 50 | 25 | 10 | |
| 150 | 22(2.2) | 20(2) | 18(1.8) | 15(1.5) | 13(1.3) | 8(0.8) |
| 100 | 18(1.8) | 17(1.7) | 15(1.5) | 13(1.3) | 10(1) | 6(0.6) |
| 75 | 15(1.5) | 14(1.4) | 13(1.3) | 11(1.1) | 9(0.9) | 5(0.5) |
| 50 | — | 11(1.1) | 10(1) | 9(0.9) | 7(0.7) | 3.5(0.35) |

The design tensile strength of masonry depends on the direction of tensile forces with respect to the joints

* Durability of the 1st degree refers to a service life of at least 100 years, durability of the 2nd degree to at least 50 years, and durability of the 3rd degree to at least 20 years.

** To masonry, the concept of characteristic strength $R_{ch}$ (see Chap. 1) is inapplicable, because its strength is neither specified by any standards, nor is subject to any tests.

(Fig. VIII.1) and also the state of stress; being $R_{ten} = 0.3$ to 2 kgf cm$^{-2}$ (0.03 to 0.2 MPa) for axial tension, and $R_{ten,\,b} = 0.4$ to 3 kgf cm$^{-2}$ (0.04 to 0.3 MPa) for tension in bending.

Sometimes, the design strength of masonry should be multiplied by a service factor $m_m$. For example, the design

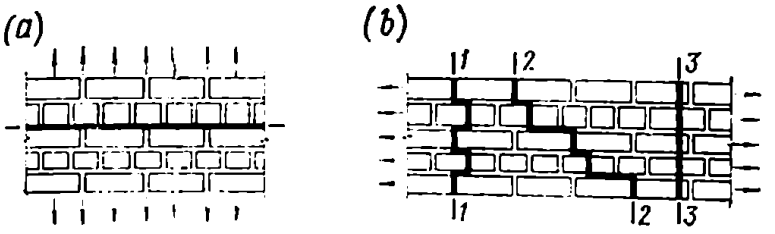



Fig. VIII.1. Brick masonry in tension
(*a*) tension across an unbroken joint; (*b*) tension across broken joints

strength of piers and partition walls smaller than 0.3 m$^2$ in cross section should be reduced by 20% ($m_m = 0.8$).

When a member of cross-sectional area $F$ is loaded over a part $F_{com}$ of its area (where it carries trusses, beams, or columns), we speak of local compression. The unloaded part of the section acts as a restraint preventing the masonry from lateral deformation within the loaded area. Therefore, the design strength over the area $F_{com}$ is increased

$$R_{cr} = \gamma R$$

where

$$\gamma = \sqrt[3]{F/F_{com}} \leqslant \gamma_1$$

If the crushing area is located nonsymmetrically, the area $F$ is reduced. The factor $\gamma_1$ depends on the type of masonry and the manner in which local loads are applied. For solid brick masonry, $\gamma_1 = 2$ if the area of crushing lies in the middle of a wall or pier, and $\gamma_1 = 1.2$ if the crushing area is near the edge of a member.

The strain of masonry under load is similar to that of concrete (see Sec. II.2):

The modulus of elasticity $E$ is given by the slope of the stress-strain ($\sigma$ — $\varepsilon$) diagram

$$E = d\sigma/d\varepsilon$$

According to an empirical relation proposed by L. I. Onishchik (USSR),

$$E = E_0 (1 - \sigma/1.1\bar{R})$$

where $E_0$ is the tangent modulus of elasticity in kgf cm$^{-2}$ (MPa).

In practical calculations, we adopt

$$E_0 = \alpha\bar{R}$$

where $\bar{R}$ is the average ultimate strength of a given masonry, taken as twice the design strength, and $\alpha$ is the elastic characteristic of the masonry, depending on the type of masonry and the brand number of the mortar used (see Table VIII.2).

**Table VIII.2. Elastic Characteristic $\alpha$**

| Type of masonry | Mortar brand number | | | Zero mortar strength |
|---|---|---|---|---|
| | 25 and over | 10 | 4 | |
| Clay brick, structural clay tile, and light-weight concrete blocks | 1 000 | 750 | 500 | 200 |
| Lime-and-sand brick | 750 | 500 | 350 | 200 |

## VIII.2. Design of Masonry Members in Compression

The load-bearing capacity of compression members in non-reinforced masonry structures depends on the strength of the masonry, member size, and eccentricity $e_0$ of the longitudinal force. The last mentioned quantity is governed by the real or accidental displacement of the force $N$ about the centroid of the member section. If a member is subjected to an axial force $N$ and a bending moment $M$, the eccentri-

city is given by

$$e_0 = M/N$$

An accidental eccentricity $e_0^{ac}$ is taken into account only in the design of load-bearing walls under 25 cm thick. It is taken as 2 cm for load-bearing walls and as 1 cm for self-bearing walls, and added to the eccentricity $e_0$.

For nonreinforced masonry subjected to basic load combinations, the eccentricity $e_0$ should not exceed $0.9y$; in walls under 25 cm thick,

$$e_0 + e_0^{ac} \leqslant 0.8y$$

where $y$ is the distance from the centroid of the section to the most compressed face (Fig. VIII.2), for a rectangular section $y = h/2$. In axial compression ($e_0 = 0$), the stress is uniformly distributed over the entire cross-sectional area of the member (Fig. VIII.2*a*). If the force is applied with a small eccentricity, the stress is distributed nonuniformly, but all of the cross section is in compression (Fig. VIII.2*b*); as the eccentricity increases, tensile stresses $\sigma_{ten}$ appear in the section (Fig. VIII.2*c*). When $\sigma_{ten}$ exceeds $R_{ten.b}$, cracks open up in the tension zone (Fig. VIII.2*d*), and only part of the section of area $F_{com}$ resists compression. For design purposes, we take the area $F_{com}$ to be symmetrical about the force $N$ and the stress ($R\omega > R$) to be uniformly distributed over this area (Fig. VIII.2*e*).

In slender compression members, we have to allow for a reduction in their load-bearing capacity due to buckling and sag build-up under long-time loads (see Chap. VII, Fig. VII.5).

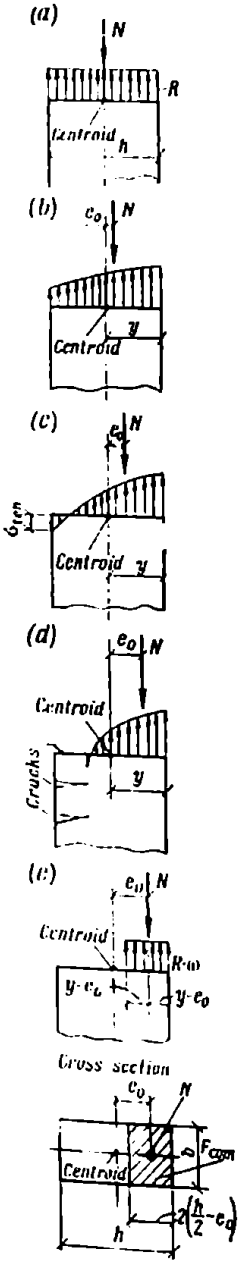



Fig. VIII.2. State of stress of compression members

From the above it follows that members in axial compression should be designed subject to

$$N \leqslant m_{l\text{-}t}\varphi FR \qquad \text{(VIII.1)}$$

where $N$ is the design longitudinal force, $F$ is the cross-sectional area of the member, $R$ is the design compression strength of the masonry, $\varphi \leqslant 1$ is the buckling coefficient, and $m_{l\text{-}t} \leqslant 1$ is a factor taking care of the effect of sag under long-time loading.

For members in eccentrical compression, the design expression takes the form

$$N \leqslant m_{l\text{-}t}\varphi_1 F_{com} R\omega \qquad \text{(VIII.2)}$$

where $N$, $m_{l\text{-}t}$, and $R$ have the same meaning as in Eq. (VIII.1), $F_{com}$ is the area in compression symmetric about the force $N$; for a rectangular section (see Fig. VIII.2*e*),

$$F_{com} = 2b\,(h/2 - e_0) \qquad \text{(VIII.3)}$$

$\omega$ is a coefficient accounting for an increase in the design strength of the masonry over the area $F_{com}$; for an arbitrary cross section,

$$\omega = 1 + e_0/3y \leqslant 1.25 \qquad \text{(VIII.4)}$$

for a rectangular cross section,

$$\omega = 1 + e_0/1.5\,h \leqslant 1.25 \qquad \text{(VIII.5)}$$

(for natural stone masonry or masonry built of stones and blocks manufactured of cellular or large-void concretes, $\omega$ is taken as 1), $\varphi_1$ is the buckling coefficient allowing for the fact that only part of the cross-sectional area resists compression.

To determine $\varphi$, $\varphi_1$, and $m_{l\text{-}t}$, it is necessary to find the slenderness ratio $\lambda$ which, as will be recalled, is defined as the ratio of the effective length of the member, $l_0$, to the radius of gyration of the cross section, $r$:

$$\lambda = l_0/r$$

The value of $l_0$ depends on how a member is fixed at the bottom. It is taken as $2H$ for free members of height $H$,

as $H$ for hinged members, and as more than or equal to $0.8H$* for partly fixed members.

The radius of gyration of a rectangular section of depth $h$ is approximately equal to $0.29h$, so the slenderness ratio may directly be expressed in terms of $l_0/h$. Accordingly, we may introduce the following notation

$$\lambda^r = l_0/r$$

and

$$\lambda^h = l_0/h$$

or

$$\lambda^h \approx 0.29\lambda^r$$

The buckling coefficient $\varphi$ depends not only on the slenderness ratio of a member, but also on the strain capacity of the masonry, expressed in terms of the elastic characteristic $\alpha$.

Table VIII.3 gives the values of $\varphi$ as a function of the reduced slenderness ratio

$$\lambda^r_{red} = (l_0/r)\sqrt{1\,000/\alpha}$$

or

$$\lambda^h_{red} = (l_0/h)\sqrt{1\,000/\alpha}$$

**Table VIII.3. Buckling Coefficient $\varphi$**

| $\lambda^h_{red}$ | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda^r_{red}$ | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 76 | 83 | 90 | 97 | 104 |
| $\varphi$ | 1 | 0.96 | 0.92 | 0.88 | 0.84 | 0.79 | 0.74 | 0.7 | 0.65 | 0.61 | 0.56 | 0.52 | 0.49 | 0.45 |

For members rectangular in cross section, the buckling coefficient $\varphi_1$ is given by

$$\varphi_1 = \varphi\,[1 - (e_0/h)\,(0.06\lambda^h - 0.2)] \qquad \text{(VIII.6)}$$

* Accordingly, the height of walls and piers in multistorey buildings of storey height $H$ is taken as $l_0 = H$. This quantity is assumed as $l_0 = 0.9H$ if the walls carry precast floors, and as $l_0 = 0.8H$ if they support in-situ floors. In one-storey industrial buildings, the effective length of walls and piers of height $H$ is taken as $1.5H$ for single-bay buildings and as $1.25H$ for multibay buildings.

For an arbitrary cross section, $\lambda^h$ in Eq. (VIII.6) is replaced by $l_0/3.5r$.

The reduction in the load-bearing capacity of compression members due to long-time loading is taken into consideration only for a small cross-sectional area: at $h < 30$ cm for members rectangular in cross section, and at $r < 8.7$ cm for members of an arbitrary cross section:

$$m_{l\text{-}t} = 1 - \eta\,(N_{l\text{-}t}/N)\,(1 + 1.2e_{0,\,l\text{-}t}/h) \quad \text{(VIII.7)}$$

where $N$ is the total design longitudinal force, $N_{l\text{-}t}$ is the design longitudinal force due to long-time loads, $e_{0,l\text{-}t}$ is the eccentricity due to long-time loads, and $\eta$ is a coefficient depending on the slenderness ratio (Table VIII.4).

At $h \geqslant 30$ cm (or $r > 8.7$ cm), $m_{l\text{-}t}$ is taken as unity.

**Table VIII.4. Coefficient $\eta$**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\lambda^h$ | Under | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 |
| $\lambda^r$ | Under | 35 | 42 | 49 | 56 | 63 | 70 | 76 | 83 | 90 |
| $\eta$ | | 0 | 0.04 | 0.08 | 0.12 | 0.15 | 0.2 | 0.24 | 0.27 | 0.31 |

## VIII.3. Reinforced Masonry Structures

Reinforced masonry structures are those congested with steel reinforcement. The steel improves the strength and stability of the masonry. The brand number of mortar in reinforced masonry structures is taken to be not less than 50 so as to protect the reinforcement against corrosion.

When the slenderness ratio is $\lambda_0^h \leqslant 15$ and longitudinal force eccentricity is $e_0 \leqslant 0.33y$, use is made of steel fabric reinforcement placed in horizontal joints (Fig. VIII.3*a* and *b*). When a member is subjected to compression, the steel fabric restrains the lateral deformation of the masonry, thereby enhancing its load-bearing capacity.

Sometimes, longitudinal reinforcement (Fig. VIII.3*d*) may be used to carry tensile forces in members working in bending or eccentrical compression with large eccentricities, and also to improve the strength and stability of thin walls. Reinforcing bars are installed either inside, in vertical

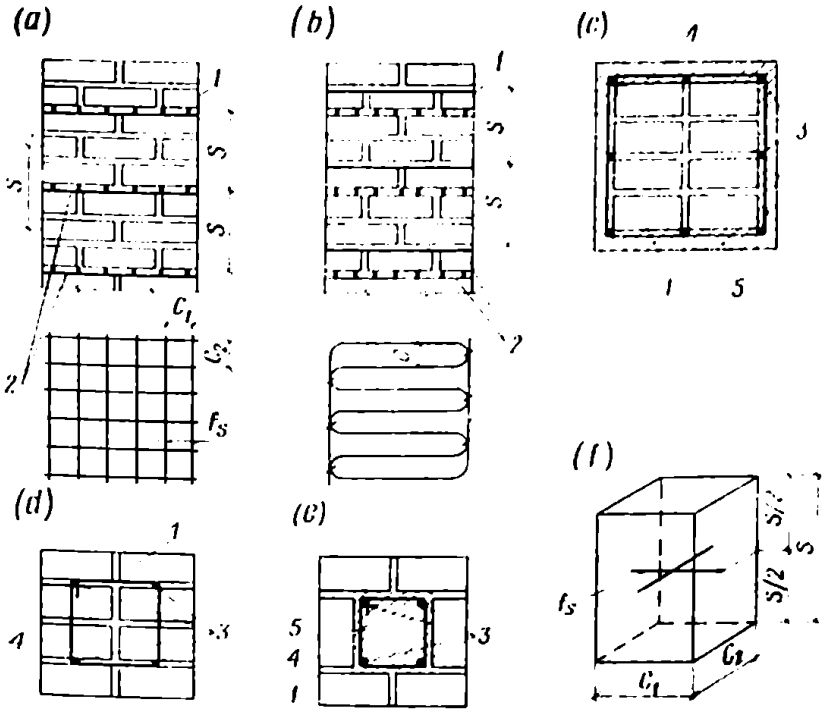



Fig. VIII.3. Reinforced masonry
*1*—masonry; *2*—steel fabric; *3*—longitudinal bars; *4*—hoops; *5*—concrete

joints, or outside, in the layer of mortar, and connected by hoops in horizontal joints.

In some cases, masonry may be strengthened with reinforced concrete (Fig. VIII.3*e*) or casing (Fig. VIII.3*c*). These and longitudinally reinforced structures are very labour consuming and have, therefore, a limited application. The state of stress of such structures is similar to that of reinforced concrete structures. The design principles for reinforced masonry structures are given in relevant standards and building codes.

As a rule, brick and structural clay tile masonry (with a course not more than 15 cm high) is reinforced with steel fabric.

In steel fabric, the wires may run in two directions (rectangular-mesh fabric), or in one direction (such as in zigzag fabric). Two pieces of zigzag fabric placed in adjacent

joints are equivalent to one piece of rectangular-mesh fabric. The wire diameter is limited by the width of joints, being from 3 to 5 mm for rectangular-mesh fabric and up to 8 mm for zigzag fabric. The spacing between the fabric wires is taken as 3 to 12 cm, and the distance between pieces of fabric should not exceed 5 courses of brick (40 cm).

The amount of fabric reinforcement is expressed in terms of the volumetric percentage of reinforcement $\mu$, defined as the ratio of the reinforcement volume to the volume of masonry

$$\mu = (V_s/V_m) \times 100$$

Referring to Fig. VIII.3*f*, the volume of masonry is $V_m = C_1C_2s$ and the volume of reinforcement is $V_s = f_1C_1 + f_2C_2$. As a rule, $C_1 = C_2 = C$ and $f_1 = f_2 = f$, so

$$\mu = (2f/Cs) \times 100 \qquad \text{(VIII.8)}$$

Relevant standards require that the minimum percentage of reinforcement should be $\mu_{min} = 0.1\%$ and the maximum percentage of reinforcement, $\mu_{max} = 1\%$.

In axial compression, the design strength of fabric-reinforced masonry is given by

$$R_{r\text{-}m} = R + 2\mu R_s/100 \leqslant 1.8R \qquad \text{(VIII.9)}$$

where $R$ is the design strength of unreinforced masonry, and $R_s$ is the design strength of the reinforcing steel, being 1 500 kgf cm$^{-2}$ (150 MPa) for class A-I bars, 2 000 kgf cm$^{-2}$ (200 MPa) for class B-I wires under 5.5 mm in diameter, and 1 800 kgf cm$^{-2}$ (180 MPa) for class B-I wires over 6 mm in diameter.

In eccentrical compression, the design strength of fabric-reinforced masonry is given by

$$R_{r\text{-}m,\,b} = R + 2\mu R_s\,(1 - 2e_0/y)/100 \qquad \text{(VIII.10)}$$

From this expression it follows that the effect of reinforcing fabric on the strength of masonry decreases with increasing longitudinal force eccentricity and reduces to zero at $e_0 = 0.5y$, that is, $R_{r\text{-}m,\,b} = R$.

The elastic characteristic of fabric-reinforced masonry is given by

$$\alpha_r = \alpha\bar{R}/\bar{R}_{r\text{-}m} = \alpha 2R/(2R + 2R'_s\mu/100) \qquad \text{(VIII.11)}$$

where $\alpha$ is the elastic characteristic of nonreinforced masonry, $\bar{R}$ is the average ultimate compressive strength of nonreinforced masonry, and $\bar{R}_{r\text{-}m}$ is the ultimate compressive strength of reinforced masonry.

When determining $\bar{R}_{r\text{-}m}$, $R'_s$ is taken as 2 400 kgf cm$^{-2}$ (240 MPa) for class A-I bars and as 3 500 kgf cm$^{-2}$ (350 MPa) for class B-I steel.

Compression members reinforced with steel fabric are designed similarly to nonreinforced members (see Sec. VIII.2), but $R$ in the design formulas is replaced by $R_{r\text{-}m}$ and the coefficient $\varphi$ (or $\varphi_1$) is determined as a function of the elastic characteristic $\alpha_r$ rather than $\alpha$.

For members in axial compression, Eq. (VIII.1) takes the form

$$N \leqslant m_{l\text{-}l}\varphi F R_{r\text{-}m} \qquad \text{(VIII.12)}$$

For members in eccentrical compression, Eq. (VIII.2) is likewise re-written as

$$N \leqslant m_{l\text{-}l}\varphi_1 F_{com} R_{r\text{-}m,\,b}\omega \qquad \text{(VIII.13)}$$

## VIII.4. Design of Masonry Structures

Any building is a space structure whose members (walls, piers, floors, etc.) work together under load.

The horizontal reactions appearing in loaded walls and piers are transferred to floors resting on transverse walls or other structures stable in the transverse direction, and working in in-plane bending.

If the distance between the transverse walls is small, being within the limits set up by relevant standards*, the floors are regarded as immovable horizontal supports for the walls and piers. In this case, the building is said to be a rigid structural system. All multistorey buildings should be rigid structural systems.

---

* The maximum distance between transverse walls or other stable transverse structures depends on the type of masonry, type of floor, building height, and wind velocity (dynamic) head. In buildings having brick walls and reinforced concrete floors, it may be as large as 54 m.

One-storey industrial buildings lacking transverse walls over a considerable length or equipped with a roof of low in-plane rigidity (for example, a roof assembled from asbestos-cement slabs installed on beams) form elastic structural systems in which the walls and piers have a movable support at the top.

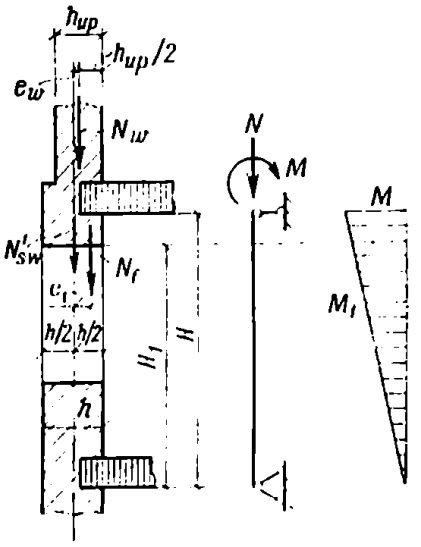



Fig. VIII.4. To the design of a masonry wall of a multistorey building of the rigid structural type

The thickness of walls in buildings is adopted so as to meet appropriate heating and strength requirements. The ratio of the wall (pier) height $H$ to its thickness $h$ should not exceed the limiting value $\beta$ specified in relevant standards. As an example, for a load-bearing wall having no openings and made of brick of brand 50 and more and mortar of brand 25, $\beta$ should be not larger than 22; for brick piers made of the same materials, the ratio is reduced by 25% to 40% depending on the cross-sectional area of the pier.

When they are designed to carry vertical loads, the walls of a multistorey building are regarded within each storey as vertical single-span beams with hinged supports at floor levels (Fig. VIII.4). Each wall is subjected to a vertical load $N_f$ due to the floor located above the storey in question, a vertical load $N_w$ due to the weight of the upper part of the wall and that of all the upper floors, and also the self-weight $N_{s.w}$.

The load $N_f$ is usually applied with an eccentricity $e_1$ about the centroid of the wall cross section and induces a moment

$$M_f = N_f e_1$$

The load $N_w$ is applied at the upper storey at the centroid of the wall cross section. If the wall has a constant thickness, the load $N_w$ produces no moment; otherwise, it has an

eccentricity

$$e_w = \pm(h - h_{up})/2$$

where $h$ is the wall thickness on the floor in question and $h_{up}$ is the wall thickness on the upper floor, and induces the following bending moment

$$M_w = \pm N_w e_w$$

The total moment at the floor bottom level is given by

$$M = M_f \pm M_w$$

The design section is chosen to be at the top of the partition between windows. At this section, the bending moment is somewhat smaller than $M$ (see Fig. VIII.4):

$$M_1 = MH_1/H$$

The normal force is given by

$$N = N_f + N_w + N'_{s\text{-}w}$$

where $N'_{s\text{-}w}$ is the weight of the wall as reckoned from the bottom of the floor to the top of the partition.

The eccentricity of the force $N$ is

$$e_0 = M_1/N$$

The partition is designed to resist eccentrical compression by Eq. (VIII.2).

## VIII.5. Worked Examples for Design of Masonry Members

**Example VIII.1.** Given: A brick pier subjected to axial compression.

Material: lime-and-sand brick.

Brick brand number: 75.

Mortar brand number: 25.

Cross section: 51 × 51 cm.

Effective length: $l_0 = 4.8$ m.

Design longitudinal force with allowance for the self-weight: $N = 18$ tf (180 kN).

Check the pier for strength.

*Solution.* Table VIII.1 gives $R = 11$ kgf cm$^{-2}$. Since the cross-sectional area of the pier

$$F = 0.51 \times 0.51 = 0.26 \text{ m}^2$$

is smaller than 0.3 m$^2$, the value of $R$ should be multiplied by the service factor $m_m = 0.8$

$$R = 11 \times 0.8 = 8.8 \text{ kgf cm}^{-2}$$

The elastic characteristic looked up in Table VIII.2 is

$$\alpha = 750$$

The reduced slenderness ratio is

$$\lambda^h_{red} = (l_0/h)\sqrt{1\,000/\alpha} = (480/51)\sqrt{1.000/750}$$
$$= 9.4 \times 1.15 = 10.6$$

According to Table VIII.3, $\varphi = 0.87$.

Since $h > 30$ cm, Eq. (VIII.1) gives for $m_{l\text{-}t} = 1$

$$N = \varphi FR = 0.87 \times 51 \times 51 \times 8.8 = 19\,900 \text{ kgf}$$
$$= 19.9 \text{ tf} = 199 \text{ kN}$$

The condition for strength is satisfied:

$$19.9 \text{ tf} > 18 \text{ tf} \quad (199 \text{ kN} > 180 \text{ kN})$$

so we infer that the pier is adequately strong.

**Example VIII.2.** Given: A brick pier in axial compression.

Cross section: 64 × 64 cm.

Effective length: $l_0 = 5.4$ m.

Material: clay brick.

Brick brand number: 100.

Mortar brand number: 50.

Reinforcement: wire fabric of class B-I steel 4 mm in diameter ($f = 0.126$ cm$^2$), placed at every three courses ($s = 23$ cm).

Wire spacing: $c = 6$ cm in either direction.

To find: The load-bearing capacity of the pier.

*Solution.* Table VIII.1 gives $R = 15$ kgf cm$^{-2}$ (1.5 MPa); $F = 0.64 \times 0.64 > 0.3$ m$^2$, so $m_m = 1$.

Equation (VIII.8) yields

$$\mu = (2f/Cs) \times 100 = 100 \times 2 \times 0.126/(6 \times 23) = 0.183\ \%$$

which is larger than $\mu_{min} = 0.1\%$.
At $R_s = 2\,000$ kgf cm$^{-2}$, Eq. (VIII.9) gives

$$R_{r\text{-}m} = R + 2\mu R_s/100 = 15$$

$$+ 2 \times 0.183 \times 2\,000/100 = 22.3 \text{ kgf cm}^{-2} < 1.8R$$

According to Table VIII.2, $\alpha = 1\,000$.
At $R'_s = 3\,500$ kgf cm$^{-2}$, Eq. (VIII.11) yields

$$\alpha_r = \alpha 2R/(2R + 2R'_s\mu/100) = 1\,000 \times 2 \times 15$$

$$/(2 \times 15 + 2 \times 3\,500 \times 0.183/100) = 700$$

The reduced slenderness ratio is

$$\lambda^h_{red} = (540/64)\sqrt{1\,000/700} = 10$$

From Table VIII.3 we get $\varphi = 0.88$.
At $m_{l\text{-}l} = 1$, Eq. (VIII.12) gives

$$N = \varphi F R_{r-m} = 0.88 \times 64 \times 64 \times 22.3$$

$$= 80\,000 \text{ kgf} = 80 \text{ tf} = 800 \text{ kN}$$

**Example VIII.3.** Given: A partition in the exterior wall of a building (Fig. VIII.5).

Storey height: 4.2 m.

Distance from the top of the partition to the bottom of the floor: 50 cm.

Wall thickness: $h = 51$ cm.

Partition width: $b = 120$ cm.

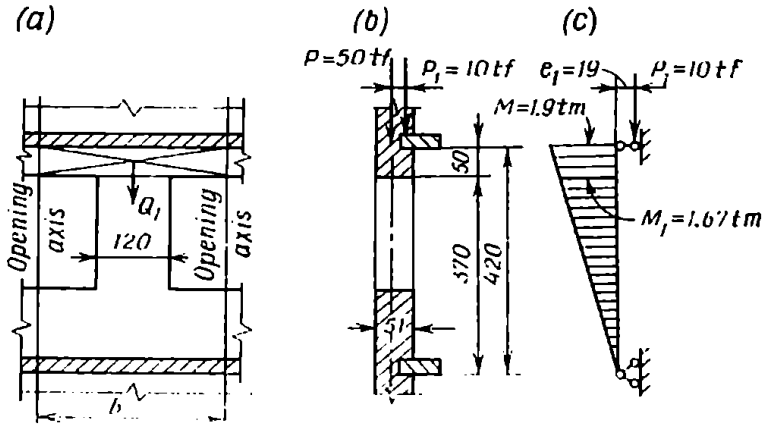



Fig. VIII.5. To Example VIII.3
(*a*) wall face; (*b*) loading system; (c) bending moment diagram

Embedment depth of the floor topping into the wall $a = 20$ cm.

Material: hollow structural clay tiles. Tile brand number: 100.

Mortar brand number: 25.

Design load: $P = 50$ tf due to the upper storeys, $P_1 = 10$ tf due to the floor above the storey in question, and $Q_1 = 1$ tf due to the weight of the wall above the window.

Check the partition for strength.

*Solution.* The eccentricity of the force $P_1$ is

$$e_1 = h/2 - a/3 = 51/2 - 20/3$$
$$= 25.5 - 6.7 = 18.8 \text{ cm} \approx 19 \text{ cm}$$

The design bending moment at the floor bottom is

$$M = P_1 e_1 = 10 \times 0.19 = 1.9 \text{ tf m}$$

The moment at the partition top is

$$M_1 = MH_1/H = 1.9 \times 3.7/4.2 = 1.67 \text{ tf m}$$

The design longitudinal force at the design section is

$$N_1 = P + P_1 + Q_1 = 50 + 10 + 1 = 61 \text{ tf}$$

The eccentricity of the longitudinal force is

$$e_0 = M_1/N_1 = 1.67/61 = 0.027 \text{ m} = 2.7 \text{ cm}$$

Table VIII.1 gives the design compressive strength of the masonry as

$$R = 13 \text{ kgf cm}^{-2}$$

According to Table VIII.2,

$$\alpha = 1\,000$$

and, according to Table VIII.3,

$$\lambda^h = 420/51 = 8.2 \text{ and } \varphi \approx 0.92$$

Equation (VIII.3) gives

$$F_{com} = 2b\,(h/2 - e_0) = 2 \times 120\,(51/2 - 2.7)$$
$$= 5\,470 \text{ cm}^2$$

Equation (VIII.5) yields

$$\omega = 1 + e_0/1.5h = 1 + 2.7/(1.5 \times 51) = 1.035$$

From Eq. (VIII.6) we get

$$\varphi_1 = \varphi\,[1 - e_0\,(0.06\lambda^h - 0.2)/h]$$

$$= 0.92\,[1 - 2.7\,(0.06 \times 8.2 - 0.2)/51] = 0.905$$

Since $h$ is larger than 30 cm, $m_{l\text{-}t}$ is taken as unity.

A check by Eq. (VIII.2)

$$N \leqslant m_{l\text{-}t}\varphi_1 F_{com} R\omega$$

gives

$$61\,000 \leqslant 1 \times 0.905 \times 5\,470 \times 13 \times 1.035$$

or

$$61\,000 < 66\,500$$

which means that the partition has an adequate strength.

# CHAPTER NINE

# METAL STRUCTURES

## IX.1. Materials for Metal Structures

The basic material for metal structures is steel. Among other things, it has an adequate strength, ductility, and weldability, and resists well dynamic loads without embrittlement. Where weight and corrosion resistance are critical, use is made of aluminium alloys.

The properties of steel depend on its chemical composition and method of manufacture. Carbon raises the strength but adversely affects the ductility and weldability of steel. Therefore, steel structures are built of low-carbon steel with a carbon content of maximum 0.22%.

The strength of steel can be improved without considerably affecting its ductility by alloying additions, such as manganese, silicon, copper, nickel, and chromium. These are known as low-alloyed steels.

The strength of some steels can be raised by heat treatment.

The impurities that may be present in steel affect its properties in different ways. For example, sulphur makes steel brittle at elevated temperatures (this is known as red shortness), whereas phosphorus causes brittleness at low temperatures (this is termed cold shortness). Because of this, the percentage of impurities in steel is strictly limited.

According to the manner of manufacture and the amount of gas evolved during solidification, low-carbon steels may be classed into rimmed (or rimming) steels (designated in the USSR by the letters кп), semi-killed steels (пс), and killed steels (сп).

Rimmed steels are only partially deoxidized low-carbon steels that evolve a considerable amount of gas during solidification. In this process a solid rim of clean metal is

rapidly formed next to the mould, while the core which solidifies more slowly is porous and has segregated impurities. Semi-killed steels are deoxidized more completely than rimmed steels but still give off a small amount of gas during solidification. Killed steels are deoxidized to the point where there is no evolution of gas and the molten metal remains quiet in the mould. Owing to this, killed steels are more reliable in structures, especially those subjected to dynamic loads and exposed to subzero temperatures.

Low-alloyed steels are manufactured only as killed steels.

The brittleness of steel is characterized by its impact strength which is defined as the work that must be done in destroying a notched specimen in an impact test. The higher the impact strength of a steel, the lower its brittleness.

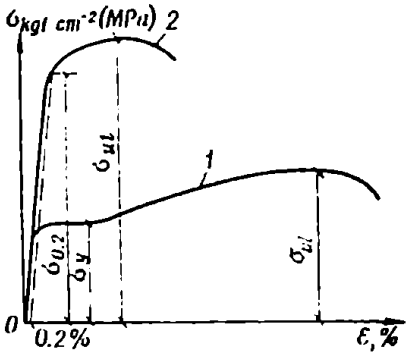



Fig. IX.1. Stress-strain curve
*1*—low-carbon steel; *2*—hard steel

The strength and deformability of steel are determined by tension tests.

Figure IX.1 shows stress-strain ($\sigma$ vs $\varepsilon$) curves for low-carbon steel (curve *1*) and high-strength steel (curve *2*). The strength of steel is characterized in terms of the yield strength $\sigma_y$ and ultimate rupture strength $\sigma_{ul}$.

For steels whose stress-strain behaviour is described by curve *2* in Fig. IX.1, it is usual to specify the proof yield strength $\sigma_{0.2}$ which is such that the permanent set is 0.2%.

When stresses in steel are sufficiently smaller than the yield point, the stress-strain relations are described by Hooke's law

$$\sigma = \varepsilon E$$

where $E$ is the modulus of elasticity equal to 2 100 000 kgf cm$^{-2}$ (210 000 MPa).

The ductility of steel can be evaluated in terms of the percent elongation at rupture, $\varepsilon_r$.

In the USSR, three groups of steels, namely group A, group Б and group B (Russian letters throughout), are available commercially. Group A refers to steels with guaranteed mechanical properties, group Б to steels with a guaranteed chemical composition, and group B to steels with guaranteed mechanical properties and chemical composition. Critical structures generally use steels falling into the last-mentioned group. Low-alloyed steels always come in group B.

The main characteristics of steels are given in relevant standards. In the Russian designation of a steel, the first letter indicates its group (Б or B). It is followed by the steel grade designation (Ст3 or 18Г) and the letters denoting the manner of manufacture (кп, сп, or пс), and the last numeral gives the steel category in terms of the impact strength: for example, ВСт3пс5, В18Гпс5, or ВСт3кп2.

The grade designation shows how much carbon in points (hundredths of a percent) a given low-alloyed steel carries. The Russian letters Г, С, Д, Н, and Х stand for manganese, silicon, copper, nickel, and chromium, respectively. If an alloying addition content exceeds 1%, the respective letter is followed by a number (rounded to the nearest integer) indicating the percentage of this particular addition. For example, in the designation 15ХСНД, the first two-digit number indicates that the carbon content is 0.15%, and the Russian letters Х, С, Н and Д stand for chromium, silicon, nickel and copper whose content does not exceed 1%. In the designation 14Г2, the first two-digit number indicates that the carbon content is 0.14%, the Russian letter Г indicates that the steel is alloyed with manganese, and the numeral 2 indicates that the manganese content may reach 2%.

In Soviet practice, all steels are divided according to their mechanical properties in tension into classes of strength (Table IX.1) which are called for brevity steel classes. In a steel class designation, the Russian letter C is followed by a fraction, with the numerator showing the minimum standard ultimate strength $\sigma_{ul}$ in kgf mm$^{-2}$, and the denominator giving the minimum yield strength $\sigma_y$ in kgf mm$^{-2}$.

Each class consists of several grades. Appropriate standards give recommendations for the use of the various grades

**Table IX.1. Classes of Steel for Metal Structures**

| Steel class | Mechanical properties in tension | | |
|---|---|---|---|
| | $\sigma_{ul}$, kgf cm$^{-2}$ (MPa) | $\sigma_y$, kgf cm$^{-2}$ (MPa) | percent elongation $\varepsilon_r$, % |
| | not lower than | | |
| C38/23 | 3 800(380) | 2 300(230) | 25 |
| C44/29 | 4 400(440) | 2 900(290) | 21 |
| C46/33 | 4 600(460) | 3 300(330) | 21 |
| C52/40 | 5 200(520) | 4 000(400) | 19 |
| C60/45 | 6 000(600) | 4 500(450) | 16 |
| C70/60 | 7 000(700) | 6 000(600) | 12 |
| C85/75 | 8 500(850) | 7 500(750) | 10 |

in steel structures according to the type of structures, exposure conditions, and expected subzero temperature. For example, at $t \geqslant -30°C$, welded floor and roof structures of class C38/23 may use steel grades ВСт3пс6, ВСт3Гпс5, and В18Гпс5; those of class C44/29, steel grade СтТпс; those of class C46/33, steel grade 14Г2; etc.

If steel structures may be exposed to corrosive environments in service, they should be protected by suitable coatings.

The characteristic strength $R^{ch}$ in tension, compression and bending is expressed in terms of the proof yield strength (for steels with a definite yield) or in terms of the proof ultimate strength (for steels having no definite yield).The design strength of steel in tension, compression and bending is given by

$$R = R^{ch}/k_s$$

where $k_s$ is the partial safety factor in terms of material, taken as 1.1 to 1.2 if the characteristic strength is given by the proof yield strength, and as 1.45 to 1.6 if the characteristic strength is the ultimate strength.

The design shear strength and end-crushing strength are obtained on multiplying $R$ by a conversion factor, namely: $R_{sh} = 0.6R$ and $R_{e-c} = 1.5R$, respectively.

For structures that may be left in service even after the metal has reached its yield point (such as pipes and circular tanks subjected to tension induced by the internal pressure), the design tensile strength of steel is uprated, being 2 600 kgf cm$^{-2}$ (260 MPa) for class C38/23, 3 000 kgf cm$^{-2}$ (300 MPa) for class C44/29, etc.

For some structural members, the value of $R$ given in Table IX.2 is multiplied by a service factor $m < 1$. This

**Table IX.2. Design Strength of Some Classes of Rolled Steel**

| Type of strength | Design strength in kgf cm$^{-2}$ (MPa) | | | |
|---|---|---|---|---|
| | steel class | | | |
| | C38/23 | C44/29 | C46/33 | C52/40 |
| Tensile, compressive and bending strength, $R$ | 2 100(210) | 2 600(260) | 2 900(290) | 3 400(340) |
| Shear strength, $R_{sh}$ | 1 300(130) | 1 500(150) | 1 700(170) | 2 000(200) |

factor is taken as 0.9 for solid beams and compressive members of trusses in civic buildings where the weight of floors exceeds the live load and for columns in residential and civic buildings and water tower supports, and as 0.75 for compressive members assembled from individual angles fastened in plane by one flange.

The standard hot-rolled sections include sheets, strips, channels, I-sections, equal and unequal angles, pipes, etc. The main section characteristics such as size, cross-sectional area, moments of inertia, moments of resistance, radii of gyration, and weight per metre run in kg are given in manufacturer's data sheets for rolled products (see Appendices X and XI).

In designing a structure, the number of section types should be kept to a minimum.

Aluminium may be alloyed with manganese (the alloy is designated АМц), magnesium (АМг), magnesium and silicon (АВ and АД), and copper and magnesium (Д). Owing to these additions, the strength of the alloys is comparable with that of steel, whereas their unit weight is one third

the unit weight of steel ($\gamma \approx 2\,600$ kg m$^{-3}$). However, they are much more expensive than steel and have a high deformability: their modulus of elasticity is $E = 710\,000$ kgf cm$^{-2}$ (71 000 MPa) which is one third that of steel. Because of this, aluminium alloys are employed only where their use is warranted.

Aluminium shapes may be rolled (sheets and strips) or extruded from small round billets through extrusion dies (angles, channels, I-sections, pipes, and box-section shapes). The flanges of aluminium angles, channels, and I-beams have bulbs at the ends, intended to raise their local stability.

The characteristics and design strength of aluminium alloys can be found in relevant standards and specifications.

## IX.2. Joints in Metal Structures

**Welded Joints.** Most steel welding is done with one of three processes which are briefly discussed here.

*Manual metal arc.* This is colloquially known as stick electrode welding or even just stick welding. The welder uses a stick or rod, which is a metal electrode with a fusible mineral coating, in a holder connected to an electrical supply. An arc is struck between the electrode and the workpiece. It melts both the electrode and a superficial area of the workpiece.

Electromagnetic forces created in the arc help to throw drops of the molten electrode on to the molten area of the workpiece where the two metals fuse to form the weld pool. The electrode coating, or flux, contributes to the content of the weld pool by direct addition of metal and by metallurgical reactions which cleanse and refine the molten metal before it solidifies into the weld bead. The flux coating also generates a local gaseous atmosphere which contributes to the stability of the arc and prevents absorption of atmospheric gases by the weld metal.

*Submerged arc.* This welding process uses a bare wire electrode and a flux added separately in the form of granules or powder over the arc and weld pool.

The flux has four principal functions:

(a) To protect the molten metal by forming a protective slag layer.

(b) To stabilize the arc.

(c) To cleanse the molten metal, particularly when there is rust on the surface.

(d) To control the composition of the weld metal.

The process is used mainly in a mechanized system feeding a continuous length of electrode from a coil on a tractor unit, which carries the welding head along the joint. Alternatively, the welding head may be fixed and the job traversed or rotated under it.

As a mechanical process, submerged arc welding is capable of greater consistency and productivity than manual welding, although this process is not suitable for areas of difficult access and multiposition work in situ.

*Gas shielded welding.* In this process, a bare wire electrode is used, and a shielding gas is fed around the arc and weld pool. This gas prevents contamination of the electrode and weld pool by air, and provides a local atmosphere giving a stable arc. The process is known as MIG (metal inert gas) when argon or helium is used—generally for nonferrous metals such as aluminium, titanium, and nickel alloys. For carbon and carbon manganese steels, the gas commonly employed is carbon dioxide ($CO_2$) or a mixture of argon and carbon dioxide, and the process is then called MAG (metal active gas).

Aluminium alloy workpieces more than 6 mm thick are welded with consumable electrodes. For thin sheet work less than 6 mm thick and precision welding of components to close tolerances, TIG (tungsten inert gas) welding is used. This process employs inert argon or helium gas to protect the weld, and the arc is struck between the work and a non-consumable tungsten electrode. Filler can be added to the joint, although most applications employ joint designs not requiring filler (autogenous welds).

Physically, two basic types of weld can be distinguished, namely the butt weld (Fig. IX.2*a*) where the deposited metal fills the gap between the workpieces lying in the same plane, and the fillet weld (Fig. IX.2*b*) where the deposited metal fills the corner made by the workpieces lying in different planes. The butt weld is easy to make and results in a good

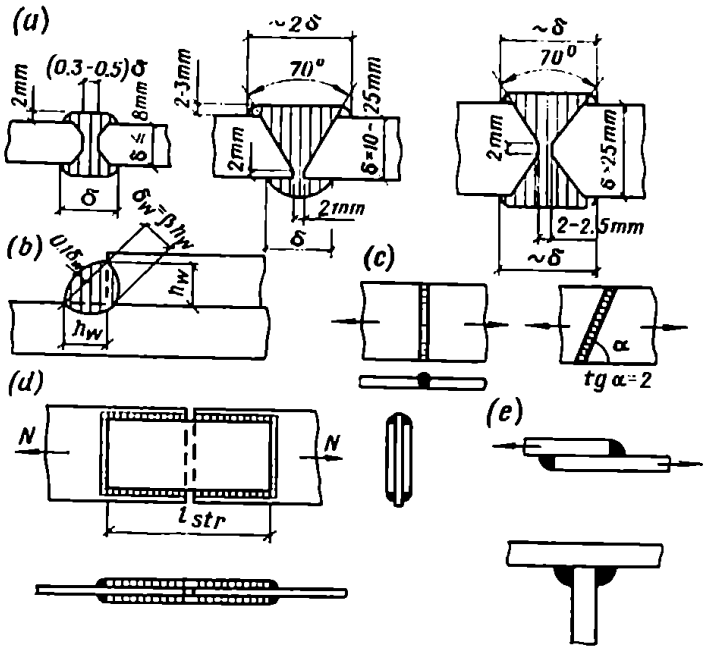



Fig. IX.2. Types of welds and welded joints

joint. However, additional labour is required for edge preparation when the workpieces are more than 8 mm thick. A normal fillet weld is an isosceles triangle* in cross section with a reinforcement 0.1 $h_w$ high on each leg.

The design thickness of a butt weld throat is taken equal to the thickness of the workpieces, δ. The design thickness of a fillet weld throat is $\beta h_w$, where β is a coefficient taken as 0.7 for manual welding, and as 0.7 to 1 for semi-automatic and automatic welding according to the number of passes. The effective length of a weld $l_w$ is taken as its full length $l$ minus 1 cm to allow for likely poor penetration and fusion at the ends.

The basic welded joints are as follows:

(1) Butt joints where the workpieces lie in the same plane and are connected by a butt weld (Fig. IX.2*c*);

* In high-strength steel structures intended to resist dynamic loads and vibrations and be exposed to subzero temperatures, the fillet weld is a right-angled triangle in cross section with a leg ratio of 1 : 1.5.

(2) Butt joints on a backing strip welded by fillet welds to the workpieces (Fig. IX.2*d*);

(3) Lap joints where the workpieces lie one on top of the other and are welded by fillet welds (Fig. IX.2*e*);

(4) T-joints where the workpieces are welded at an angle to each other by fillet welds (Fig. IX.2*f*).

The strength of welds is characterized in terms of their design strength as given in Table IX.3.

Butt welds are designed so as to satisfy the following condition

$$N/l_w\delta \leqslant R^w \tag{IX.1}$$

where $N$ is the design longitudinal force, $R^w$ is the design weld strength, $l_w$ is the effective weld length, and $\delta$ is the effective thickness of the weld throat.

Fillet welds must meet the following condition

$$N/\beta h_w l_w \leqslant R_f^w \tag{IX.2}$$

where $\beta h_w$ is the effective thickness of the weld throat, and $R_f^w$ is the design strength of the fillet weld.

**Example IX.1.** Given: A butt joint on backing strips (see Fig. IX.2*d*).

Steel: class C38/23.

Backing strip thickness: 8 mm.

Backing strip width: 160 mm.

Design tensile force: $N = 80$ tf $= 800$ kN.

To find: The effective length of the weld $l_w$ and the necessary length of the backing strips $l_{st}$.

*Solution.* Table IX.3 gives

$$R_f^w = 1\ 500 \text{ kgf cm}^{-2}$$

The throat thickness is taken equal to the strip thickness

$$h_w = 0.8 \text{ cm}$$

Equation (IX.2) where $\beta$ is taken as 0.7 gives the necessary weld length for the backing strips

$$l_w = N/R_f^w\, 0.7h_w = 80\ 000/(1\ 500 \times 0.7 \times 0.8) = 94 \text{ cm} \tag{IX.3}$$

**Table IX.3. Design Strength of Welds $R^w$ in kgf cm$^{-2}$ (MPa)**

| Type of weld | State of stress | Designation | $R^w$ for | | | |
|---|---|---|---|---|---|---|
| | | | C38/23 | C44/29 | C46/33 | C52/40 |
| Butt | Compression * | $R^w_{com}$ | 2 100 (210) | 2 600 (260) | 2 900 (290) | 3 400 (340) |
| Butt | Tension in manual and semi-automatic welding * | $R^w_{ten}$ | 1 800 (180) | 2 200 (220) | 2 500 (250) | — |
| Butt | Shear | $R^w_{sh}$ | 1 300 (130) | 1 500 (150) | 1 700 (170) | 2 000 (200) |
| Fillet | Shear | $R^w_f$ | 1 500 (150) | 1 800 (180) | 2 000 (200) | 2 200 (220) |

* In automatic, semi-automatic and manual welding where the welds are tested by X-rays or ultrasound, $R^w_{ten} = R^w_{com}$.

Since we have two backing strips, the weld length for each backing strip is

$$l_w = 94/2 = 47 \text{ cm}$$

With the backing strip 16 cm wide, the necessary length of the longitudinal welds is

$$(47 - 16)/2 = 15.5 \text{ cm}$$

and, with allowance for poor penetration and fusion at the ends, it is

$$15.5 + 1 = 16.5 \text{ cm}$$

With the gap between the workpieces 1 cm wide, the total length of the backing strips is

$$l_{st} = 2 \times 16.5 + 1 = 34 \text{ cm}$$

**Riveted and Bolted Joints.** Rivets (Fig. IX.3*a*) are used rather seldom, mostly in structures carrying dynamic loads or made of steel and aluminium alloys having poor weldability. Bolts (Fig. IX.3*b*) are mainly used for field connections. The two basic types of bolts are rough-finished or black bolts and bright bolts. The former are forged from rolled bars, and the latter are turned on a lathe to fit the hole diameter.

Holes in plates forming parts of riveted and bolted structures are punched, punched and reamed, or drilled. Drilling, although more expensive, is more precise. As a rule,

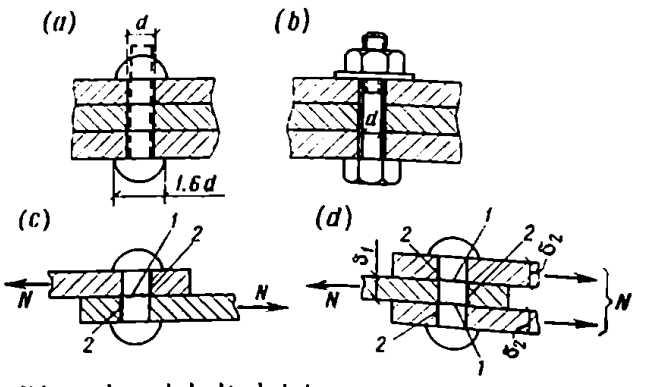



Fig. IX.3. Riveted and bolted joints
*1*—shear; *2*—bearing

punched holes are used for rivets and black bolts, and drilled holes for bright bolts.

Rivets and bolts resist shear across and bearing along their axes (Fig. IX.3*c* and *d*). The area in shear is equal to the cross-sectional area of a rivet (bolt), and the area in bearing is taken as the rivet (bolt) diameter $d$ multiplied by the minimum total thickness $\Sigma\delta$ of members subjected to bearing in one direction.

Rivets and bolts are designed subject to the following conditions:

— in shear

$$N/nn_{sh}0.25\pi d^2 \leqslant R_{sh} \qquad \text{(IX.4)}$$

— in bearing

$$N/nd\Sigma\delta \leqslant R_b \qquad \text{(IX.5)}$$

Here, $N$ is the design longitudinal force imposed on a given connection, $n$ is the number of rivets (bolts) in the connection, $n_{sh}$ is the number of shear planes in one rivet (bolt), and $R_{sh}$ and $R_b$ are respectively the shear strength and the bearing strength depending on the grade of steel used in the rivets (bolts), the parts being connected, and the manner of hole manufacture.

The design shear strength of rivets made of Ст2 steel is 1800 kgf cm$^{-2}$ for drilled holes and 1 600 kgf cm$^{-2}$ for punched holes; that of black bolts is 1 500 kgf cm$^{-2}$. In a riveted joint, the design bearing strength of members made of class C38/23 steel is 4 200 kgf cm$^{-2}$ for drilled holes and 3 800 kgf cm$^{-2}$ for punched holes. In a joint using a group of bolts, $R_b$ = = 3 400 kgf cm$^{-2}$.

Nowadays, riveting is rapidly being supplanted, both in the shop and in the field, by bolting with high-strength bolts. Tightened bolts give rise to friction forces in the plane of contact between the parts being fastened, so they can resist considerable shearing forces. High-strength bolts may be set in holes whose diameter is several millimetres larger than the bolt diameter, which markedly simplifies erection.

The design force carried by each friction surface (owing to one bolt) is given by

$$N_b = Pfm$$

where $f = 0.25$ to $0.55$ is the coefficient of friction varying with the class of surface finish and steel class, $m = 0.9$ is the service factor, and $P$ is the axial force of bolt tightening

$$P = 0.65\sigma_{ul}F_{net}$$

Here, $\sigma_{ul}$ is the ultimate strength of a high-strength bolt ($\sigma_{ul} = 8\,000$ to $13\,500$ kgf cm$^{-2}$ or 800 to 1 350 MPa), and $F_{net}$ is the net cross-sectional area of the bolt.

In butt joints, the pitch of rivets and bolts (the centre-to-centre distance between rivets and bolts) is taken as $3d$. The distance from the centre of an outer rivet (or bolt) to the edge of the member is taken as $2d$ along the force or $1.5d$ across the force.

## IX.3. Design and Proportioning of Beams

Steel beams* are generally assembled from rolled I-sections which have a high flexural rigidity in the plane of the web. Sometimes, use is also made of rolled channels. When large spans are to be bridged or heavy loads are to be carried, beams are made as built-up I-beams consisting of three sheets welded to form the web and the flanges.

According to their static performance, beams may be classed into single-span, continuous, and cantilever types. As a rule, use is made of single-span beams as the least labour-consuming in manufacture and erection, although they are less economical of material than continuous beams.

The base for a floor is constituted by a system of beams known as the grillage. Loads from the floor are transmitted to the supports (columns or walls) by girders, and the floor decking is supported by beams (Fig. IX.4). The floor decking may be made as an in-situ reinforced concrete slab or assembled from prefabricated reinforced concrete slabs or steel sheets welded to the beams. The arrangement of the grillage depends on the column grid size adopted, disposition of openings in the floor, size of the equipment to be accommo-

* Unless explicitly qualified, beams refer to both girders and beams proper. When any distinction is intended, the beam is the smaller member and may be supported by the girder.—Translator's note.

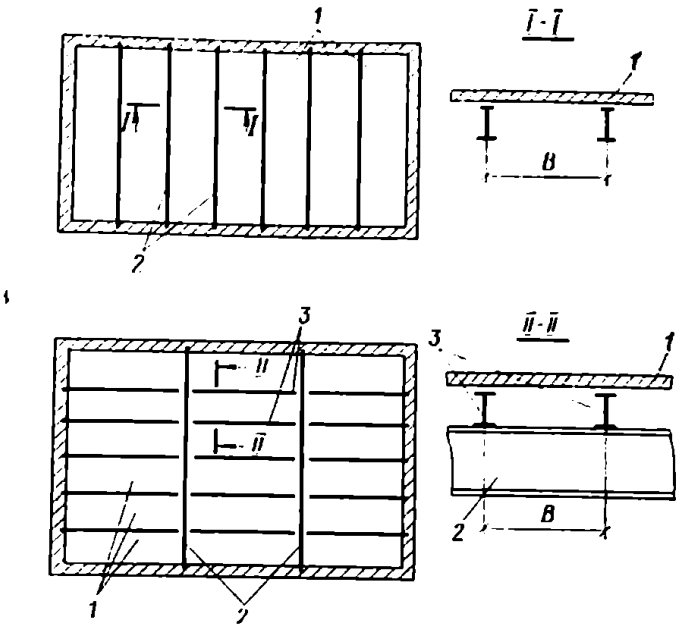



Fig. IX.4. Types of grillage
*1*—decking; *2*—girders; *3*—beams

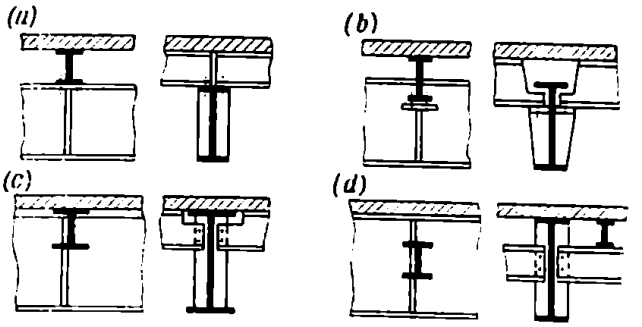



Fig. IX.5. Beam and girder connections in a grillage

dated, maximum allowable floor height, and economic considerations.

In height, connections of beams and girders may be classed into four types, namely the beam on girder connection (Fig. IX.5*a*), raised beam and girder connection (Fig. IX.5*b*), coped beam and girder connection (Fig. IX.5*c*), and lowered beam and girder connection (Fig. IX.5*d*). The beam on gir-

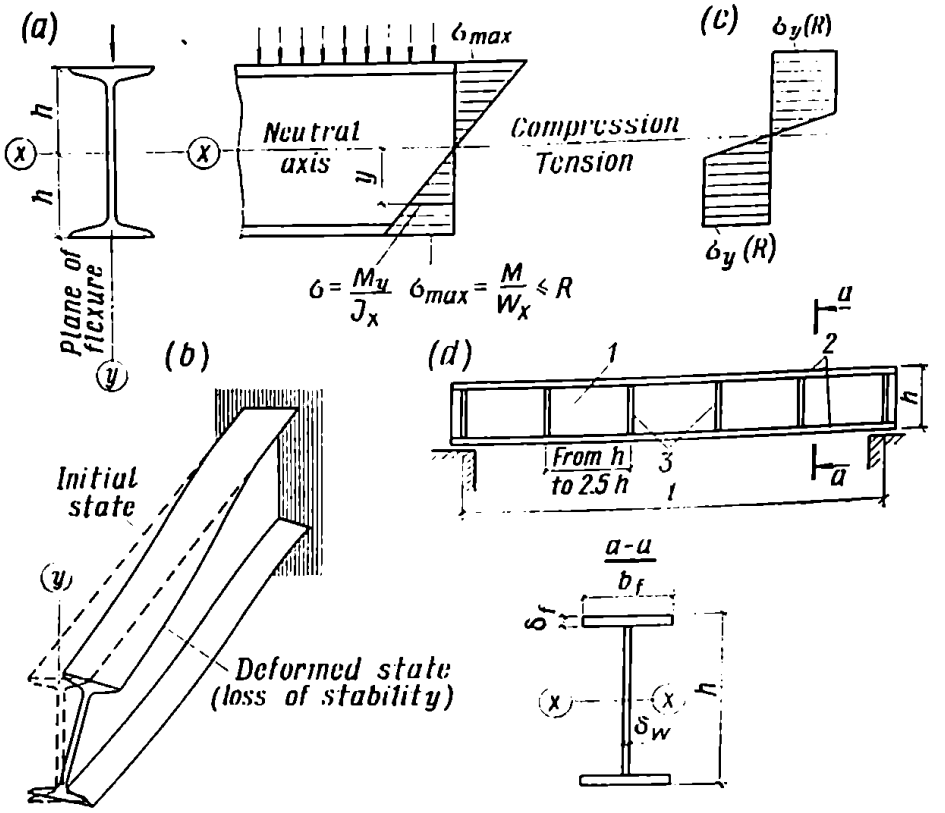



Fig. IX.6. To the design of steel beams
*1*—web; *2*—flange; *3*—stiffeners

der type is the simplest, but the floor height required is the largest. In the coped beam and girder type, the floor decking can be supported on four sides. The lowered beam and girder type is a good choice when the floor decking must be thick.

Beams are checked for strength, sag, and, sometimes, stability.

Rolled beams are only checked for normal-stress strength. Tangential- and principal-stress strength is assumed to be ensured by an appropriate distribution of material, so it is not subject to any check in rolled shapes.

The strength analysis of beams consists in determining edge normal stresses in a section subjected to the bending moment induced by design loads. These stresses should not

exceed the design strength $R$ (Fig. IX.6*a*)

$$M/W_{net} \leqslant R \qquad \text{(IX.6)}$$

where $M$ is the bending moment induced by the design loads, $R$ is the design strength of the material, and $W_{net}$ is the moment of resistance of the net cross-sectional area.

For rolled shapes, the value of $W$ is looked up in the manufacturer's data sheets. For built-up beams, one should first determine the moment of inertia $J$, and then the moment of resistance $W = J/y$ (where $y$ is the distance from the centroid to the farthest edge of the section). In symmetrical sections, $y = h/2$. The sag is found as caused by the action of characteristic loads and as a function of the span $l$, manner of load application, and beam rigidity $EJ$. For a single-span beam carrying a uniformly distributed load $q^{ch}$,

$$f_{max} = 5q^{ch}l^4/384EJ$$

For a single-span beam resisting concentrated loads $P$ applied at third-points

$$f_{max} = (23/648)\,(Pl^3/EJ)$$

Relevant standards specify the limiting values of the relative sag defined as the ratio of the sag $f$ to the beam span $l$: $f/l \leqslant 1/400$ for floor girders, and $f/l \leqslant 1/250$ for floor beams.

If the compressive flange of a beam is not fixed along the span, the beam may warp and bend out of the plane of flexure—a condition known as the loss of stability, even before all of its load bearing capacity has been used (Fig. IX.6*b*). The reduction in the load-bearing capacity due to the loss of stability is taken care of in Eq. (IX.6) by an additional factor $\varphi_b < 1$ which depends on the section depth, beam span, and manner of loading, and is determined using the data of Appendix XII.

Local 'weak spots' have no effect on the critical load that causes a beam to lose its stability, so the design formula is written to include the gross moment of resistance $W_{gr}$ of the cross section:

$$M/\varphi_b W_{gr} \leqslant R \qquad \text{(IX.7)}$$

Beams are presumed to be stable by nature of their configuration, so they are not checked for stability if the compres-

sive flange gives support to a rigid floor decking (such as reinforced concrete slabs, metal sheets, or corrugated steel) imposing a distributed static load on the beam, and in the case of built-up I-beams, if the ratio of the effective length $l_0$ of the compressive flange to its width $b_f$ is within 13:1 to 17:1 (provided the load is applied to the top flange).

In most cases, the above conditions are satisfied, so stability analysis is only carried out for beams whose compressive flange is not fixed over a considerable length (monorails, crane beams, etc.).

Beams carrying a static load and fixed against the loss of stability may be checked for strength under the assumption of a plastic hinge. The point is that after the stress in the outer fibres has reached its ultimate value, such a beam is still able to carry some additional load because stresses are still likely to build up in the fibres removed from the edge. The load-bearing capacity of the beam is exhausted when the stress reaches its ultimate value all over the cross section (Fig. IX.6*c*).

The associated moment of resistance is known as the plastic moment of resistance $W^{pl}$. Accordingly, Eq. (IX.6) takes the form

$$M/W^{pl} \leqslant R \qquad \text{(IX.8)}$$

Here, $W^{pl} = 2S \leqslant 1.2W$, and $S$ is the static moment of the half-section about the section centroid. For rolled I-beams and channels,

$$W^{pl} = 1.12W \qquad \text{(IX.9)}$$

For a symmetric built-up I-beam welded from three sheets (Fig. IX.6*d*),

$$W^{pl} \approx h\,(b_f\delta_f + \delta_w h/4) \qquad \text{(IX.10)}$$

The procedure for determining rolled beam sections is as follows. Calculate the bending moment $M$ and find the necessary moment of resistance from Eq. (IX.6) or Eq. (IX.8) if the beam may be designed under the assumption of a plastic hinge. Then, look up a suitable shape in a listing of rolled products, determine the respective moment of inertia $J$, find the sag, and compare it with the maximum safe value.

The procedure for determining the section of a built-up I-beam welded of three sheets is as follows. Calculate the necessary moment of resistance $W$ using Eq. (IX.6) and find the optimal beam depth

$$h \approx 1.15 \sqrt{W/\delta_w} \qquad \text{(IX.11)}$$

where $\delta_w$ is the web thickness in cm.

In determining $h$, the web should be taken 8 to 10 mm thick and then checked for shear strength under the action of a shearing force (without taking into account the contribution from the flanges), using the following expression

$$\delta_w \geqslant 1.5 Q_{max}/R_{sh} h \qquad \text{(IX.12)}$$

It should be remembered that when $h/\delta_w > 100\sqrt{2\,100/R}$ the web should be strengthened with transverse stiffening ribs against the loss of stability*.

The necessary moment of inertia of the entire beam section is given by

$$J = Wh/2 \qquad \text{(IX.13)}$$

Since the moment of inertia of the web is known, the moment of inertia of the flanges is

$$J_f = J - J_w = J - \delta_w h^3/12 \qquad \text{(IX.14)}$$

The moment of inertia of the flanges about the central axis of the section (leaving out the moments of inertia about their own axes, which may be neglected) is

$$J_f \approx 2F_f (h/2)^2$$

Hence, the required area of one flange is given by

$$F_f = 2J_f/h^2 \qquad \text{(IX.15)}$$

Once $F_f$ is known, assume the flange cross section such that $b_f = h/3$ to $h/5$ and $\delta_f = F_f/b_f$.

Using the above data, proportion the beam cross section, find the moment of inertia $J$ and the moment of resistance $W$, check the section for strength using Eq. (IX.6), and determine the sag.

---

* If the beam carries a live load, stiffening ribs must be installed when $h/\delta_w > 70\sqrt{2\,100/R}$.

The stiffening ribs that reinforce the beam web against the loss of stability (see Fig. IX.6*d*) are placed where concentrated loads are applied at the supports, and are spaced not more than $2h$ apart if $h/\delta_w > 100$ or not more than $2.5\,h$ if $h/\delta_w \leqslant 100$ in the span.

The rib thickness should be at least 1/15 of their width $b_r \approx b_f/2$. The ribs at the supports together with the adjacent web portions $15\delta_w$ long on either side of the rib should additionally be checked by calculation as compressive struts carrying the support reaction.

Built-up beams need not be checked for stability if $h/\delta_w \leqslant 110\sqrt{2\,100\ /R}$ or $h/\delta_w \leqslant 80\sqrt{2\,100/R}$. The latter case holds when a load applied between the stiffening ribs gives rise to a local stress.

The welds joining the web and flanges of built-up beams must take up a shearing force $T$ appearing when a beam is subjected to bending:

$$T = QS_f/J \qquad \text{(IX.16)}$$

where $Q$ is the design shearing force, $S_f$ is the static moment of the flange about the neutral axis of the beam, and $J$ is the moment of inertia of the entire beam section.

This shearing force must be resisted by two fillet welds, that is,

$$T = 2\beta h_w R_f^w$$

whence

$$h_w = T/2\beta R_f^w \geqslant \delta_w/2 \geqslant 6 \text{ mm} \qquad \text{(IX.17)}$$

Built-up beams may be designed under the assumption of a plastic hinge only when they are reliably fixed against the loss of stability. For this purpose, the ratio of the effective length $l_0$ of the compressive flange to the flange width $b_f$ should be anywhere between 9:1 and 12:1, the ratio of the width of the compressive flange overhang to its thickness should not exceed $10\sqrt{2\,100/R}$, and the ratio of the web depth to the web thickness (with only transverse ribs present) should not exceed $70\sqrt{2\,100/R}$.

**Example IX.2.** Given: A maintenance platform assembled from rolled steel I-beams carrying precast reinforced concrete slabs 8 cm thick with ceramic tiles laid on a layer of cement mortar with a total thickness of 4 cm.

Beam span: $l = 4.6$ m.
Centre-to-centre distance between beams: 2.2 m.
Characteristic live load imposed on floor: 400 kgf $cm^{-2}$.
Overload factor: $n = 1.2$.
Maximum safe relative sag: $f/l = 1/250$.
Beam material: class C38/23 steel.
To find: The beam section.

*Solution.* The beam is regarded as a single-span beam carrying a uniformly distributed load.

The loads imposed on the beam are as follows:

— weight of the reinforced concrete slabs

$$0.08 \times 2\,500 = 200 \text{ kgf m}^{-2} (2 \text{ kN m}^{-2})$$

— weight of the floor

$$0.04 \times 2\,000 = 80 \text{ kgf m}^{-2} (0.8 \text{ kN m}^{-2})$$

— approximate self-weight of the beam

$$30 \text{ kgf m}^{-1} (0.3 \text{ kN m}^{-1})$$

— characteristic load per metre run of the beam

(a) dead load

$$g^{ch} = (200 + 80) \times 2.2 + 30 = 646 \text{ kgf m}^{-1} (6.46 \text{ kN m}^{-1})$$

(b) live load

$$p^{ch} = 400 \times 2.2 = 880 \text{ kgf m}^{-1} (8.8 \text{ kN m}^{-1})$$

(c) total load

$$q^{ch} = g^{ch} + p^{ch} = 646 + 880$$
$$= 1\,526 \text{ kgf m}^{-1} (15.26 \text{ kN m}^{-1})$$

The design load per metre run of the beam is

$$q = g^{ch} n_1 + p^{ch} n_2 = 646 \times 1.1 + 880 \times 1.2$$
$$= 1\,766 \text{ kgf m}^{-1} (17.66 \text{ kN m}^{-1})$$

The maximum bending moment is

$$M = ql^2/8 = 1\,766 \times 4.8^2/8$$
$$= 5\,100 \text{ kgf m} (51 \text{ kN m})$$

Table IX.2 gives the design strength $R = 2\,100$ kgf cm
The beam is subjected to a uniformly distributed sta

load and its top compressive flange is fixed by the rigid floor decking against the loss of stability, so the beam may be designed under the assumption of a plastic hinge. Equation (IX.8) yields the required plastic moment of resistance

$$W^{pl} = M/R = 510\,000/2\,100 = 242 \text{ cm}^3$$

Recalling that $W^{pl} = 1.12W$, we obtain

$$W = 242/1.12 = 216 \text{ cm}^3$$

In the listing of rolled products (see Appendix X), this corresponds to a size 22 I-beam ($W_x = 232$ cm$^3$ and $J_x = 2\,550$ cm$^4$).

The sag caused at midspan by the characteristic load is

$$f = 5q^{ch}l^4/384EJ = 5 \times 15.26 \times 480^4/(384 \times 2.1 \times 10^6 \times 2\,550) = 1.93 \text{ cm}$$

The relative sag is

$$f/l = 1.93/480 = 1/250$$

**Example IX.3.** Given: An immovable support for a heat pipeline, made as a single-span steel beam built up of two channels.

Concentrated load at mid-span: $P = 8$ tf (80 kN).

Beam span: $l = 2.4$ m.

Material: class C38/23 steel.

To find: The beam cross section.

*Solution.*

$$M = Pl/4 = 8 \times 2.4/4 = 4.8 \text{ tf m } (48 \text{ kN m})$$

$$W_{req} = 480\,000/2\,100 = 228 \text{ cm}^3$$

In the listing of rolled products (see Appendix XI), we look up two size 18 channels

$$W_x = 2 \times 121 = 242 \text{ cm}^3$$

which is larger than 228 cm$^3$.

**Example IX.4.** Given: A steel built-up beam assembled from welded components.

Beam span: $l = 9$ m.

Concentrated loads at third-points $P^{ch} = 26$ tf, $P = 32$ tf.

Maximum safe relative sag: $f/l = 1/400$.

Material: class C44/29 steel.
To find: The beam cross section.
*Solution.* The design forces are

$$M = Pl/3 = 32 \times 9/3 = 96 \text{ tf m}$$
$$Q = P = 32 \text{ tf}$$

The moment of resistance required for $R = 2\,600$ kgf cm$^{-2}$ (see Table IX.2) is

$$W = M/R = 9\,600\,000/2\,600 = 3\,700 \text{ cm}^3$$

Assuming the web to be 8 mm thick, Eq. (IX.11) gives the beam depth to be

$$h = 1.15\sqrt{W/\delta_w} = 1.15\sqrt{3\,700/0.8} = 78 \text{ cm}$$

A check to see if the web thickness meets the condition for shear resistance at the support section according to Eq. (IX.12), with $R_{sh} = 1\,500$ kgf cm$^{-2}$, yields

$$\delta_w = 1.5Q/R_{sh}h = 1.5 \times 32\,000/(1\,500 \times 78) = 0.41 \text{ cm}$$

which is smaller than 0.8 cm.
Equation (IX.13) gives the required moment of inertia

$$J = Wh/2 = 3\,700 \times 78/2 = 144\,000 \text{ cm}^4$$

The moment of inertia of the flanges is given by Eq. (IX.14)

$$J_f = J - \delta_w h^3/12 = 144\,000 - 0.8 \times 78^3/12$$
$$= 111\,000 \text{ cm}^4$$

The flange cross-sectional area is found from Eq. (IX.15)

$$F_f = 2J_f/h^2 = 2 \times 110\,000/78^2 = 37 \text{ cm}^2$$

We adopt

$$b_f \approx 0.25\,h = 20 \text{ cm}$$

and

$$\delta_f = 37/20 \approx 2 \text{ cm}$$

Thus, the following cross-sectional dimensions are adopted:

$$h = 80 \text{ cm}$$
$$b = 20 \text{ cm}$$
$$\delta_f = 2 \text{ cm}$$
$$h_w = 76 \text{ cm}$$
$$\delta_w = 0.8 \text{ cm}$$

The moment of inertia and the moment of resistance for this cross section are

$$J_x = 0.8 \times 76^3/12 + 2 \times 20 \times 2 \times 39^2$$
$$= 29\,400 + 121\,600 = 151\,000 \text{ cm}^4$$
$$W_x = J_x/0.5h = 151\,000/40 = 3\,780 \text{ cm}^3$$

The beam is checked for strength using Eq. (IX.6), because the conditions for stability of the flange and web are not met and the assumption of a plastic hinge does not apply:

$$l_0/b_f = 300/20 = 15 > (9{:}1 \text{ to } 12{:}1)$$

$$h_w/\delta_w = 76/0.8 = 95 > 70\sqrt{2\,100/2\,600} = 63$$

$$M/W_x = 9\,600\,000/3\,780 = 2\,540 \text{ kgf cm}^{-2} < 2\,600 \text{ kgf cm}^{-2}$$

As is seen, the beam has an adequate strength.

The sag induced at mid-span by the characteristic load $P^{ch} = 26$ tf is

$$f = 23 \times 26\,000 \times 900^3/(648 \times 2.1 \times 10^6 \times 151\,000)$$
$$= 2.1 \text{ cm}$$

The relative sag is

$$f/l = 2.1/900 = 1/428 < 1/400$$

Equation (IX.16) gives the shearing force $T$ at the welds

$$T = QS_f/J = 32\,000 \times 20 \times 2 \times 39/151\,000 = 332 \text{ kgf cm}^{-1}$$

According to Eq. (IX.17), the weld throat thickness for $\beta = 0.7$ and $R_f^w = 1\,800$ kgf cm$^{-2}$ is

$$h_{weld} = T/(2\beta R_f^w) = 332/(2 \times 0.7 \times 1\,800)$$
$$= 0.14 \text{ cm} < 0.6 \text{ cm}$$

We adopt $h_{weld} = 6$ mm.

A check to see if the web needs to be checked for stability gives

$$h_w/\delta_w = 76/0.8 = 95 < 110\sqrt{2\,100/2\,600} = 99$$

which means that a stability check may be omitted.

Transverse stiffeners are installed at the supports, at third-points, and, since $h_w/\delta_w = 95 > 100\sqrt{2\,100/2\,600} = 90$, at every 1.5 m which is smaller than $2.5h$. The stiffeners are taken to be 8 mm thick, which exceeds 1/15 of their width $b_r \approx b_f/2 = 10$ cm.

## IX.4. Design and Proportioning of Columns

**Columns in Axial Compression.** Columns may be made either as solid structures built of sheets, rolled shapes, and pipes (Fig. IX.7*a*), or as open-work structures consisting of individual side members connected to one another by rectangular strips or diagonal lacing lightweight angles (Fig. IX.7*b* and *c*). Open-work columns are more economical of material than their solid counterparts, but require more labour to fabricate.

At the top, a column has a head whose construction depends on how the beams that the column is to support rest on the head (Fig. IX.8*a*, *b*, and *c*). The load at the lower end of a column must be transferred to a concrete footing or pier which in turn transfers the load to the ground. This is done by means of the column base. Bases for steel columns may consist of a single base plate (Fig. IX.8*d*), a base plate and stiffeners (Fig. IX.8*e*), or a base plate and vertical wing-plates transferring the load to the base plate (Fig. IX.8*f*). The base plate has holes to receive binding bolts from the footing, that join them together.

The column is checked for strength at a section weakened by holes subject to

$$N/F_{net} \leqslant R \qquad \text{(IX.18)}$$

where $N$ is the design load, $F_{net}$ is the net area of the section, and $R$ is the design strength of the steel used.

The cross-sectional dimensions of a column are small as compared with its length, so columns are likely to lose stability long before the stress in the material reaches its ulti-

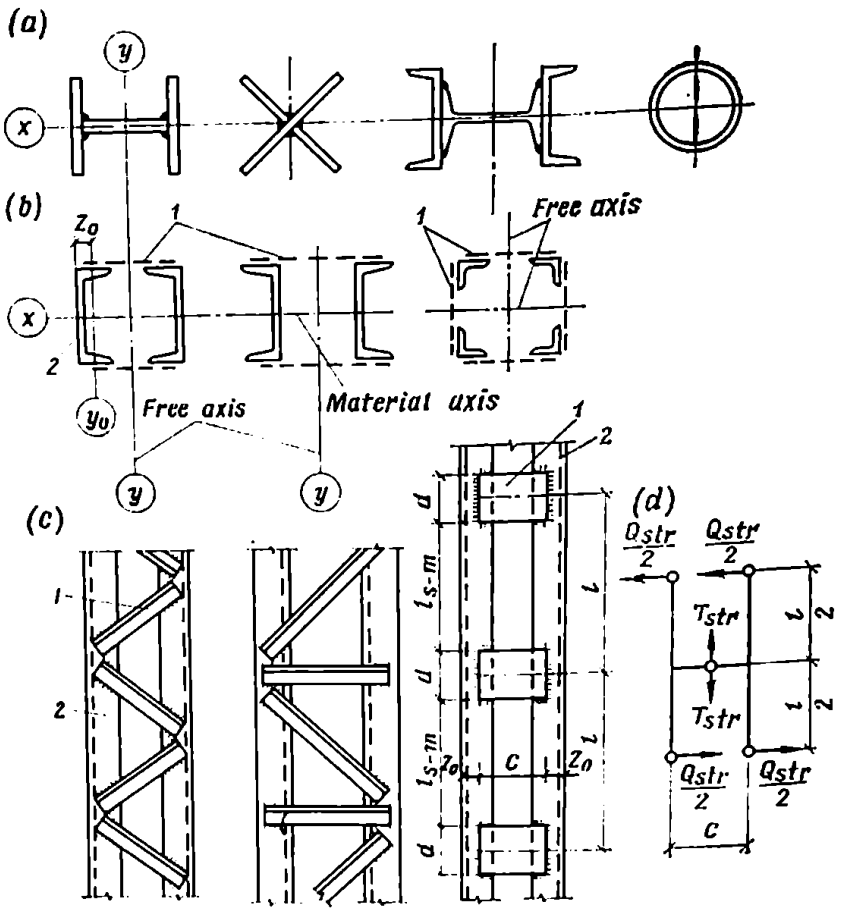



Fig. IX.7. Steel columns in axial compression
(*a*) cross sections of solid columns; (*b*) cross sections of open-work columns; (*c*) types of lacing in open-work columns; (*d*) to the design of lacing strips; *1*—lacing; *2*—side member

mate value (what is known as buckling takes place). Therefore, columns are designed for stability subject to

$$N/F_{gr}\varphi \leqslant R \qquad \text{(IX.19)}$$

where $F_{gr}$ is the gross area of the section in question, and $\varphi$ is a coefficient which accounts for the reduction in the load-bearing capacity of the column due to buckling, and varies with the column slenderness ratio $\lambda$ and class of steel. For members in axial compression made of class C38/23

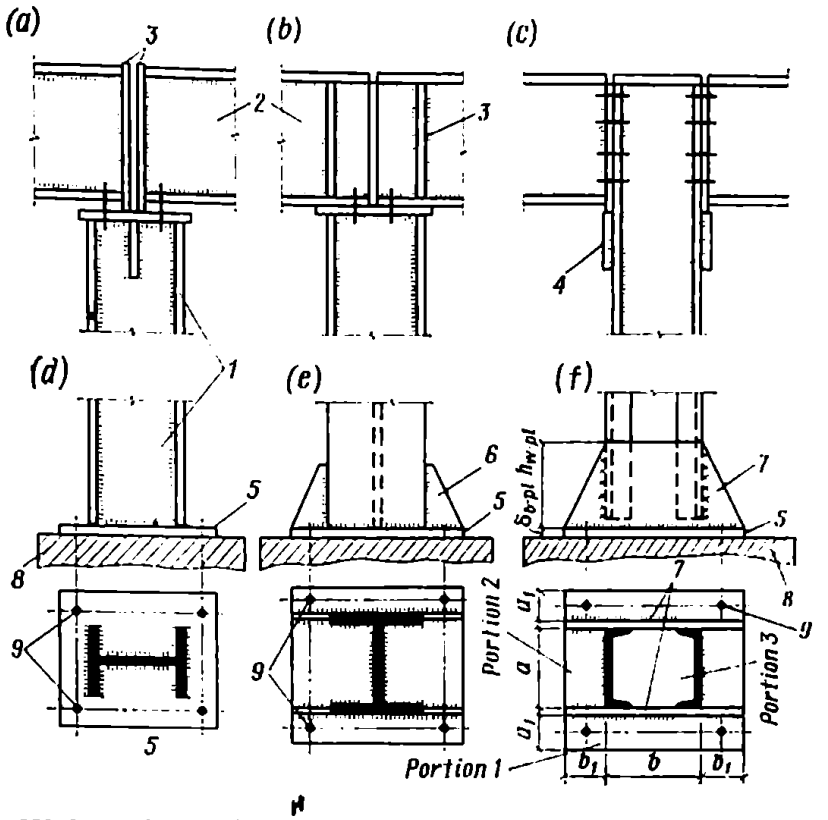



Fig. IX.8. Column head and base

*1*—column core; *2*—beams; *3*—support ribs in beam; *4*—table; *5*—base plate; *6*—stiffeners; *7*—wing-plates; *8*—footing; *9*—binding bolts

steel, φ may take on the following values:

| λ | 0 | 20 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 120 | 150 | 200 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| φ | 1 | 0.97 | 0.9 | 0.87 | 0.82 | 0.77 | 0.71 | 0.65 | 0.58 | 0.45 | 0.3 | 0.17 |

The slenderness ratio of a column, λ, is defined as the ratio of its effective length $l_0$ (Fig. IX.9) to the radius of gyration of the cross section, $r$, which is a function of the moment of inertia and the cross-sectional area:

$$\lambda_x = l_{0x}/r_x$$

$$\lambda_y = l_{0y}/r_y$$

$$r_x = \sqrt{J_x/F_{gr}}$$

$$r_y = \sqrt{J_y/F_{gr}}$$

One ought to design a column so that it has equal stability about either axis, that is, $\lambda_x \approx \lambda_y$.

In open-work columns, the axis crossing the side members (the $x$-$x$ axis in Fig. IX.7*b*) is known as the material axis, and that crossing the lacing (the $y$-$y$ axis in the figure), as the free axis.

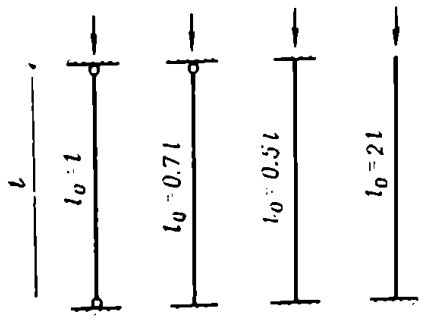



Fig. IX.9. Effective length of columns in axial compression

The slenderness ratio about the material axis is given by

$$\lambda_x = l_0/r_x \quad \text{(IX.20)}$$

In a column built up of two similar shapes, the radius of gyration $r_x$ of the entire section is equal to $r_x$ of one side member.

Because the side member portions between the joints in the lacing are compliant about the free axis, $y$-$y$, in design calculations we adopt a reduced slenderness ratio $\lambda_{red} < \lambda_y$.

For a column built up of two side members laced together by rectangular strips

$$\lambda_{red} = \sqrt{\lambda_y^2 + \lambda_{s-m}^2} \quad \text{(IX.21)}$$

where

$$\lambda_y = l_{0y}/r_y \text{ and } \lambda_{s-m} = l_{s-m}/r_{s-m} \leqslant 40 \quad \text{(IX.22)}$$

Here, $l_{s-m}$ is the clear space between the lacing strips (see Fig. IX.7*c*), and $r_{s-m}$ is the radius of gyration of the side member about the $y_0$-$y_0$ axis passing through the centroid of the side member (to be looked up in the listing of rolled products).

The maximum slenderness ratio of columns is $\lambda_{max} = 120$.

In buckling, the lacing strips should take up a fictitious shearing force which for columns made of class C38/23 steel is assumed as

$$Q_{fict} = 20F_{gr}$$

The strips on either side of a column resist the shearing force given by

$$Q_{str} = Q_{fict}/2 = 20F_{gr}/2 = 10F_{gr} \quad \text{(IX.23)}$$

From the condition for equilibrium of a column portion (see Fig. IX.7*d*), we may write

$$Q_{str}l/2 = T_{str}c/2$$

where $T_{str}$ is the shearing force acting in the strip, $l$ is the centre-to-centre distance between the lacing strips, and $c$ is the distance between the column axes.

Hence,

$$T_{str} = Q_{str}l/c \qquad \text{(IX.24)}$$

The bending moment in the strip is given by

$$M_{str} = T_{str}c/2 = (Q_{str}l/c)\,(c/2) = Q_{str}l/2 \qquad \text{(IX.25)}$$

The thickness $\delta$ of the lacing strips usually ranges between 6 and 10 mm, and their width $d$ is taken as 0.5 to 0.75 of the column section width. The flexural strength of a strip is assumed to be adequate if the strength of the fillet weld fastening the strip to the column meets appropriate requirements.

The length of the fillet weld is taken equal to the strip width $d$, and the effective length of the weld (with allowance for poor penetration and fusion at the ends) as $l'_{weld} = d - 10$ mm. The weld carries normal stresses $\sigma_{weld}$ induced by the moment $M_{str}$ and tangential stresses $\tau_{weld}$ due to the shearing force $T_{str}$:

$$\sigma_{weld} = M_{str}/W_{weld} = 3Q_{str}l/0.7h_{weld}l^2_{weld} \qquad \text{(IX.26)}$$

$$\tau_{weld} = T_{str}/F_{weld} = Q_{str}l/c0.7h_{weld}l_{weld} \qquad \text{(IX.27)}$$

The weld is checked for strength subject to

$$\sigma_{des} = \sqrt{\sigma^2_{weld} + \tau^2_{weld}} \leqslant R^w_f \qquad \text{(IX.28)}$$

As previously mentioned, the column base is intended to spread the column load over the area of the footing.

The area of the base plate, $F_{b\text{-}pl}$, is chosen so that the stress beneath the plate cannot exceed the design strength of the concrete in bearing:

$$N/F_{b\text{-}pl} \leqslant R_b \qquad \text{(IX.29)}$$

The value of $R_b$ depends on the ratio of the area in bearing (the base plate area) to the total area $F_f$ of the footing

carrying the column base, and also on the design strength of concrete $R_{pr}$:

$$R_b = R_{pr} \sqrt{F_f/F_{b\text{-}pl}} \leqslant 2R_{pr}$$

The base plate may be taken larger in size than is required by design calculations so as to accommodate the wing-plates, stiffeners, and binding bolts. The minimum height of the wing-plates depends on the length of the welds fastening them to the column core. With a certain margin of safety*, the welds are designed to withstand the total longitudinal force $N$.

The contact footing pressure $q = N/F_{b\text{-}pl}$ causes the base plate to work in bending. The bending moment depends on the aspect ratio of the base plate in the portions bounded by the wing-plates, stiffeners, and the column core. If a portion of the base plate is fixed at one side, it works in bending only in one direction as a cantilever. If one side of a portion fixed at three or four sides is not larger than twice the other side, the plate is assumed to be in bending in two directions. Otherwise, we may neglect that the base plate is subjected to bending in the direction of the longer span, and assume that the plate is to resist bending only in the direction of the shorter span.

Portion *1* in the base plate shown in Fig. IX.8*f* behaves as a cantilever, so the bending moment is given by

$$M_1 = qa_1^2/2$$

Portion *2* is supported on three sides; the maximum bending moment is given by

$$M_2 = \beta q a^2$$

where $a$ is the length of the free side and $\beta$ is a coefficient depending on the aspect ratio:

| $b_1/a$ | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 | 1.2 | 1.4 | 2.0 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $\beta$ | 0.060 | 0.074 | 0.088 | 0.097 | 0.107 | 0.112 | 0.120 | 0.126 | 0.132 | 0.133 |

Portion *3* is supported on four sides; the maximum bending moment is given by

$$M_3 = \alpha q a^2$$

* We do not take into consideration that some of the longitudinal force is transferred by the welds fastening the column core directly to the base plate.

where $a$ is the length of the shorter side and $\alpha$ is a coefficient depending on the aspect ratio:

| $b/a$ | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.8 | 2.0 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 0.048 | 0.055 | 0.063 | 0.069 | 0.075 | 0.081 | 0.086 | 0.094 | 0.100 | 0.125 |

The necessary thickness of the base plate is determined as

$$\delta_{b\text{-}pl} = \sqrt{6M/R}$$

where $M$ is the maximum bending moment in the base plate and $R$ is the design bending strength of the steel.

The wing-plates and stiffeners should be placed in such a way that the design effective thickness of the base plate is about the same (usually from 16 to 40 mm) in all portions.

**Example IX.5.** Given: An H-section steel column (Fig. IX.19*a*).

Material: welded sheets of class C38/23 steel.

Design load: $N = 105$ tf (1 050 kN).

Column length: $l = 4.1$ m.

Effective length (with hinged ends): $l_0 = l$.

To find: The load-bearing capacity of the column.

*Solution.* The geometrical characteristics of the column cross section are

$$F = 24 \times 1 \times 2 + 18 \times 1 = 66 \text{ cm}^2$$

$$J_x = 2 \times 1 \times 24^3/12 + 18 \times 1^3/12 = 2\,305 \text{ cm}^4$$

$$J_y = 1 \times 18^3/12 + 2\,(24 \times 1^3/12 + 24 \times 1 \times 9.5^2)$$
$$= 4\,800 \text{ cm}^4$$

$$r_x = \sqrt{2\,305/66} = 5.9 \text{ cm}$$

$$r_y = \sqrt{4\,800/66} = 8.5 \text{ cm}$$

$$\lambda_{max} = l_0/r_{min} = l_0/r_x = 410/5.9 = 70 < 120$$

For $\lambda = 70$, $\varphi = 0.77$.

A check on column stability according to Eq. (IX.19) gives

$$N/(F_{gr}\varphi) = 105\,000/(66 \times 0.77) = 2\,060 \text{ kgf cm}^{-2} \text{ (206 MPa)}$$
$$< 2\,100 \text{ kgf cm}^{-2} \text{ (210 MPa)}$$

As is seen, the column has an adequate load-carrying capacity.

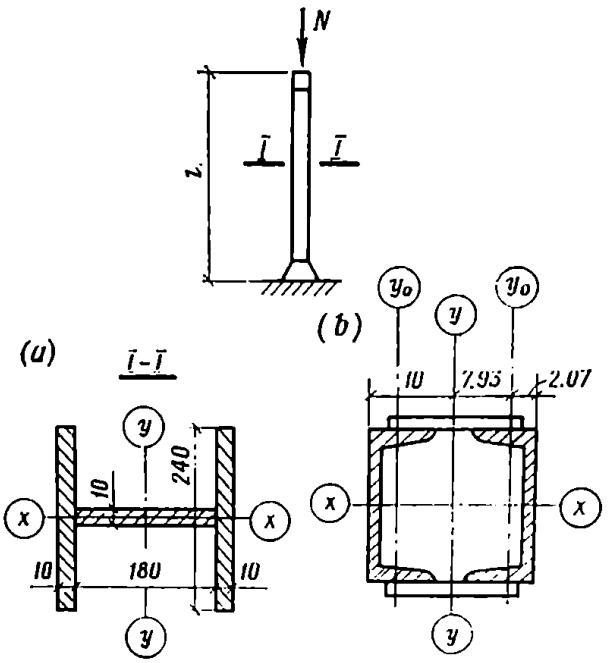



Fig. IX.10. Column design examples

**Example IX.6.** Given: A column built up of two size 20 channels with a fixed lower end and a hinged upper end (Fig. IX.10*b*).

Column width: $b = 20$ cm.

Column length: $l = 6$ m.

Material: class C38/23 steel.

Clear space between lacing strips: $l_{str} = 70$ cm.

Strip dimensions: $d = 15$ cm, $\delta = 6$ mm.

Throat thickness of welds fastening lacing strips to side members: $h_{weld} = 6$ mm.

Effective length: $l_0 = 0.7 \times l = 0.7 \times 6 = 4.2$m.

To find: The design load.

*Solution*: The listing of rolled products (see Appendix XI) gives the channel characteristics

$$F = 23.4 \text{ cm}^2$$
$$J_x = 1\,520 \text{ cm}^4$$
$$J_{y_0} = 113 \text{ cm}^4$$
$$r_x = 8.07 \text{ cm}$$
$$r_{y_0} = 2.2 \text{ cm}$$

The distance from the outer face of the web to the section centroid is 2.07 cm.

Equation (IX.20) gives the slenderness ratio about the material axis to be

$$\lambda_x = l_0/r_x = 420/8.07 = 52$$

To find the reduced slenderness ratio about the free axis $y$-$y$, we calculate

$$J_y = 2\,(113 + 23.4 \times 7.93^2) = 3\,220 \text{ cm}^4$$

$$r_y = \sqrt{3\,220/(23.4 \times 2)} = 8.3 \text{ cm}$$

Hence,

$$\lambda_y = 420/8.3 = 50.5$$

Equation (IX.22) yields the slenderness ratio for the side member

$$\lambda_{s\text{-}m} = l_{s\text{-}m}/r_{y_0} = 70/2.2 = 31.8$$

Equation (IX.21) gives the reduced slenderness ratio

$$\lambda_{red} = \sqrt{\lambda_y^2 + \lambda_{s\text{-}m}^2} = \sqrt{50.5^2 + 31.8^2} = 60$$

which is larger than $\lambda_x = 52$.

For $\lambda = 60$, $\varphi = 0.82$.

Recalling Eq. (IX.19), we find the design load imposed on the column

$$N = RF\varphi = 2\,100 \times 2 \times 23.4 \times 0.82$$
$$= 80\,500 \text{ kgf (805 kN)}$$

Now, we check the lacing strips for strength. The centre-to-centre distance between the strips is

$$l = 70 + 15 = 85 \text{ cm}$$

Equation (IX.23) gives

$$Q_{str} = 10\,F_{gr} = 10 \times 2 \times 23.4 = 468 \text{ kgf (4.68 kN)}$$

Equation (IX.24) gives

$$T_{str} = Q_{str}l/c = 468 \times 85/15.86 = 2\,490 \text{ kgf (24.9 kN)}$$

Equation (IX.25) gives

$$M_{str} = Q_{str}l/2 = 468 \times 85/2$$
$$= 19\,900 \text{ kgf cm (199 000 N m)}$$

Next, we check the fillet weld for strength. The weld length is

$$l_{weld} = l - 1 = 15 - 1 = 14 \text{ cm}$$

The moment of resistance of the weld is given by

$$W_{weld} = 0.7 h_{weld} l_{weld}^2/6$$
$$= 0.7 \times 0.6 \times 14^2/6 = 13.7 \text{ cm}^3$$

The cross-sectional area of the weld is

$$F_{weld} = 0.7 h_{weld} l_{weld} = 0.7 \times 0.6 \times 14 = 5.9 \text{ cm}^2$$

Equation (IX.26) yields the normal stress in the weld

$$\sigma_{weld} = M_{str}/W_{weld} = 19\,900/13.7 = 1\,453 \text{ kgf cm}^{-2}$$

Equation (IX.27) gives the tangential stress resisted by the weld

$$\tau_{weld} = T_{str}/F_{weld} = 2\,490/5.9 = 422 \text{ kgf cm}^{-2}$$

Equation (IX.28) gives the design stress in the weld

$$\sigma_{des} = \sqrt{\sigma_{weld}^2 + \tau_{weld}^2} = \sqrt{1\,453^2 + 422^2}$$
$$= 1\,513 \text{ kgf cm}^{-2} (151.3 \text{ MPa}) \approx R_f^w$$
$$= 1'500 \text{ kgf cm}^{-2} (150 \text{ MPa})$$

As is seen, the weld is sufficiently strong.

**Columns in Eccentrical Compression.** Columns in eccentrical compression are simultaneously loaded by a longitudinal force $N$ and a bending moment $M$. In single-storey industrial buildings, columns are assumed to work in eccentrical compression. They are combined by the roof to form a transverse frame of a building. Such columns may be solid or open-work. The cross section of an eccentrically loaded column may be constant or it may vary along the height (as in stepped columns). Most frequently, use is made of stepped columns (Fig. IX.11*b* and *c*). A stepped column has a built-up H-section welded of three sheets above the crane beam it carries. Below the crane beam, it is built up of two side members joined either by cover plates (in which case, we speak of a solid column) or lacing strips (in which case, we speak of an open-work column). The side members are generally made of rolled sections, such as I-beams, channels, or sheets.

If a side member is intended to carry heavy loads, its I-section may be welded of three sheets, and the channel section may be welded of a sheet and two angles.

The web of solid columns is relatively thin, being 1/100 to 1/120 of the column length. The local stability of the web is enhanced by stiffeners welded pairwise on either side of the web and spaced a distance $2.5h$ to $3h$ apart (see Fig. IX.11*b*).

The lacing of open-work columns is placed in two planes. It is usually assembled from single angles forming a triangular mesh with additional struts (see Fig. IX.11*c*).

Low buildings equipped with overhead travelling cranes having a lifting power of 15 to 20 t may use constant-section solid columns (Fig. IX.11*a*). Such columns are assembled from wide-flange rolled I-sections or welded of three sheets. In open-work columns, one side member extends throughout the entire height of a building, giving support to the roof, and the other goes as far as the bottom of the crane beam (Fig. IX.11*d*). The two side members are joined together by horizontal strips spaced so as to make the crane side member equally slender both in and across the transverse frame of the building.

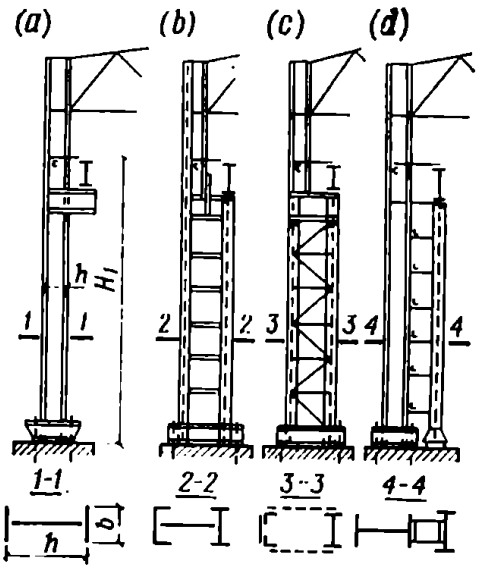



Fig. IX.11. Steel columns in eccentrical compression

(*a*) constant-section solid column; (*b*) stepped solid column; (*c*) stepped open-work column; (*d*) composite column

As a rule, columns in eccentrical compression are checked for stability.

The critical longitudinal force $N_{cr}$ at which an eccentrically loaded column may lose stability, and the respective critical stresses $\sigma_{cr}^{ext} < \sigma_y$ depend on the initial eccentricity of the longitudinal force, $e = M/N$, the column slenderness ratio $\lambda$, and the geometrical characteristics of the section.

Eccentrically loaded columns are checked for stability in the plane of flexure (in the plane of the transverse frame) subject to

$$N/(\varphi^{ext} F_{gr}) \leqslant R \qquad \text{(IX.30)}$$

where $\varphi^{ext} < 1$ can be found in Appendix XIII according to the assumed slenderness ratio $\bar{\lambda} = \lambda\sqrt{R/E}$ and the reduced eccentricity $m_1$ of the longitudinal force.

Across the plane of flexure (across the plane of the transverse frame), the column is checked for stability subject to

$$N/(c\varphi_y F_{gr}) \leqslant R \qquad \text{(IX.31)}$$

where $\varphi_y$ is the buckling factor for an eccentrically compressed column, depending on the slenderness ratio $\lambda_y$ (across the plane of the frame), and $c$ is a coefficient accounting for the effect that the moment acting in the frame plane has on the column stability across this plane; it is found as prescribed in Appendices XIV and XV.

In open-work columns, it is also necessary to check the side members for stability. In this case, the side members are regarded as columns in axial compression. For a symmetrical cross section, the longitudinal force acting in the side member is given by

$$N_{s\text{-}m} = N/2 + M/h \qquad \text{(IX.32)}$$

where $h$ is the distance between the centroids of the side members.

The web members are designed to carry a shearing force $Q^*$ and the force induced by the contraction of the side members under the action of the longitudinal force**.

The force in a diagonal of cross-sectional area $F_d$ is

$$N_d = Q/2 \sin\alpha + F_d N \sin^2\alpha/F \qquad \text{(IX.33)}$$

where $\alpha$ is the diagonal angle (usually equal to 45°) and $F$ is the total cross-sectional area of the side members.

Compressive diagonals are checked for stability subject to Eq. (IX.19) where the slenderness ratio is determined

---

* If $Q < Q_{fict}$, we adopt $Q = Q_{fict} = 20F_{gr}$ (for class C38/23 steel).

** The longitudinal compressive force likewise shortens the web members.

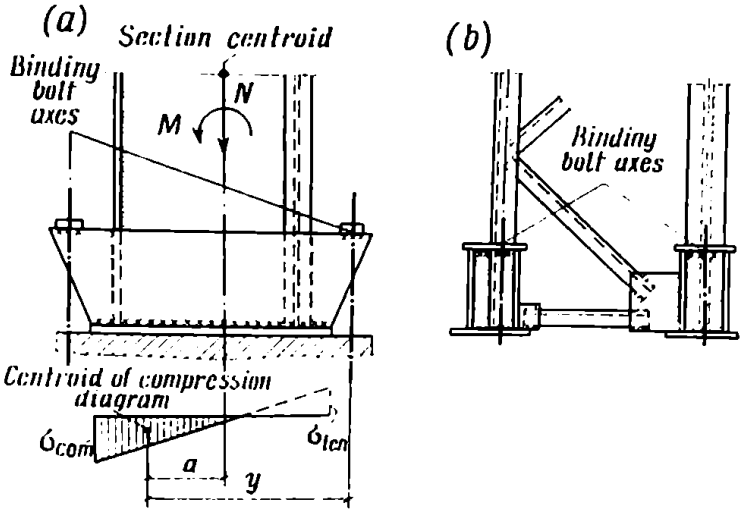



Fig. IX.12. (*a*) Base of a solid column in eccentrical compression and (*b*) base of an open-work column in eccentrical compression

according to the least radius of gyration and $R$ is multiplied by the service factor $m = 0.75$, because the diagonals are fastened to the side members by one flange.

The base plate dimensions should be such that the stress in the concrete footing cannot exceed $R_{pr}$. For the base of a solid column (Fig. IX.12*a*), the marginal stresses beneath the base plate are given by

$$\sigma = N/F_{b\text{-}pl} \pm M/W_{b\text{-}pl} \qquad \text{(IX.34)}$$

where $W_{b\text{-}pl}$ is the moment of resistance of the base sheet about the axis perpendicular to the plane of flexure. With the plus sign, we get maximum compressive stresses, and with the minus sign, maximum tensile stresses. As already noted, $\sigma_{com}$ may not exceed $R_{pr}$.

The tensile forces $Z$ should be taken up by the binding bolts:

$$Z = (M - Na)/y \qquad \text{(IX.35)}$$

where $M$ is the design bending moment at the bottom section of the column (see Fig. IX.12*a*), $N$ is the design longitudinal force, $a$ is the distance from the centroid of the compressive stress diagram to the column axis, and $y$ is the distance from

the axis of the binding bolts to the centroid of the compressive stress diagram.

The necessary cross-sectional area of the binding bolts is found as

$$F_b = Z/(nR^b_{ten}) \qquad \text{(IX.36)}$$

where $n$ is the number of binding bolts, and $R^b_{ten}$ is the design tensile strength of binding bolts, being equal to 1 400 kgf cm$^{-2}$ (140 MPa) for bolts made of grade ВСт3кп2 steel.

In open-work columns, each side member should preferably have a base of its own (Fig. IX.12*b*). Each base is designed to carry the maximum compressive force $N_{s\text{-}m}$ similarly to columns in axial compression.

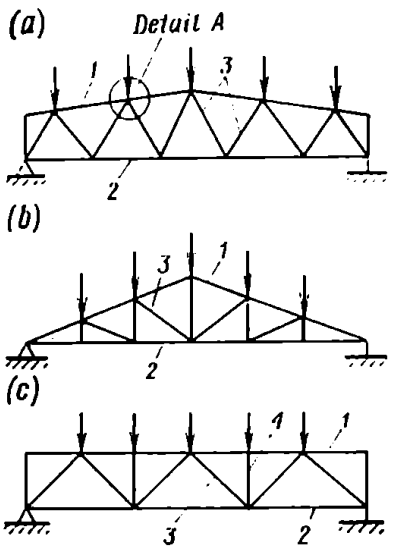



Fig. IX.13. Steel trusses
*1*—top chord; *2*—bottom chord; *3*—diagonals; *4*—verticals; *5*—gusset

## IX.5. Design and Proportioning of Trusses

A truss is an open-work (lattice) structure which is often used as a bearing member for building roofs. The members of a truss are commonly called chords and web members. The uppermost longitudinal members are referred to as the top chord, and the lower ones the bottom chord. The interior members are called web members; these may be classed as verticals or diagonals depending upon their orientation. The joints are often referred to as panel points. There are many types of steel trusses used in building construction, and they are usually designated by the names of the men who introduced the particular system of web bracing. Of these the quadrangular Warren (Fig. IX.13*a*), Howe (Fig. IX.13*b*), and flat-top Warren (Fig. IX.13*c*) are the most common. The system of web bracing depends on the truss configuration, depth, and necessary panel size. The

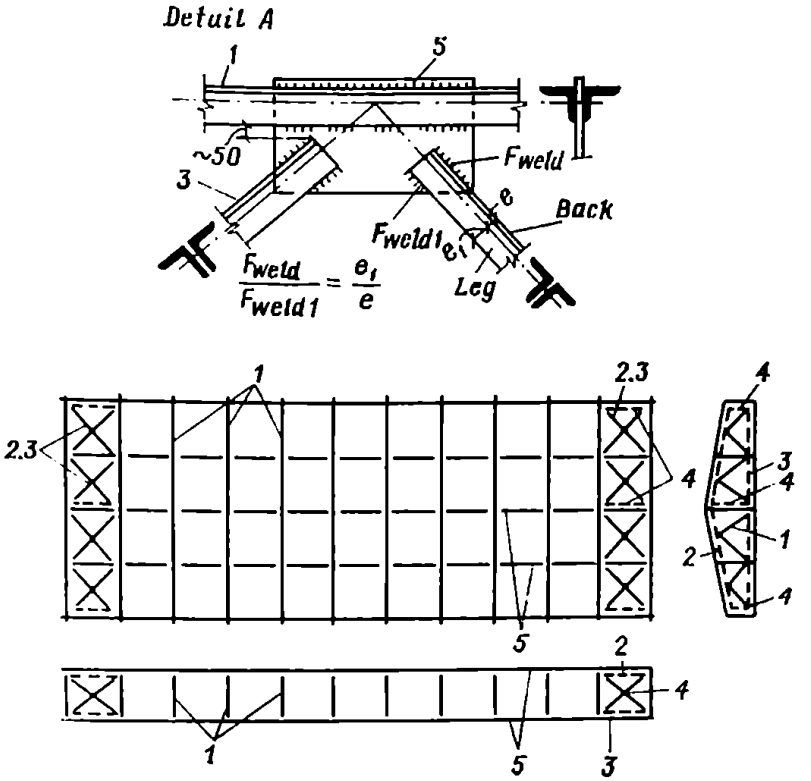



Fig. IX.14. Lateral truss bracing

*1*—trusses; *2*—lateral bracing in plane of top chord; *3*—lateral bracing in plane of bottom chord; *4*—lateral bracing in vertical plane; *5*—spacers

angle of inclination of web diagonal usually ranges between 35° and 45°. Truss members are usually made of twin angles connected at the joints by gussets to which the angles are welded (see Fig. IX.13, Detail *A*).

Across its plane, a truss owes its stability to lateral bracing. Braced trusses form a spatially stable block (Fig. IX.14).

In abstract diagrams, each member of a truss is shown by a single line passing through the centroid of its section. The axes of all members meeting at one joint should intersect at the same point. For design purposes, the members are considered to be hinged at the joints. The member will then be subjected to only a direct stress (either compressive or tensile) which is constant throughout the length of the member, provided the loading is applied only to the joints. The value and sign of the internal forces are found by design

calculations which may be carried out either graphically (by the Maxwell diagram) or analytically (by the method of sections). The top chord of a truss is in compression, and the bottom chord in tension. When a load is applied between the panel points, the axial force in the member is accompanied by a bending moment. Because of this, the cross-sectional area of the member must considerably be increased, so this type of loading should be avoided.

After the internal forces are found, the cross sections of the truss members are determined. The compressive members are designed subject to the condition defined by Eq. (IX.19). The slenderness ratio $\lambda$ of the principal compressive members (chords and support diagonals) should not exceed 120, whereas that of the other compressive members may be as high as 150. When finding the slenderness ratio, the effective length $l_0$ of the chords, support diagonals, and struts in the plane of the truss is taken equal to their actual length $l$ between the centres of the joints (the panel size). For the other web members, the effective length is taken as $l_0 = 0.8l$. Across the truss plane, the effective length of a member is assumed as the distance between the joints fixed against displacement across the truss plane by bracing, roof slab ribs welded to the top chord, etc.

Tensile members are checked subject to

$$N/F_{net} \leqslant R \qquad \text{(IX.37)}$$

In choosing the sections, we should seek to reduce the number of different shapes to a minimum (usually to about 5 or 7).

The size of the welds fastening the truss members to the gussets depends on the design force $N$ and is found by Eq. (IX.2).

In proportioning the joints between the truss members, we should remember that the centroid of the twin angles is closer to the weld on the angle back side and, as a consequence, the force applied to this weld is larger than that applied to the weld on the angle leg side. The total cross-sectional area of the weld, necessary to fasten the member, is inversely proportional to the distance from the weld centroid to the edges (see Fig. IX.13, Detail *A*).

Once the necessary weld length is determined, we choose the gusset size, bearing in mind that the members should be spaced at the joint at least 50 mm apart, and the gusset shape should be as simple as possible.

## IX.6. Prestressed Steel Structures

To prestress is to artificially induce compressive stresses in a structural element before it is loaded so that any tensile stresses which might be caused by the external dead and live loads are automatically canceled. This extends the range of the elastic behaviour of the member and makes it possible to use high-strength cables and wire strands which are economical of material.

Prestressing may be accomplished by inducing a hog or by tensioning high-strength strands which are part of a beam. Figure IX.15*a* shows a prestressed beam built up of two hogged elements welded in the hogged condition by longitudinal welds. The stress curve after manufacture is shown in Fig. IX.15*b*. This curve is added together with the curve of stresses induced by the external load (Fig. IX.15*c*). As the external load increases, the resultant stress curve (Fig. IX.15*d*) becomes almost rectangular. Thus, the load-bearing capacity of the beam is considered to be exhausted when the stresses reach the ultimate strength over the entire section, rather

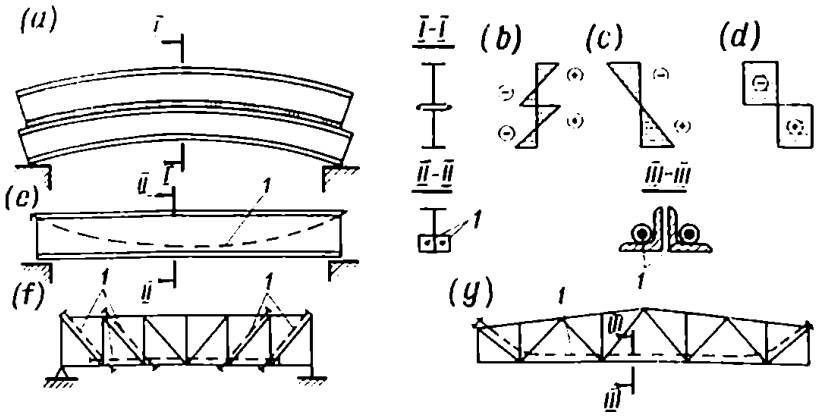



Fig. IX.15. Prestressed structures
1—prestressing strands

than only in the outer fibre as is assumed in non-prestressed beams. Figure IX.15*e* shows a beam prestressed by high-strength wire strands. The beam and the wire strands form a combined system in which the high tensile strength of the strands is used to improve the load-bearing capacity of the beam.

A similar manner of prestressing applies to steel trusses which are prestressed by wire strands. Here, the prestressing strands are placed individually along the main tensile members as shown in Fig. IX.15*f*, or a common wire strand is used for the entire truss (Fig. IX.15*g*).

# CHAPTER TEN

# DESIGN OF BUILDINGS

## X.1. Principles of Building Layout

Any building is made up of individual members combined in a single space structure. The requirements that the various structural members are to meet depend on their purpose and behaviour under load. For example, floor members should sufficiently be strong and rigid so that their sag cannot disturb the behaviour of the entire building; walls and columns supporting the floor should likewise be strong and stable. In addition, reinforced concrete members should sufficiently be crack-resistant so that cracks developing in the tension zone cannot exceed a certain limiting width. Moreover, some prestressed concrete elements are not allowed to show any cracks at all.

A building as a whole must have an adequate spatial rigidity, that is, it must stand up well to all kinds of vertical and horizontal loads.

In construction, buildings may be of the skeleton type or of the bearing-wall type.

Buildings of the bearing-wall construction owe their spatial rigidity to the co-operation between the longitudinal and transverse walls combined into a single space structure by floors. Brick- and blockwork buildings of the bearing-wall construction have been discussed in Sec. VIII.4.

Skeleton-type buildings may, in turn, be classed into frame and frame-shear wall types. Frames are constituted by girders and columns which are generally rigidly fixed at the joints. Shear walls are usually masonry walls, reinforced concrete walls, or steel braces. In frame-type buildings, all external loads are taken up by transverse and longitudinal frames combined into a space structure by floors and a roof (Fig. X.1*a* and *b*). In frame-shear wall buildings, the frames work together with the shear walls (Fig. X.1*c*).

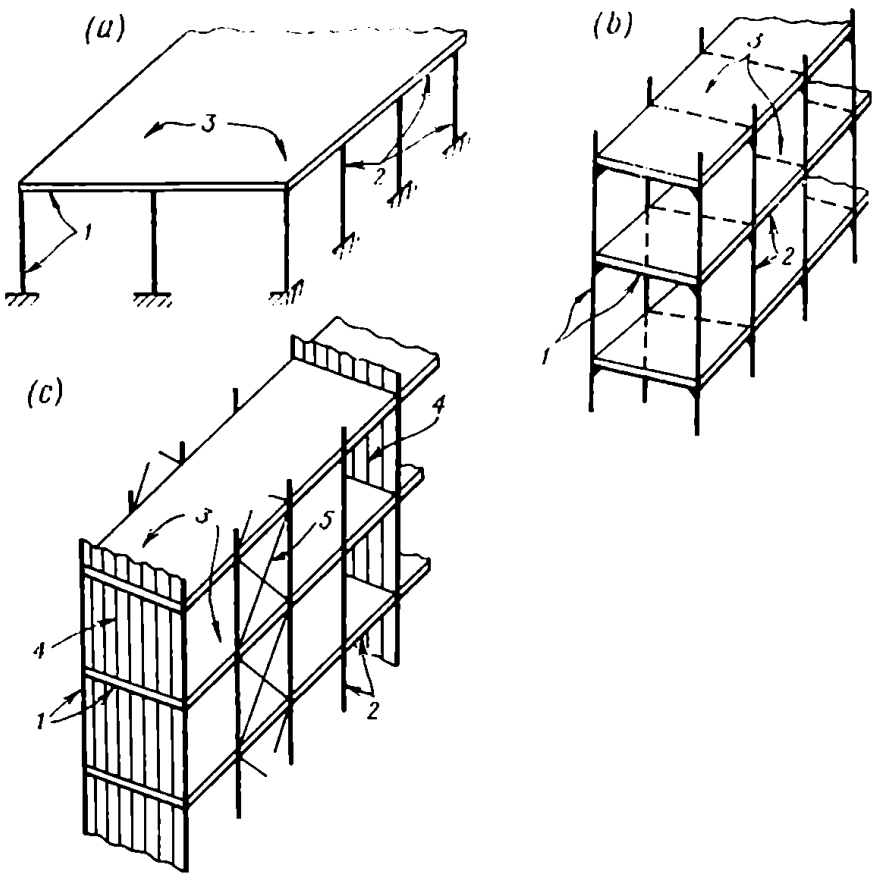



Fig. X.1. Construction types of buildings
*1*—transverse frame; *2*—longitudinal frame; *3*—floor or roof; *4*—shear wall; *5*—steel braces

It is important that the individual structural members and a building as a whole should be strong, rigid, and stable at all stages of erection. For this purpose, temporary braces, spreaders, and similar elements are provided by design.

In a spatially rigid building, a load imposed on a building member is shared by the other members.

Whatever construction type, buildings should be designed so as to make the most of machines and mechanisms in the erection, and to keep labour requirements and the cost of materials to a minimum. From this point of view, the most suitable are prefabricated structures.

Prefabrication of structural members is most efficient when a factory can produce each type of elements on a repe-

titive basis. In view of this, it pays to limit the number of member types used in a building, and adapt them to as large a number of building designs as possible. For this purpose, the main overall dimensions and design types of buildings are standardized and the main dimensions in plan and height are reduced to a limited range of standard sizes on the basis of a unified modular system*.

For example, one-storey industrial buildings including water supply and sewage disposal buildings (such as buildings for pumping stations, blowers, chlorinating stations, and the like) and boiler houses have dimensions taken as multiples of 3 m (or 6 m) in plan and as multiples of 1.2 m in height. Their standardized design type is based on hinged joints between the roof and the walls or columns, so that the roof can be assembled from elements of the same type irrespective of the height of the building and presence of overhead travelling cranes or buried sections (see Secs. X.4 and X.5).

For each element of a building, the most rational type is selected as a standard. Type-design elements are manufactured in quantity and constitute the main type of products turned out by construction industry plants in the USSR. The necessary data on standard prefabricated members can be found in appropriate parts kits which are intended as guides in the design of buildings and structures. With progress in technology and experience, standard elements are continually improved, so updated kits containing advanced design types are published from time to time.

The members for prefabricated buildings must be designed as large as may be permitted by the lifting power of available erection mechanisms, load-carrying capacity of vehicles, and methods of transportation. Large-size units cut down both the number of units to be installed and the time necessary to erect a building, and this requirement must be provided for at the time of design.

The designer should also remember that prefabricated members should readily be adaptable to streamlined manufacture and erection.

---

* Member sizes are chosen to form a modular system based on a 100-mm module or a larger module which is a multiple of 100 mm.

When lifted, moved, or placed in permanent position, prefabricated members are subjected to loads induced by their self-weight. In any of these situations, the disposition of loads may considerably differ from that existing in service. For instance, columns are intended to work in compression in service, whereas in transit and erection they work in bending as beams under the action of the self-weight; trusses and beams are supported at their ends in service, but at the points where slings are applied when lifted.

Prefabricated members should be designed to carry their self-weight multiplied by a dynamic factor of 1.5 for lifting and erection and 1.8 for transit*.

For the dimensional coordination of standard building members in the USSR, three dimension categories have been set up, namely nominal, structural, and actual. Nominal dimensions refer to the overall dimensions of a member as applied to the standardized dimensions of a building. Structural dimensions (those given in drawings) differ from nominal by the width of gaps and joints. Actual dimensions depend on the accuracy of manufacture and may differ from structural dimensions by a certain value, known as tolerance, which may range between 3 and 10 mm. For example, a reinforced concrete roof slab in an industrial building has nominal dimensions of 6 × 3 m and structural dimensions of 5.96 × 2.98 m.

## X.2. Reinforced Concrete Floors

**General.** Reinforced concrete floors may be built of precast members, cast in-situ, or erected as a combination of precast and in-situ members. There are two basic types of floor design, namely beam and girder construction and flat-slab construction. The former refers to floors formed by slabs or floor panels supported by and acting together with beams running in one or two directions. In the flat-slab construction, flat slabs are carried directly by columns flared at the top to form what are called capitals.

---

* In this case, the load induced by the self-weight is taken without any overload factor applied.

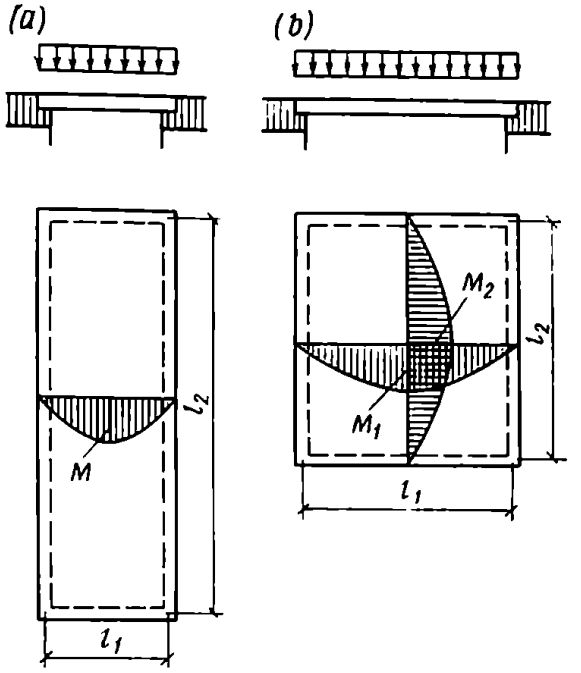



Fig. X.2. Slab classification according to the aspect ratio
(*a*) one-way slab; (*b*) two-way slab

The type of floor and its layout (column grid and the direction of and spacing between beams) depend on the purpose of a building, its layout, and design, and also on the live load imposed on the floor, which is given by relevant standards for residential and office buildings, or is chosen according to the production process for industrial buildings. The values of the characteristic live loads and respective overload factors are given in Appendix II.

When choosing the design type of a floor, it is necessary to compare several design alternatives in terms of the material consumption, labour requirements, and cost.

According to the aspect ratio, floor slabs may be classed as follows. When $l_2/l_1 \geqslant 2$ (Fig. X.2*a*), we speak of a one-way slab subjected to bending along its shorter side. When $l_2/l_1 \leqslant 2$ (Fig. X.2*b*), we speak of a two-way slab subjected to bending in two directions.

Statically indeterminate beams and slabs for reinforced concrete floors are designed with allowance for the redist-

ribution of internal forces which may be caused by cracking, inelastic properties of concrete and steel, and partial loss of bond between the steel and the concrete. This makes it possible to distribute reinforcement in the most rational way and sometimes cut down the amount of reinforcing steel. Let us discuss it in more detail.

As the external load increases, the stresses in the sections of a reinforced concrete member rise, too. When the tensile reinforcement at the most stressed section reaches its yield point, the highly stressed concrete in the compression zone undergoes a considerable inelastic deformation. This state of stress appearing in one of the sections of a statically determinate member causes its portions lying to the left and right of the section in question to rotate. As a result, the member fails before the stress in the concrete can reach the ultimate compressive strength [Stage III of the stress-strain state (see Chap. VI)].

If the stress in the tensile steel reaches the ultimate strength in any section of a statically indeterminate member, no failure takes place at this section, because the fixed ends prevent the member portions from rotation. The section in question undergoes a considerable local deformation, but it is still able to take up the moment

$$M_{pl} = \sigma_y F_s z_c$$

where $F_s$ is the steel area and $z_c$ is the arm of the internal couple.

In design calculations, this moment is taken as

$$M = R_s F_s z_c$$

The beam portion experiencing the above state of stress is customarily called the plastic hinge. As the load rises further, the moment remains unchanged at the plastic hinge and increases at the other sections—what is known as redistribution of moments takes place.

Plastic hinges may develop consecutively at several sections of a member until it becomes statically determinate. After that, the appearance of one more plastic hinge will cause failure.

As an example, let us consider a single-span statically indeterminate beam of span $l$ with fixed ends, subjected to a uniformly distributed load $q$ (Fig. X.3).

The ultimate bending moments taken up by the span and support sections of the beam are

$$M_{sp} = \sigma_y F_s^{sp} z_c^{sp} \quad \text{and}$$

$$M_{sup} = \sigma_y F_s^{sup} z_c^{sup}$$

In statically determinate beams, the sum of moments in the span and at the support is equal to the moment $M_0$ that would be taken up by the same beam if it were free-supported:

$$M_{sp} + M_{sup} = M_0 \quad \text{(X.1)}$$

With a uniformly distributed load,

$$M_0 = ql^2/8$$

For our case, we may write

$$\sigma_y F_s^{sp} z_c^{sp} + \sigma_y F_s^{sup} z_c^{sup} = ql^2/8$$

Assuming different values of $F_s^{sp}$ and $F_s^{sup}$, we obtain different values of the moments in the span and at the support.

With elastic design, the moments at the supports are

$$M_{sup} = ql^2/12$$

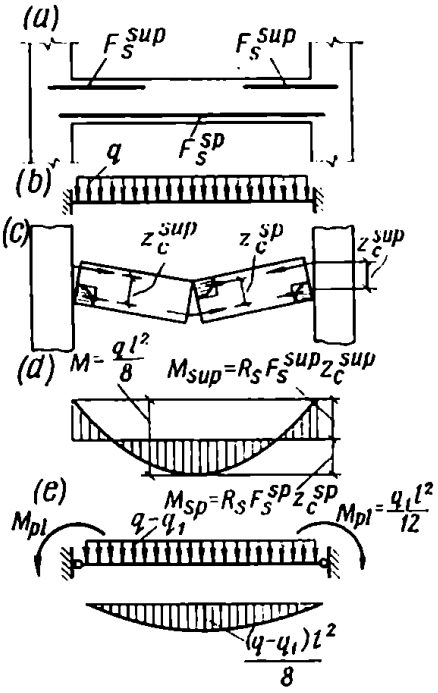



Fig. X.3. To the design of a single-span statically indeterminate reinforced concrete beam with allowance for the redistribution of forces

(a) distribution of reinforcement; (b) loading system; (c) formation of plastic hinges; (d) bending moment diagram; (e) loading system and bending moment diagram due to load $q - q_1$ after formation of plastic hinges at supports

and the moment at mid-span is

$$M_{sp} = ql^2/24$$

Thus,

$$M_{sup} + M_{sp} = ql^2/8 = M_0$$

If the amount of reinforcement placed at the beam supports is less than that required to carry the above support

moment, plastic hinges will develop already at $q_1 < q$, and an additional load $(q - q_1)$ will build up moments only in the span. For instance, if the end portions of a beam are reinforced to carry the moment $M_{sup} = ql^2/16$, the amount of steel in the span should be enough to carry the moment

$$M_{sp} = M_0 - M_{sup} = ql^2/8 - ql^2/16 = ql^2/16$$

As is seen, the manner in which the redistribution of moments takes place depends on the amount and distribution of reinforcement and may be specified in advance at the design stage.

To limit the width of cracks at the section likely to develop a plastic hinge, the redistributed moment $M$ should not differ from the respective elastic moment by more than 30%. The reinforcement ratio at sections containing a plastic hinge is subject to

$$\xi = x/h_0 \leqslant 0.35$$

**Precast Slab and Girder Floors.** A precast slab-and-girder floor consists essentially of floor slabs (or panels) supported by girders. Single- or multispan girders may be longitudinal (directed along the building) and transverse (Fig. X.4). In civic buildings, the spans are taken as multiples of a 400-mm module; in industrial buildings, the column grid is any one of the following: 6 × 6 m, 6 × 9 m, or 6 × 12 m.

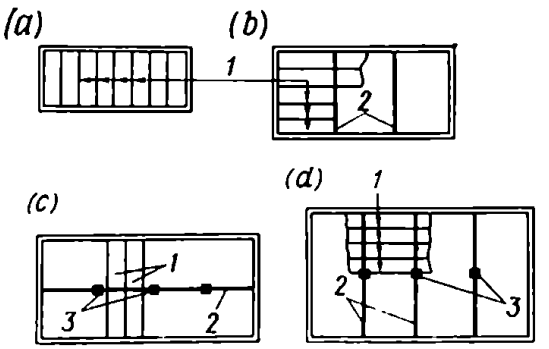



Fig. X.4. Precast slab and girder floors

(*a*) without girders; (*b*) with single-span transverse girders; (*c*) and (*d*) with multispan longitudinal or transverse girders; *1*—slabs; *2*—girders; *3*—columns

Multispan girders are generally assembled from appropriately joined single-span girders. Single-span girders may have hinged supports at the columns. Girders can be rectangular, T-sections with the flange facing up, and T-sections with flange facing down (Fig. X.5*a*). The depth $h$ of precast girders usually ranges between $l/10$ and $l/14$, where $l$ is the girder span, and the width $b$ ranges between $0.3h$ and $0.4h$.

According to their cross section, floor slabs may be classed into hollow-core slabs, ribbed slabs, and solid slabs (Fig. X.5*b*, *c* and *d*). Hollow-core slabs are used in civic buildings where live loads on the floors are low and the ceiling should be smooth. Ribbed slabs are preferably used in industrial buildings. Solid slabs are heavy, so their use is warranted for small spans only.

Panels may be made in either ordinary reinforced concrete or prestressed (usually pretensioned) concrete. The depth of non-prestressed slabs is taken from $l/15$ to $l/20$, and that of prestressed slabs, from $l/20$ to $l/30$, where $l$ is the slab span.

Floor slabs are supported by girders or walls. In design calculations, they are regarded as single-span beams with free supports, subjected to a uniformly distributed load which is the sum of the self-weight, the weight of the topping, and the live load (Fig. X.6*a*). The load $q$ per metre run of a slab is given by

$$q = q_1 b_{sl}$$

where $q_1$ is the load per square metre and $b_{sl}$ is the slab width.

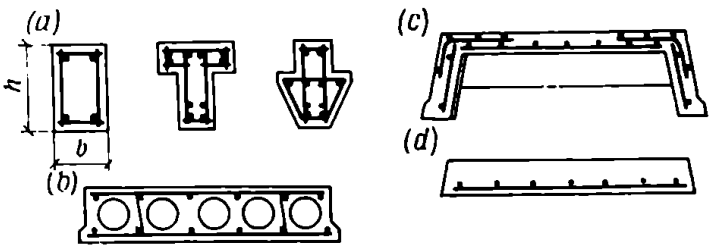



Fig. X.5. Cross sections of prefabricated members of slab and girder floors

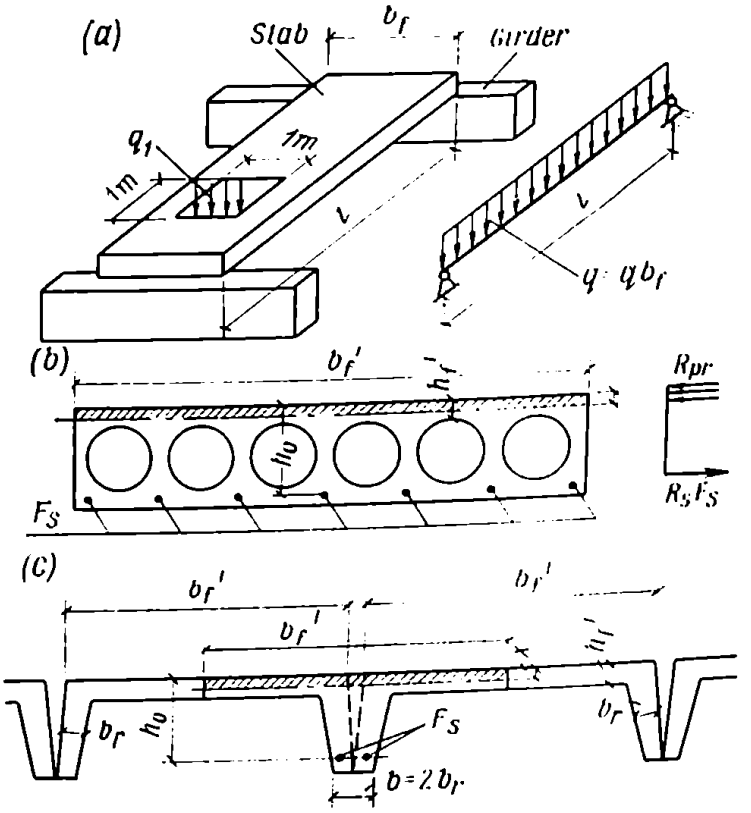



Fig. X.6. Loading system and design sections of floor slabs

The maximum bending moment at mid-span is $M = 0.125\, ql^2$, and the maximum shearing force at the support is $Q = 0.5\, ql$.

The area $F_s$ of the longitudinal tensile steel necessary to ensure the normal-section strength is determined as prescribed in Sec. VI.3. The design cross section of slabs is taken to be rectangular of width $b'_{sl}$ because the depth of the compression zone $x$ at Stage III of the stress-strain state is usually smaller than the depth of the upper flange $h'_f$ (Fig. X.6*b* and *c*).

The inclined-section strength of slabs is determined as prescribed in Sec. VI.4.

If

$$Q \leqslant k_1 R_{t\,en} b h_0$$

where $b$ is the total width of ribs, transverse reinforcement is placed for structural reasons. Otherwise, the area of the

transverse bars in the mats placed in the ribs is found by design calculations. In round-void slabs, $Q$ is generally smaller than $k_1 R_{ten} b h_0$, so bar mats are placed only at the slab ends over the length $l/4$.

As already mentioned, the behaviour of the flange between ribs depends on the aspect ratio of a slab. If a slab has no ribs or the distance $l_2$ between the transverse ribs is twice the distance $l_1$ between the longitudinal ribs, the slab behaves as a one-way slab (Fig. X.7*a*). With allowance for the redistribution of internal forces, the bending moment is defined as

$$M = q^1 l_1^2/11$$

where $q^1$ is the load per square metre of the flange area.

If $0.5 < l_2/l_1 < 2$ (Fig. X.7*b*), the slab behaves as a two-way slab supported on four sides. The design procedure for slabs will be discussed later. When transverse ribs are spaced close together so that $l_1/l_2 > 2$ (Fig. X.7*c*), the slab

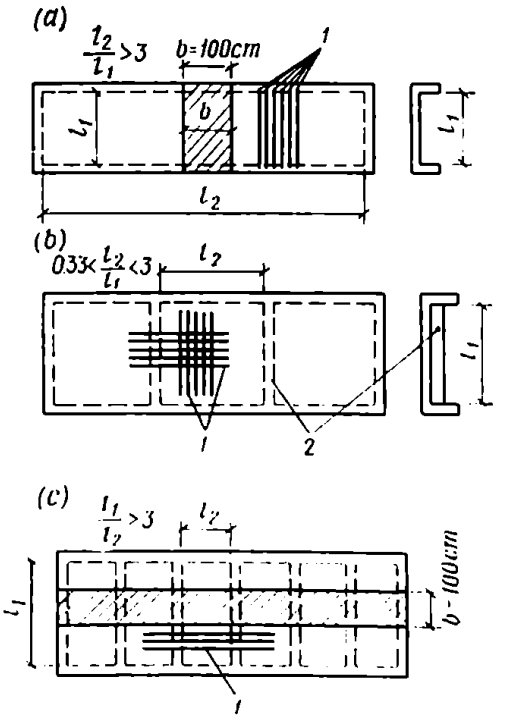



Fig. X.7. To the strength analysis of the slab flange for local bending between the ribs

*1*—slab load-bearing reinforcement; *2*—slab transverse ribs

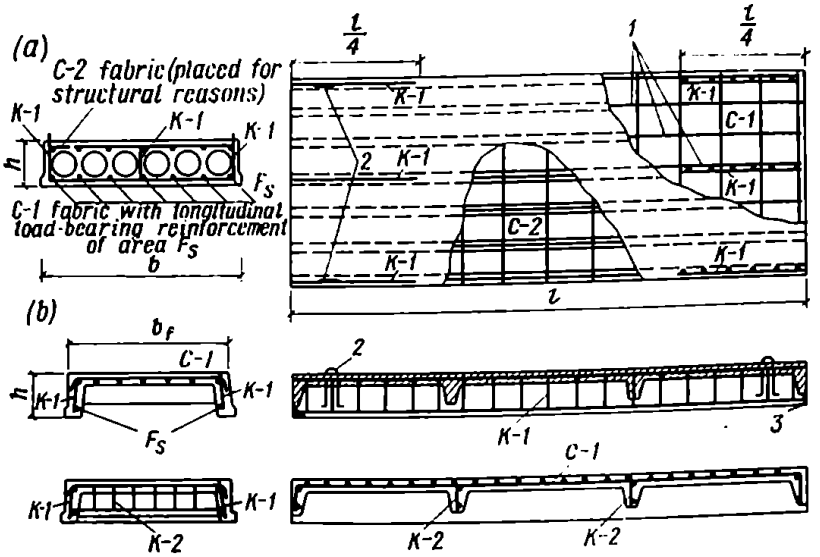



Fig. X.8. (*a*) Distribution of reinforcement in a hollow-core slab and (*b*) in a ribbed slab

*1*—load-bearing reinforcement of area $F_s$; *2*—lifting eyes; *3*—anchor of longitudinal load-bearing reinforcement of area $F_s$ (prestressed reinforcement of area $F_s$ consists of individual bars which are not members of bar mats)

works as a one-way slab in the direction $l_2$. Transverse ribs (or joints) are designed as single-span beams supported by the longitudinal ribs to carry the load imposed by the flange. For design purposes, the transverse ribs are taken to be T-sections. They are usually reinforced with one flat bar mat.

The distribution of reinforcement in hollow-core and ribbed slabs is shown in Fig. X.8.

Once the strength of slabs is determined, it is necessary to carry out a crack-opening analysis and to find the sag as prescribed in Secs. VI.6 and VI.7. This is done with ribbed slabs reduced to a T-section, and hollow-core slabs to an I-section as shown in Fig. X.9.

The design of a girder consists in determining the load imposed on it, choosing the cross-sectional dimensions, and finding the necessary amount of reinforcement.

The load imposed on a girder is taken as the load imposed on a strip of width $k$ equal to the distance between the girders. If we designate as $g_1$ the load due to the self-weight of the slabs and topping per square metre, and as $g_{s\text{-}w}$ the

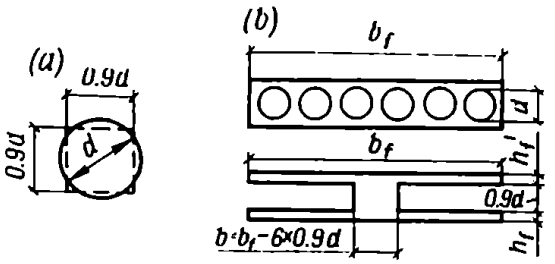



Fig. X.9. To finding slab deformations
(a) substitution of round hole by equivalent rectangular hole; (b) design section of hollow-core slab with round voids

load due to the self-weight of the girder, then the design load per metre run of the girder will be $g = g_1 k + g_{s\text{-}w}$. Accordingly, the live load per metre run of the girder will be given by $p = p_1 k$ (where $p_1$ is the design live load per square metre of floor area).

The total load imposed on the girder will, thus, be $q = g + p$.

In a single-span girder, $M_{max} = 0.125\ ql^2$ and $Q_{max} = 0.5\ ql$.

In continuous girders, bending moments and shearing forces are determined with allowance for the redistribution of internal forces. This means that the elastic moments at the supports are reduced by 30% and the moments in the span are raised subject to the condition of Eq. (X.1). Owing to the reduced moments at the supports, the joint between the girder and a column is much simplified.

The elastic design of a continuous girder, assuming the worst combination of live loads*, is carried out by the general methods of structural mechanics. In the case of equispan girders, it is convenient to use tabulated data**. To demonstrate, for a girder subjected to a uniformly distributed

* The dead load $g$ is considered to be applied simultaneously to all spans. The maximum bending moment in the span is found with the live load $p$ applied to the span in question and then at every other span. The maximum bending moment at the support is determined with the live load applied to the two spans adjoining the support and to every other span.

** See Appendix XV.

load, the bending moment is given by

$$M = (\alpha g + \beta p)\, l^2$$

where $\alpha$ and $\beta$ are tabular coefficients.

If the live load is very small as compared with the dead load, the moments may be determined at the elastic stage with all of the spans simultaneously subjected to the total load $q = g + p$

$$M = \alpha\, ql^2$$

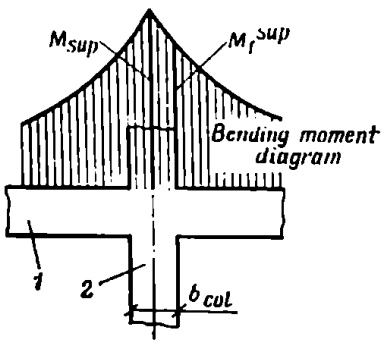



Fig. X.10. Design girder moment at the support
1—girder; 2—column

After the redistribution of bending moments has been found, it is necessary to determine shearing forces. For this purpose, each span is regarded as a simply supported beam carrying its share of load together with the support moments.

On moving away from the support, the negative bending moments in the girder decrease (Fig. X.10). The design section of the girder at the support is that at the face of the column. Here, the bending moment is

$$M' = M_{sup} - Qb_{col}/2$$

Since the above approach envisages the possibility of a plastic hinge developing at the support, the cross-sectional dimensions of the girder should be re-defined so that at the support section

$$\xi = x/h_0 \leqslant 0.35$$

The adopted bending moments are then used to find the area of longitudinal load-bearing steel in the spans and at the supports necessary to obtain the desired normal-section strength. The amount of transverse reinforcement (transverse bars in the mats) must be sufficient to give the desired inclined-section strength.

A joint (at the support) in a multispan girder should ensure the continuity of the girder. Referring to the joint of Fig. X.11*a*, the upper tensile reinforcement of the girder

is welded (during manufacture) to embedded steel parts. The connection bars which are installed during erection are passed through holes in the column and welded to the embedded parts of the girder. At the joint, compressive forces are transferred from the girder to the column cantilever through embedded steel parts welded to one another. After welding, the joint is concreted.

The joint of Fig. X.11*b* is made as follows. The exposed ends of the support reinforcement of the girder are connected to the mating stick-outs in the column with the help of inserts welded in a pool of molten metal contained in a reusable mould. After that, the joint is concreted. The inserts make it possible to reduce the length of stick-outs, thereby avoiding their damage in transit and erection, and also to compensate for a likely misalignment between the girder and the column stick-outs. In a roof girder, connection bars are welded to the girder reinforcement exposed ends in a pool of molten metal and then welded to an embedded part at the column head (see Fig. XII.7).

The tensile force in the connection bars is given by

$$N = M^{I}/z$$

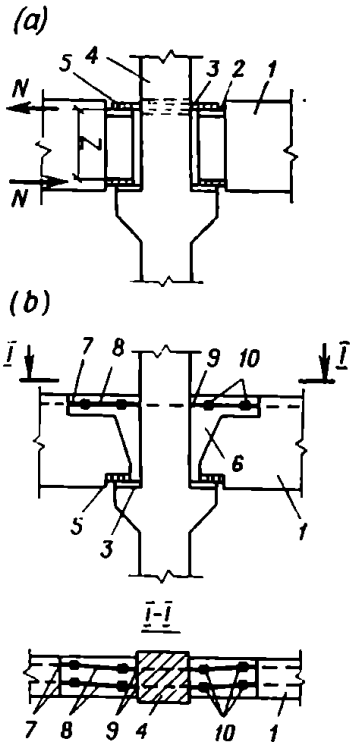



Fig. X.11. Joints in a multi-span girder at the support

*1*—girder; *2*—embedded steel parts; *3*—connection bars; *4*—column; *5*—to be field-welded; *6*—in-situ concrete; *7*—girder reinforcement stick-outs; *8*—connection inserts; *9*—column reinforcement stick-outs; *10*—to be welded in pool of molten metal

where $z$ is the arm of the internal couple (see Fig. X.11*a*). The cross-sectional area of the connection bars necessary to carry this force is

$$F_s = N/R_s$$

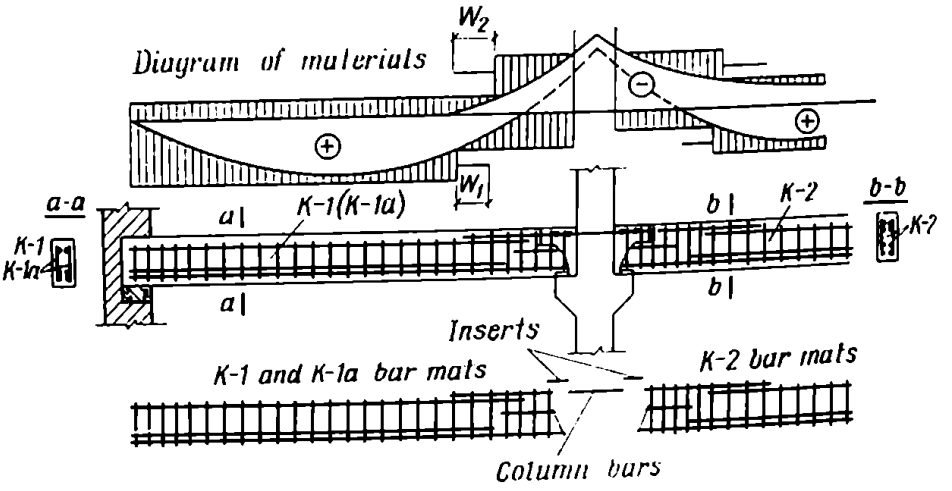



Fig. X.12. Distribution of reinforcement in a multispan girder

The welds should be designed to take up this force increased by 30%. This increase will guarantee that the welds will not break, should a plastic hinge develop in the girder section at the support.

The distribution of reinforcement in a continuous girder is shown in Fig. X.12. In order to save reinforcing steel, it is advisable to cut off some longitudinal bars in proportion to the decrease in the bending moment. Cutoff points are located using appropriate moment diagrams (see Sec. VI.4).

**In-Situ Beam and Girder Floors.** A beam-and-girder floor consists essentially of girders, beams, and a one-way slab* (Fig. X.13) cast in-situ in a common form and combined into a single system.

The load from the slab is transferred to the beams, then from the beams to the girders, and finally from the girders to columns or bearing walls. The minimum slab thickness is 6 cm for civic buildings and 7 cm for industrial buildings. The centre-to-centre distance between the beams is usually 1.8 to 2.5 m, and that between the girders is from 5 to 7 m. The beam depth is taken as $l_b/12$ to $l_b/16$, and that of girders

* The aspect ratio of the slab is $l_1/l_2 \geqslant 3$, so it works in bending across the beams and, consequently, may be regarded as a one-way slab.

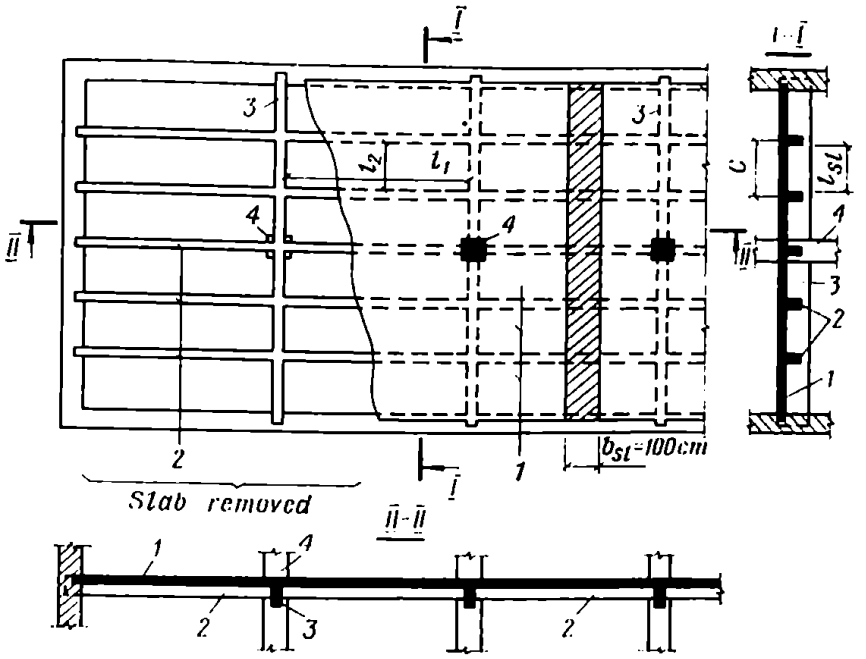



Fig. X.13. In-situ beam and girder floor
*1*—slab; *2*—beams; *3*—girders; *4*—columns

as $l_g/10$ to $l_g/12$. The width of girders and beams may range between $h/2$ to $h/3$.

The design of a one-way slab is carried out for strip 1 m wide cut from the slab parallel to its shorter sides (Fig. X.13). The slab is subjected to a dead load $g$ due to its self-weight and the weight of the topping, and a live load $p$. The total load is $q = g + p$. Bending moments (Fig. X.14*a* and *b*) are determined with allowance for the redistribution of forces:

—in all middle spans and at intermediate supports

$$M = ql^2/16$$

—in the outer spans and above the first intermediate support

$$M = ql^2/11$$

The design cross section of the slab is a rectangle of width $b = 100$ cm.

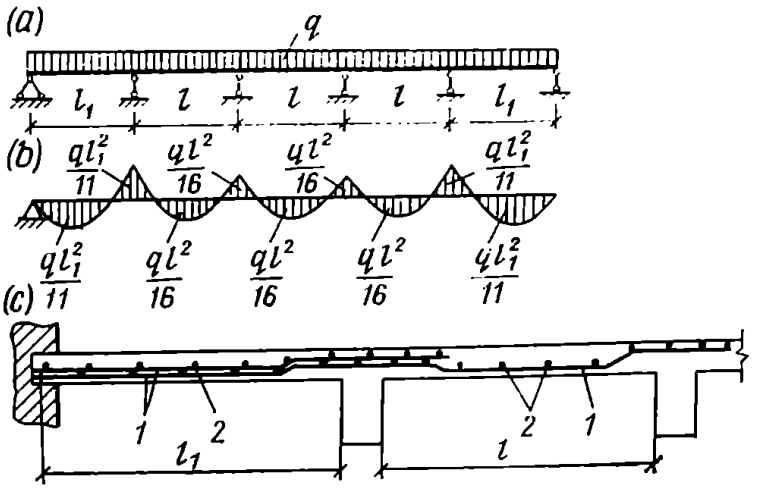



Fig. X.14. One-way slab of an in-situ beam and girder floor
(*a*) loading system; (*b*) bending moment diagram; (*c*) distribution of reinforcement; *1*—load-bearing reinforcement; *2*—distribution reinforcement

Slabs are reinforced with welded-wire fabric in rolls. The reinforcement is placed near the bottom face in the span and near the top face at the supports, that is, according to the respective curve of bending moments.

In the middle panels of a slab, surrounded by beams and girders, the design steel area may be reduced by 20% owing to the positive effect of the thrust appearing due to the redistribution of forces.

The slab is not analyzed for inclined-section shear strength because the condition of Eq. (VI.35) is satisfied always.

The beams carry their self-weight $g_1$ and the uniformly distributed load transferred by the slab from the strip of a width equal to the centre-to-centre distance $c$ between the beams. The load per metre run of the beam is

$$q_b = (g + p)\, c + g_1$$

where $(g + p)$ is the sum of the design dead and live loads per square metre of the slab surface.

The loading system for multispan beams is similar to that for the slab. The bending moments and shearing forces are determined with allowance for the redistribution of forces as given in Fig. X.15.

The effective depth of the beam is chosen such that $\xi \leqslant 0.35$ at the first intermediate support, as reckoned from

the edge (where a plastic hinge is likely to develop because of the redistribution of forces).

In the spans, the necessary area of longitudinal tensile steel is determined as for a T-section with a flange of width $b_f' = c$, because the flange (slab) lies in the compression zone. At the supports, the steel area is found as for a rectangular section of a width equal to the web width $b$, because the flange is in the tension zone.

The reinforcement in the spans consists of welded bar mats whose longitudinal bars are chosen to carry the bending moment in the span, and the transverse bars are taken to ensure an adequate inclined-section shear strength (provided the width of the cross section is equal to the web width $b$). At the supports, the reinforcement is made up of two rolls

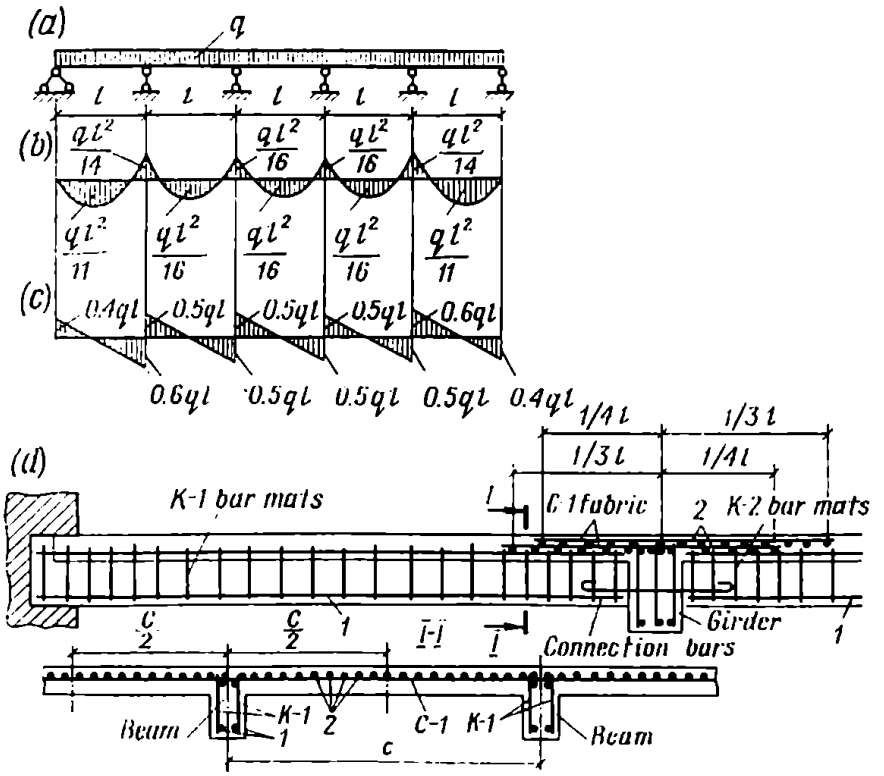



Fig. X.15. Multispan beam of a beam and girder floor

(a) loading system; (b) bending moment diagram; (c) shearing force diagram; (d) distribution of reinforcement; *1*—load-bearing reinforcement in span; *2*—load-bearing reinforcement at support

of welded-wire fabric offset from each other and having transverse load-bearing wires. The fabric is placed above the girder. The steel area of the beam at the support is equal to the total area of the transverse wires within the portion *c*.

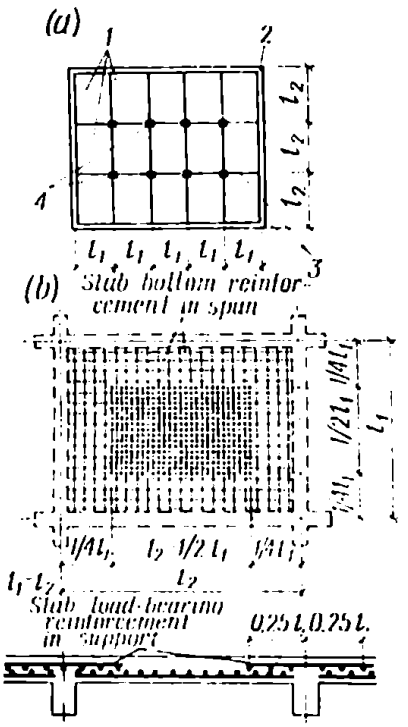



Fig. X.16. In-situ floor with slabs spanning in two directions
(*a*) plan; (*b*) slab reinforcement; *1*—slabs; *2*—wall; *3*—girders; *4*—columns

The distribution of reinforcement in a beam is shown in Fig. X.15.

The girders carry concentrated loads applied at the points where they support the beams and, naturally, their self-weight. Bending moments and shearing forces are determined with allowance for the redistribution of forces as in the girders of precast floors.

**In-Situ Floors with Slabs Spanning in Two Directions.** This type of floor consists essentially of two-way slabs and girders resting on columns running in two directions (Fig. X.16*a*). The column grid is usually from 4 to 6 m square. The slab thickness ranges between 8 and 14 cm. The slabs are subjected to bending in two directions, and they are reinforced with welded-wire fabric placed near the bottom face in the span and near the top face at the supports (over the girders).

Since bending moments in the span decrease on moving away from the mid-span to the supports, the bottom reinforcement of the slab is made up of two pieces of welded-wire fabric with load-bearing wires running in two directions. The area of the load-bearing reinforcement in each piece of fabric is the same. One of the fabric pieces is cut off within $l_1/4$ of the intermediate supports and within $l_1/8$ of the outer free supports (where $l_1$ is the shorter span of the slab). The top pieces of fabric have the load-bearing wires placed in

one direction. The load-bearing reinforcement is distributed across the girders and reaches into the span for a distance $l_1/4$ (Fig. X.16*b*).

Under load, the bottom surface of the slab develops cracks along the bisectors of the slab corners (Fig. X.17*a*), and the top surface develops cracks along the girders (Fig. X.17*b*). As the load increases, the slab breaks at the cracks. In the limit state, the slab is regarded as a system of members connected by plastic hinges along the lines of breakdown (Fig. X.17*c*). The design bending moment at the hinge per metre run of the hinge depends on the cross-sectional area $F_s$ of the load-bearing reinforcement per metre of slab length and is found by the following formula

$$M = R_s F_s z_c$$

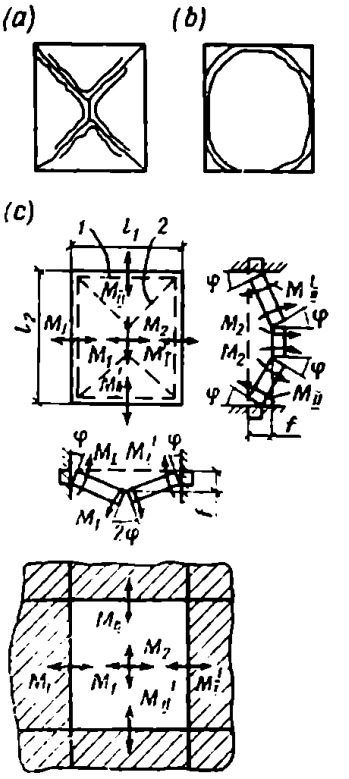



Fig. X.17. To the design of a two-way slab (fixed at four sides)

*1*—plastic hinge at the support (opens at the top); *2*—plastic hinge in the span (opens at the bottom)

If, in the general case, the steel areas at the supports and in all spans (irrespective of their direction) differ from one another, the design bending moments will be different, too. For our discussion, let us designate as $M_1$ and $M_2$ the design moments in the span, and as $M_I$, $M_I'$, $M_{II}$, and $M_{II}'$ the design moments at the supports (see Fig. X.17*c*).

A two-way slab is calculated in the state of ultimate equilibrium, assuming that the system of slab members connected by plastic hinges moves under load, and the external load $q$ moving together with the slab does work $W_q$ equal to work $W_M$ done by the internal forces, which is the work done by the bending moments in producing angular deflections $\varphi$.

Since half of the bottom bars are cut off within $l_1/4$ of

the supports (see Fig. X.16*b*), the design equation takes the form

$$ql_1^2 (3l_2 - l_1)/12 = (2M_1 + M_I + M'_I)\, l_2 + (3M_2/2 - M_1/2 + M_{II} + M'_{II})\, l_1 \qquad (X.2)$$

If half of the bottom bars are cut off within $l_1/8$ of the supports, then

$$ql_1^2 (3l_2 - l_1)/12 = (2M_1 + M_I + M'_I)\, l_2 + (7M_2/4 - M_1/4 + M_{II} + M'_{II})\, l_1 \qquad (X.3)$$

The above design equations include six unknown moments. Taking five of them in an assumed proportion, the problem can be reduced to one unknown, and the others will be found from the proportion assumed.

Calculations are much simplified if $l_1 = l_2 = l$ and the bending moments are assumed to be the same in all intermediate panels. In this case, Eq. (X.2) yields

$$ql^2/6 = 7M$$

whence

$$M = ql^2/42 = M_{sp} = M_{sup}$$

In a simply supported slab, all moments at the supports are equal to zero. Recalling that half of the bottom bars are cut off within $l_1/8$ of the supports and assuming that $l_1 = l_2 = l$ and $M_1 = M_2 = M_{sp}$, Eq. (X.3) gives

$$M_{sp} = ql^2/21$$

The necessary area of the tensile steel $F_s$ is determined as for a rectangular section 100 cm wide. Owing to the thrust arising in two-way slabs connected monolithically with the girders supporting them, the load-bearing capacity of the slabs increases. Therefore, the steel area may be reduced by 20%.

Two-way slabs are designed to carry the load distributed over areas taking the shape of triangles and trapezoids bounded by the bisectors of the slab corners (Fig. X.18*a*). With a square column grid, the girders in either direction are subjected to loads distributed over a triangular load area.

The bending moments with allowance for their redistribution are as follows:

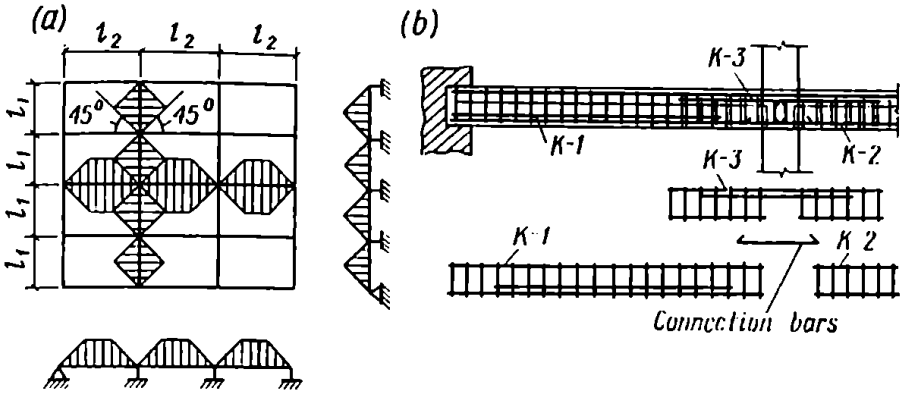



Fig. X.18. To the design of the girders of floors using two-way slabs (*a*) loading system; (*b*) distribution of reinforcement

—in the first span and at the first intermediate support

$$M = 0.7M_0$$

—in the middle spans and at the middle supports

$$M = 0.5M_0$$

where $M_0$ is the bending moment induced by the load distributed over a triangular or trapezoidal load area.

Two-way slabs may sometimes be used as part of ribbed panels in precast slab-and-girder floors having transverse ribs (see Fig. X.7*b*) and also as part of precast square panels surrounded by ribs (see Figs. XII.4 and XII.5). The portions between the ribs of such slabs are designed using the technique discussed above, and the ribs as single-span beams subjected to a triangular load.

**Flat-Slab Floors.** An in-situ flat-slab floor is shown in Fig. X.19*a*. The column grid size is usually 6 m square and the slab thickness ranges between 18 and 22 cm. Flat-slab floors have no protruding ribs, so their height is small and the space below them can easily be ventilated. Owing to this, flat-slab floors are widely used in buried tanks (see Figs. XI.1 and XI.25), cold-storage houses, warehouses, and garages.

The faces of capitals are inclined at 45°. Their base side length is $c = 0.2l$ to $0.3l$ at the bottom of the drop panel,

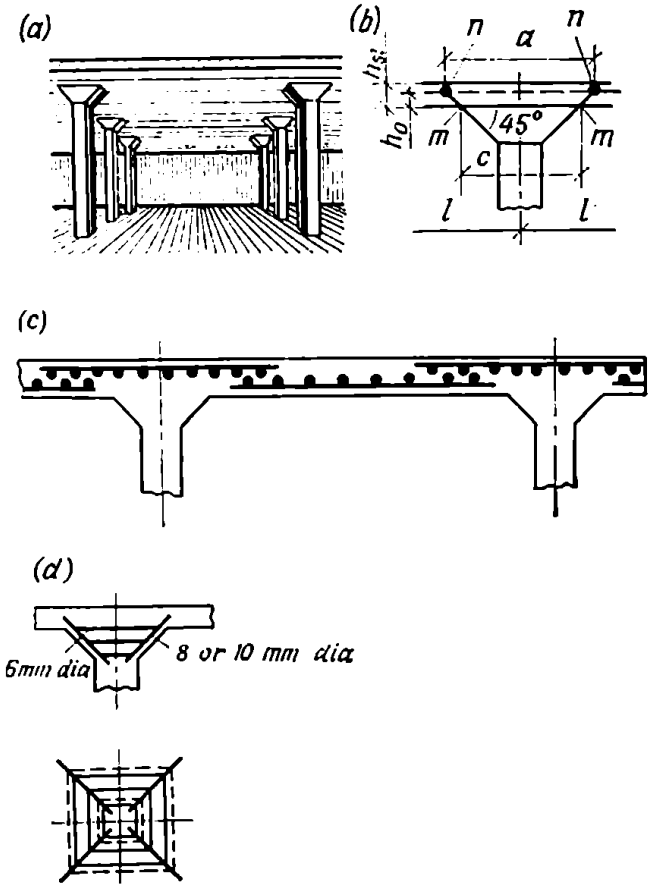



Fig. X.19. Flat-slab floor
(*a*) general view; (*b*) capital dimensions; (*c*) distribution of slab reinforcement; (*d*) distribution of capital reinforcement

and $a = c + 2h_0$ at the top reinforcement of the drop panel (Fig. X.19*b*). The dimensions of the capitals should be chosen so as to prevent the flat slab from crushing along the faces of the pyramid designated *mm nn* in Fig. X.19*b*.

The crushing strength is adequate if the following condition is met

$$P \leqslant R_{ten} b_{med} h_0$$

where $P$ is the total load imposed on the floor slab minus the load applied to the top of the crushing pyramid:

and $b_{med}$ is the median perimeter of the crushing pyramid:

$$b_{med} = (4a + 4c)/2 = 2(a + c)$$

A flat slab is designed by the ultimate equilibrium method, according to the manner in which it is likely to break down. With a strip load, the middle panels develop two longitudinal cracks (spaced a distance $c_1 \approx 0.1l$ apart) on the top surface and one crack on the bottom surface in the mid-span (Fig. X.20*a*). At the limit state, linear plastic hinges connecting two fragments of the slab form along these cracks (Fig. X.20*b*). The bending moments at the plastic hinges are

$$M_{sup} = R_s F_s^{sup} z_c^{sup}$$

and

$$M_{sp} = R_s F_s^{sp} z_c^{sp}$$

The sum of the bending moments in the span and at the supports (Fig. X.20*c*) is assumed equal to the moment $M_0$ in an equivalent beam

$$M_{sup} + M_{sp} = M_0 \quad (X.4)$$

Assuming the design load $q$, strip width $l_2$, and effective span $l_1 - 2c_1$, we have

$$M_0 = ql_2 (l_1 - 2c_1)^2/8$$

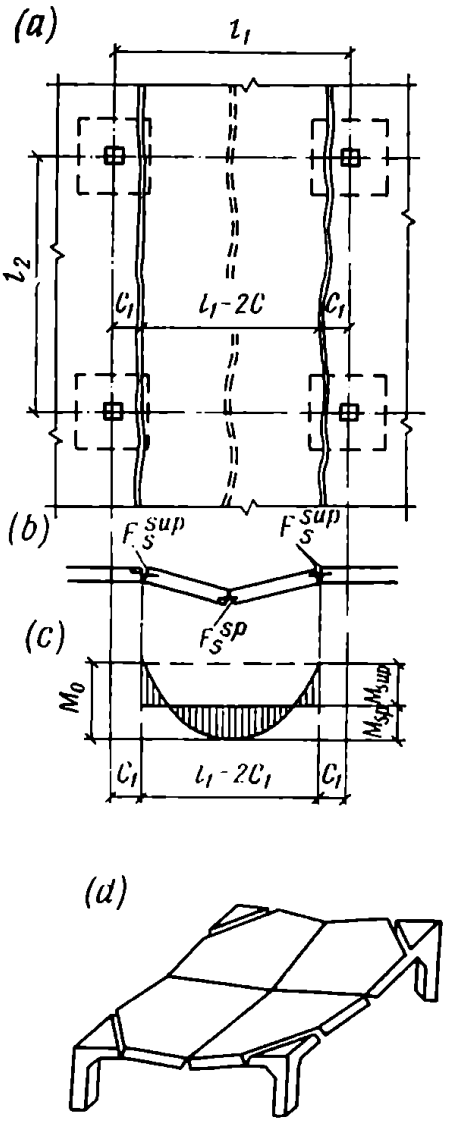



Fig. X.20. To the design of a flat slab in the state of ultimate equilibrium

(*a*) and (*b*) slab breakdown under strip load; (*c*) bending moment diagram; (*d*) slab breakdown when entire slab area is loaded

Substituting the values of $M_{sp}$, $M_{sup}$, and $M_0$ into Eq. (X.4) yields the following design formula

$$ql_2 (l_1 - 2c_1)^2/8 \leqslant R_s (F_s^{sup} z_c^{sup} + F_s^{sp} z_c^{sp}) \quad (X.5)$$

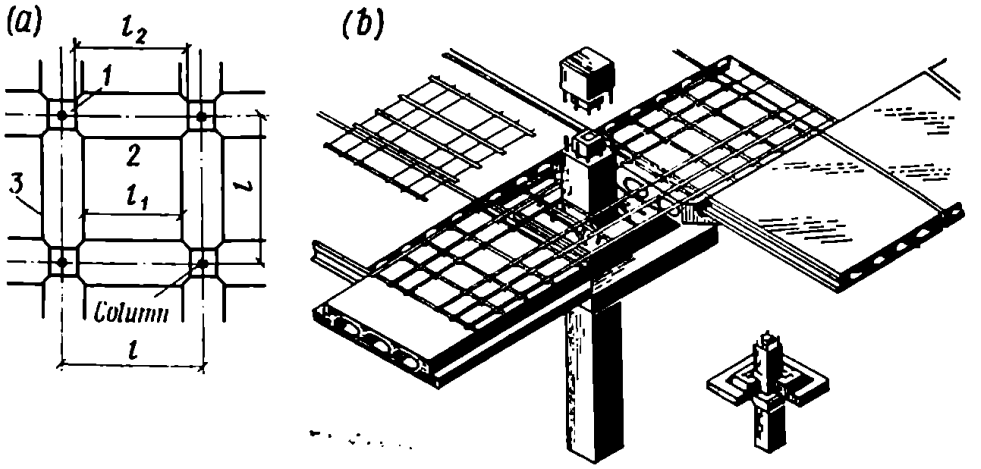



Fig. X.21. (*a*) Precast flat-slab floor and (*b*) precast/in-situ flat-slab floor

*1*—capital; *2*—slab; *3*—drop panel

On specifying $F_s^{sup}/F_s^{sp} = 0.5$ to 0.67, Eq. (X.5) can be reduced to one unknown.

The above design procedure is the basic one for flat-slab floors and must be carried out in all cases. Sometimes, it proves necessary to check flat slabs for strength when the load is simultaneously applied to adjacent panels. The plastic hinges that form in this case are shown in Fig. X.20*d*.

Precast flat-slab floors consist essentially of three groups of members, namely capitals, drop panels, and slabs. The capitals are carried by the flared portions of the columns and take up the load from the drop panels which run in two mutually perpendicular directions and act as girders. The slabs are supported on four sides by the flanges of the drop panels, and act as two-way slabs (Fig. X.21*a*).

The capitals may be rectangular or have the shape of a truncated pyramid. The drop panels and slabs may be ribbed, hollow-core, or solid.

The component members are connected by welding together their embedded steel parts, after which the joints are concreted.

Bending moments are determined with allowance for the partial fixation at the joints:

—for a square slab having a side of $l_1$

$$M = ql_1^2/27$$

—for a square drop panel of span $l_2$

$$M_{sp} = q_{eq} l_2^2/12$$

and

$$M_{sup} = q_{eq} l_2^2/24$$

where $q_{eq}$ is the equivalent uniformly distributed load.

The capital is designed as a cantilever carrying the reaction of the drop panel and the distributed load within the capital area.

In precast/in-situ flat-slab floors (Fig. X.21*b*), the precast capitals, drop panels, and slabs are covered with additional reinforcement and in-situ concrete. As a result, they work similarly to in-situ floors, but need neither scaffolding nor formwork.

## X.3. Reinforced Concrete Column Footings

Foundations are intended to transmit loads from columns to soil and distribute the loads in such a way that the pressure beneath the bottom of a footing is low and will not cause undue settlement of the foundation material.

There are three basic types of foundations, namely an isolated or independent benched footing which supports a single column (Fig. X.22*a*), a strip or continuous footing (Fig. X.22*b*), and a mat or raft foundation (Fig. X.22*c*). The two last-mentioned types are used with weak soils or considerable loads. According to the manner of manufacture,

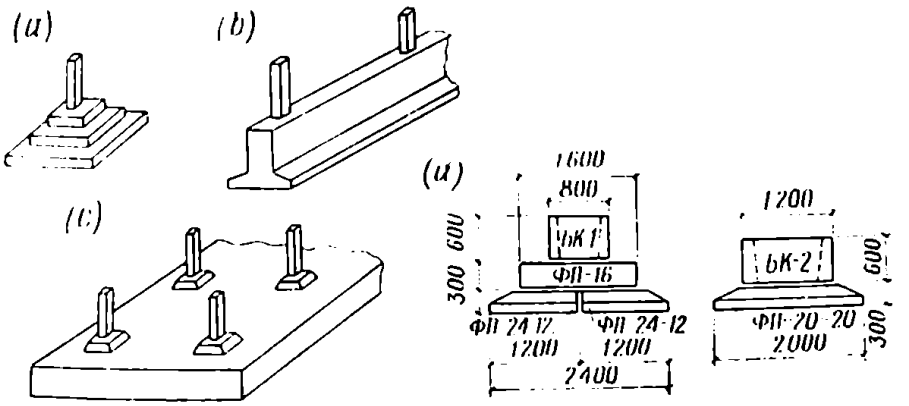



Fig. X.22. Types of reinforced concrete footings

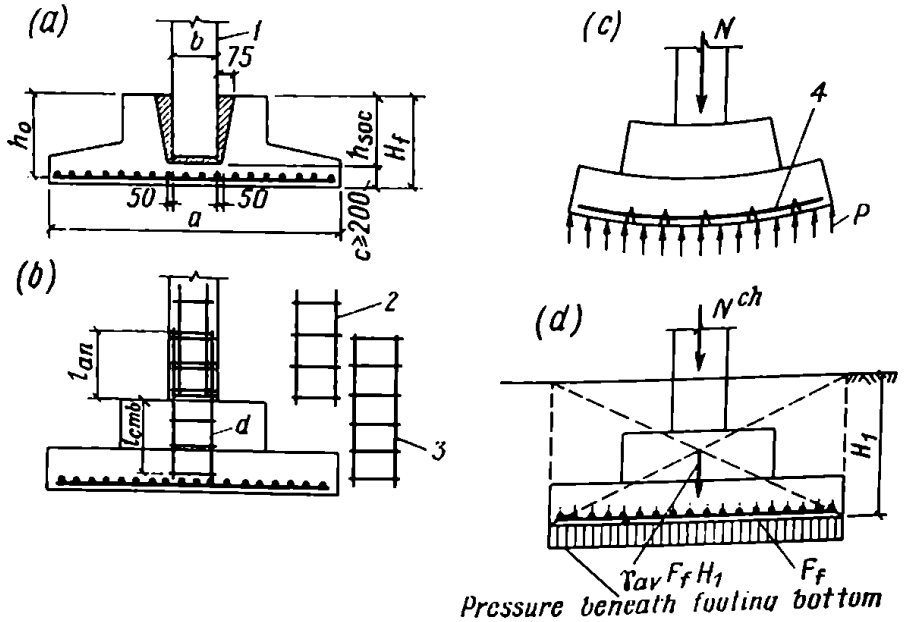



Fig. X.23. Benched footings

(*a*) footing for a precast column; (*b*) footing for an in-situ column; (*c*) footing bent under load; (*d*) to determining footing area; *1*—column; *2*—column reinforcement; *3*—footing dowels; *4*—footing load-bearing reinforcement

we may distinguish precast footings consisting of a single prefab member or assembled from several members (Fig. X.22*d*), and in-situ foundations. The latter are easier to manufacture, so they are used rather frequently, especially if they are large.

Precast columns are built in special sockets provided in footings for a depth of not less than the side of the column cross section (with solid single-piece columns), nor less than the anchorage length of the column reinforcement. The dimensions of a socket in plan should be larger than the respective dimensions of the column by 100 mm at the bottom and 150 mm at the top (Fig. X.23*a*). After installation and a check against a plum-bob, the gaps between the column and the socket are filled with concrete. If a footing is to carry an in-situ column, it has dowels which are welded to the column reinforcement (Fig. X.23*b*).

The reinforcement of a benched footing is a bar mat with the load-bearing bars running in two directions. The mat is placed at the bottom with a concrete cover of 3.5 cm (if a layer of sand and gravel is provided beneath the bottom

of the footing) or 7 cm (if no layer is provided). This reinforcement takes up tensile forces at the bottom of the footing when the footing is subjected to bending due to the contact soil pressure (Fig. X.23*c*).

If a footing is subjected to an axial load only, we speak of an axially loaded footing. If a footing resists a normal force, an overturning moment, and a shearing force applied simultaneously, it is regarded as an eccentrically loaded footing.

Owing to the high rigidity of isolated benched footings, the pressure beneath their bottom (and, as a consequence, the contact soil pressure) is assumed to be distributed linearly. The bottom size is chosen such that the settlement does not exceed the maximum safe value specified for a given type of structure. For some industrial and civic buildings resting on low-compressibility soils*, no settlement analysis needs be carried out, if the pressure beneath the bottom of the footing does not exceed the design soil bearing capacity $R_s$ determined on the basis of siting data.

Relevant standards (see Appendix XVI) give the nominal design bearing capacity $R_0$ for various soils, which may be adopted as the design soil bearing capacity $R_s$ for footings of width $b_1 = 1$ m and embedment depth $h_1 = 2$ m. For $h \geqslant 2$ m and $b \leqslant 1$ m,

$$R_s = R_0 [1 + k_1 (b - b_1)/b_1] [(h + h_1)/2h_1]$$

For $h > 2$ m and $b \geqslant 1$ m,

$$R_s = R_0 [1 + k_1 (b - b_1)/b_1] + k_2 \gamma_{II} (h - h_1)$$

where

$k_1 = 0.125$ for gravel and sandy soils (except fine sands);
$k_1 = 0.05$ for clay and fine sand;
$k_2 = 0.25$ for gravel and sandy soils;
$k_2 = 0.2$ for loam and sand loam;
$k_2 = 0.15$ for clay;
$\gamma_{II}$ = bulk unit weight of the backfill over the footing steps (in tf m$^{-3}$).

To prevent undue settlement of the foundation material (the second group of limit states), the pressure beneath the

* The modulus of elasticity is $E \geqslant 300$ to 500 kgf cm$^{-2}$ (30 to 50 MPa).

bottom of a footing should not exceed the design soil bearing capacity $R_s$. For this purpose, the appropriate loads should be taken equal to their characteristic values, that is, multiplied by the overload factor $n = 1$.

**Design of Axially Loaded Footings.** Here, the loads responsible for the pressure beneath the bottom of a footing are the normal force $N^{ch}$ transmitted from the column to the footing, the self-weight of the footing, and the weight of the backfill over its steps (Fig. X.23*d*).

If we take the average load due to the unit volume of the footing and the backfill over its steps as $\gamma_{av} = 2$ tf m$^{-3}$, then their total weight with a footing area $F_f$ and embedment depth $H_1$ will be $\gamma_{av}F_fH_1$ and the pressure beneath the bottom of the footing, caused by this weight, will be

$$\gamma_{av}F_fH_1/F_f = \gamma_{av}H_1 \qquad (X.6)$$

Reducing $R_s$ by $\gamma_{av}H_1$ gives the design pressure which may be carried by the soil if only the force $N^{ch}$ transmitted by the column is acting. As a consequence,

$$N^{ch}/F_f \leqslant R_s - \gamma_{av}H_1 \qquad (X.7)$$

whence

$$F_f = N^{ch}/(R_s - \gamma_{av}H_1) \qquad (X.8)$$

Axially loaded footings are generally square in plan, so the side of the footing is given by

$$a = \sqrt{F_f} \qquad (X.9)$$

The contact soil pressure responsible for the footing sag is produced only by the load transmitted by the column. The self-weight of the footing and the respective contact soil pressure balance each other without causing any footing sag. The design contact soil pressure caused by the design normal force $N$ is given by

$$p_s = N/F_f \qquad (X.10)$$

The minimum depth of a footing is chosen so as to prevent punching, assuming that the punching can take place over the surface of a pyramid whose side faces begin at the columr base and make an angle of 45° with the base (Fig. X.24*a*). The design punching force $P$ is assumed to be equal to the design force $N$ transmitted by the column, minus the total

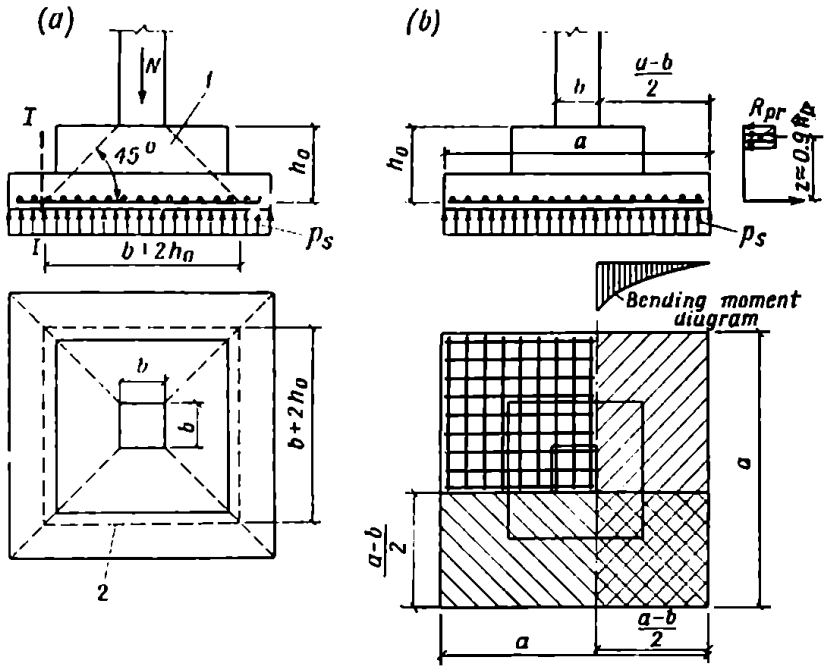



Fig. X.24. To the punching strength analysis of axially loaded footings (*a*) and (*b*) to determining the steel area of the footing
*1*—punching pyramid; *2*—bottom base of punching pyramid

contact soil pressure $p_s$ within the base of the punching pyramid. If the column is of square cross section and has a side $b$, the top base area of the punching pyramid is $b^2$ and that of the bottom base is $(b + 2h_0)^2$. So,

$$P = N - p_s (b + 2h_0)^2 \qquad (X.11)$$

The condition for punching strength (for heavy-concrete footings) is as follows

$$P \leqslant R_{ten} b_m h_0 \qquad (X.12)$$

where $R_{ten}$ is the design tensile strength of the concrete and $b_m$ is the arithmetic mean of the perimeters of the top and bottom bases of the punching pyramid

$$b_m = [4b + 4 (b + 2h_0)]/2 = 4 (b + h_0) \qquad (X.13)$$

Substituting $P$ and $b_m$ given by Eqs. (X.11) and (X.13) into Eq. (X.12) yields the footing depth required to prevent

punching

$$h_0 = -b/2 + 0.5\sqrt{N/(R_{ten} + p_s)} \qquad (X.14)$$

$$H_f = h_0 + a \qquad (X.15)$$

Sometimes, the depth of a footing may be chosen according to the depth of the socket $h_{soc}$

$$H_f = h_{soc} + 200$$

In height, column footings are divided into steps (or benches) 30 to 50 cm high. The height of the bottom step should be checked for inclined-section shear strength at section *I-I* (see Fig. X.24*a*) as if it contained no transverse reinforcement.

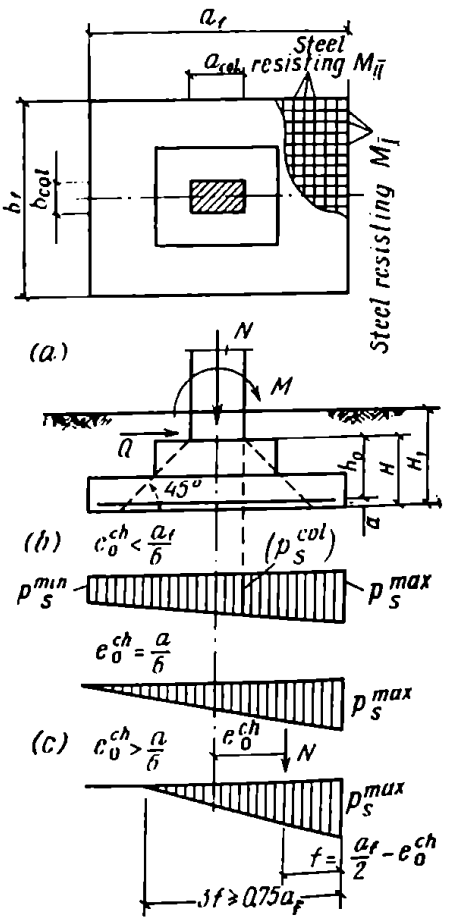



Fig. X.24A. Eccentrically loaded reinforced concrete footings

After the dimensions in plan and the depth of the footing have been selected, it is necessary to determine the cross-sectional area of the reinforcing steel. For this purpose, the external parts of the footing extending past the column face are regarded as cantilevers of arm $(a - b)/2$ and width $a$, subjected to a uniformly distributed load $p_s a$ (Fig. X. 24*b*).

The bending moment at the face of the column is

$$M = p_s a\,[(a - b)/2]^2/2$$
$$= p_s a\,(a - b)^2/8 \qquad (X.16)$$

The required cross-sectional area of the reinforcement is found from the condition

$$M = R_s F_s z_c$$

Assuming that $z_c \approx 0.9\ h_0$, we have

$$F_s = M/0.9 R_s h_0 \qquad (X.17)$$

The area found above gives the steel area for the bottom bar mat which generally consists of bars 10 to 16 mm in diameter and spaced 10 to 20 cm apart.

**Design of Eccentrically Loaded Footings.** If they are to carry a relatively small overturning moment, eccentrically loaded footings (Fig. X.24A) may be square in plan. Otherwise, it is good practice to make them with the base extended in the plane of the applied overturning moment. The base dimensions are determined assuming that the pressure is distributed linearly, so that the maximum pressure exerted by the edge of the footing does not exceed $1.2R_s$.

As a first trial, the necessary base area is found from Eq. (X.8) where a coefficient of 1.2 to 1.8 is introduced to allow for the action of the overturning moment. The footing depth is chosen similarly to axially loaded footings. Then, the overall dimensions of the footing are specified, and the pressure beneath the bottom of the footing is checked using the expressions given below.

For our discussion, let $a_{col}$ designate the column side in the plane of the overturning moment, $a_f$ the footing side in the plane of the overturning moment, $b_{col}$ the column side in the direction normal to the plane of the overturning moment, $b_f$ the footing side in the direction normal to the plane of the overturning moment, and $H$ the depth of the footing.

The moment caused by the characteristic loads at the bottom of the footing is

$$M_s^{ch} = M^{ch} + Q^{ch}H \qquad \text{(X.18)}$$

The normal force caused by the characteristic loads at the bottom of the footing is

$$N_s^{ch} = N^{ch} + F_f \gamma_{av} H_1 \qquad \text{(X.19)}$$

where $H_1$ is the embedment depth.

The eccentricity is

$$e_0^{ch} = M_s^{ch} / N_s^{ch} \qquad \text{(X.20)}$$

If $e_0^{ch} \leqslant a_f/6$ (that is, the normal force is applied within the kern), the pressure beneath the bottom of the footing takes only one sign (Fig. X.24A*a* and *b*). The pressure exerted

by the edge of the footing is given by

$$p^{ch} = N_s^{ch} (1 \pm 6e_0^{ch}/a_f)/a_f b_f \qquad (X.21)$$

With the plus sign in Eq. (X.21), we have $p_{max}^{ch}$; with the minus sign, we get $p_{min}^{ch}$. As previously stated,

$$p_{max}^{ch} \leqslant 1.2R_s$$

If this condition is not met, the base dimensions should be refined and the analysis repeated.

If $e_0^{ch} > a_f/6$, the normal force is applied outside the kern and the pressure is distributed only within a portion of the base area. Within the remaining part of the base area, the footing loses contact with the soil (Fig. X.24 A*a* and *c*). In this case, the edge pressure $p_{max}^{ch}$ is determined assuming that the centroid of the pressure triangle coincides with the direction of the force $N_s^{ch}$ applied eccentrically.

The distance from the force $N_s^{ch}$ to the base edge is $f = a_f/2 - e_0^{ch}$. The centroid of the pressure triangle should be at this point, so the length of the pressure triangle is $3f$. It is recommended that $3f \geqslant 0.75a_f$.

The resultant of the pressure triangle, equal to $0.5p_{max}^{ch} 3fb_f$, should balance the force $N_s^{ch}$. So,

$$p_{max}^{ch} = 2N_s^{ch}/3b_f f \leqslant 1.2R_s \qquad (X.22)$$

Columns intended to carry crane loads may not be installed on footings having a triangular pressure diagram. To reduce the eccentricity, it sometimes proves expedient to offset the footing from the column for a distance *c*. At $c = e_0^{ch}$, the footing becomes axially loaded. As a rule, *c* is taken as $0.5\ e_0^{ch}$.

To determine the cross-sectional area of the reinforcement in an eccentrically loaded footing, it is necessary to find the contact soil pressure due to the design loads *M*, *N*, and *Q* transferred by the column (without taking into consideration the self-weight of the foundation and the backfill over its steps). The design moment at the bottom of the footing is $M_s = M + QH$ and the eccentricity is $e_0 = M_s/N$.

If $e_0 \leqslant a_f/6$,

$$p_s = N (1 \pm 6e_0/a_f)/a_f b_f \qquad (X.23)$$

If $e_0 > a_f/6$,

$$p_s = 2N/3b_f f \tag{X.24}$$

Referring to the diagram of $p_s$, one finds the contact soil pressure $p_s^{col}$ at the column face, then its mean value

$$p_s^m = (p_s^{\max} + p_s^{col})/2 \tag{X.25}$$

The bending moment at the face of the column in the plane of the overturning moment is determined as for an axially loaded footing

$$M_{\mathrm{I}} = p_s^m b_f (a_f - a_{col})^2/8 \tag{X.26}$$

The bending moment $M_{\mathrm{II}}$ in the perpendicular direction is found from the contact soil pressure $p_s^0$, without taking into account the overturning moment:

$$p_0^s = N/a_f b_f \tag{X.27}$$

$$M_{\mathrm{II}} = p_s^0 a_f (b_f - b_{col})^2/8 \tag{X.28}$$

After the bending moments have been determined, the necessary cross-sectional area of reinforcing steel for each direction is found from Eq. (X.17).

## X.4. Prefab Reinforced Concrete One-Storey Industrial Buildings

The structural elements for one-storey skeleton-construction industrial buildings (Fig. X.25) include columns built into their footings, roof girders (such as beams, trusses, and arches), roof slabs, crane beams (if the building is to accommodate an overhead travelling crane), and (if necessary) skylights and air monitors. The column grid size is taken a multiple of 6 m, that is, 12, 18, 24, 30, or 36 m in the transverse direction (the span of the building), and 6 or 12 m in the longitudinal direction (the bay of the building). When the bay is 12 m and the wall panels are 6 m long, intermediate (trellis-work) columns are additionally installed along the sides.

In the transverse direction, a one-storey skeleton-construction building owes its spatial rigidity to transverse frames made up of the columns fixed in their footing and the

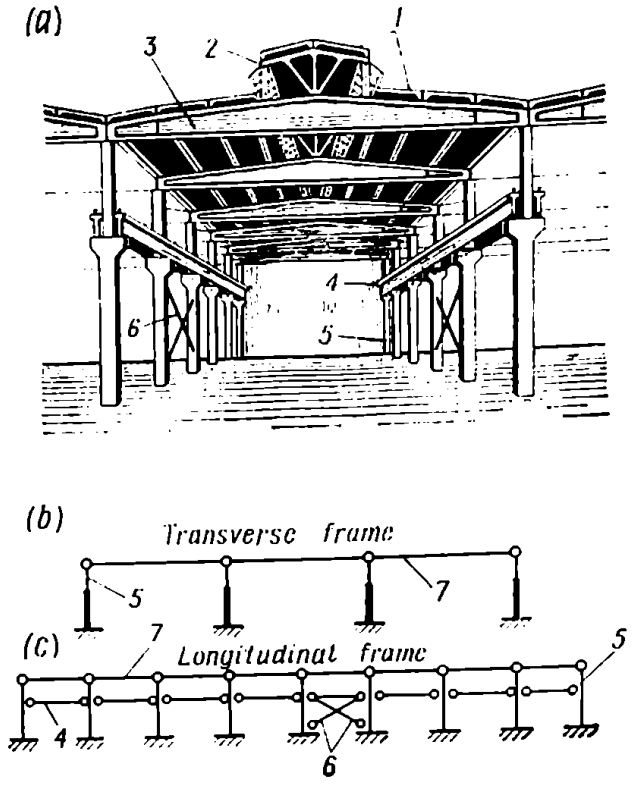



Fig. X.25. Prefab reinforced concrete one-storey industrial building with overhead travelling cranes
*1*—roof slab; *2*—monitor; *3*—roof beam; *4*—crane beams; *5*—columns; *6*—braces; *7*—roof

roof girders hinged to the columns (Fig. X.25*b*). In the longitudinal direction, spatial rigidity is ensured by longitudinal frames composed of the same columns, roof girders, crane beams, and lateral braces (Fig. X.25*c*). Hinged joints between the girders and columns result in higher bending moments in the girders as compared with rigid joints (Fig. X. 26), but, as already mentioned in Sec. X.1, with hinged joints, the girders and columns may be standardized separately and can therefore be readily adapted to industrialized manufacture.

With spans of 12 and 18 m, roof girders are most frequently made in the form of prestressed (mostly pretensioned) concrete I-beams (Fig. X.27). The concrete brand number is

usually M-300 to M-500. The prestressed steel $A_{pr}$ is either class A-IV to AT-VI deformed bars or high-strength wire. The steel area is determined by normal-section strength analysis.

To prevent the compression zone of a beam from cracking during the transfer of the prestress, prestressed reinforcement ($A'_{pr}$) is sometimes added in the compression zone, too. The beam web is reinforced with one or two non-prestressed bar mats whose transverse bar area is chosen so as to ensure the desired inclined-section shear strength. Longitudinal cracking which might appear at the supports during the transfer of the prestress is prevented by additional transverse bars welded to an embedded steel part in the support region. The top compressive flange is reinforced with a welded-bar mat.

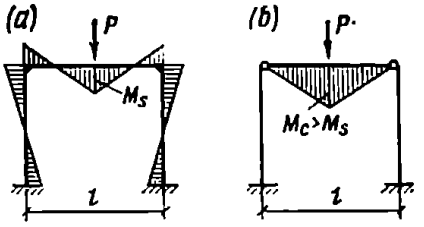



Fig. X.26. Bending moment diagrams in the frame of a one-storey industrial building (*a*) rigid joints; (*b*) hinged joints

In design calculations, the beams are regarded as single-span beams subjected to a uniformly distributed load induced by the weight of the roof, the weight of snow on the roof, and the self-weight of the beams.

If a building is equipped with an overhead-track hoist suspended from the roof beams, the design bending moment and shearing force should be determined with allowance for the load induced by the hoist. This load is assumed to be concentrated at the suspension points of the hoist. The beams should also be analyzed for the second group of limit states, that is, crack width and strains.

*Reinforced concrete trusses* (Fig. X.28) are used as roof girders for spans of 18, 24, and 30 m. The main types of trusses are the camel-back truss with a polygonal top chord, and the quadrangular truss which is used in flat roofs. Whatever the type of truss, the depth at the midspan is usually taken as 1/7 to 1/9 of their span. The top chord panels are designed to be 3 m long in order that all of the roof slab load can be transmitted directly to the joints. Then, the top chord is solely in axial compression, the bot-

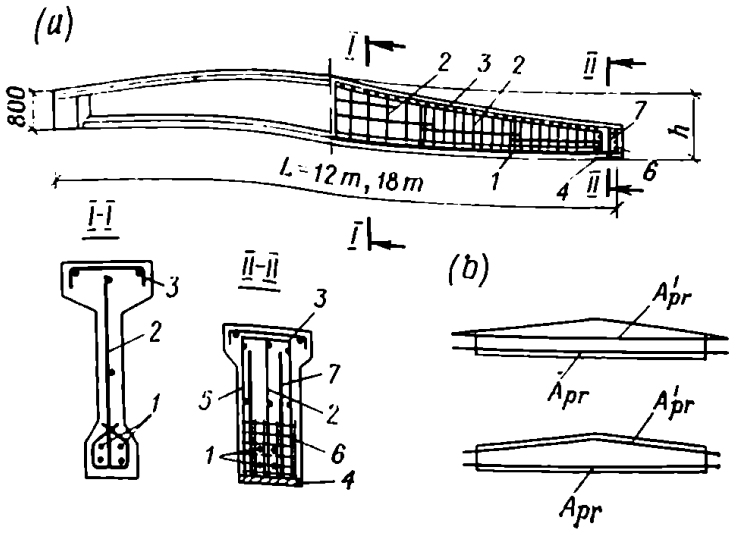



Fig. X.27. Prestressed roof beams

(*a*) general view and distribution of reinforcement; (*b*) distribution of prestressed reinforcement; *1*—longitudinal prestressed reinforcement; *2*—web welded-bar mats; *3*—top flange welded-bar mats; *4*—embedded part at support; *5*—additional welded-bar mats at support; *6*—welded-wire fabric; *7*—transverse bars welded to embedded part at support

tom chord is in axial tension, and the web members (struts and diagonals) are in either axial compression or tension.

The forces induced in the truss members by loads applied at the joints are determined by the methods of structural mechanics (either graphically, using the Maxwell diagram, or analytically, by the method of sections) on the assumption that the truss members are connected by hinges. This is an approximate approach which does not allow for the rigidity of truss joints. As compared with precise methods, however, the error is insignificant.

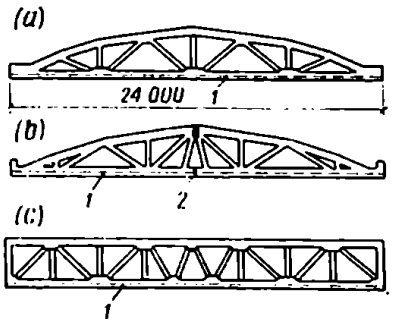



Fig. X.28. Reinforced concrete roof trusses

(*a*) camel-back single-piece truss; (*b*) camel-back two-piece truss; (*c*) flat-top truss; *1*—prestressed reinforcement; *2*—erection joint

The top chords, struts, and diagonal members of trusses are reinforced with welded-bar mats. The bottom chords may be either pretensioned or posttensioned. In pretensioning, the prestressed reinforcing steel is uniformly distributed over the chord cross section and embraced by stirrups.

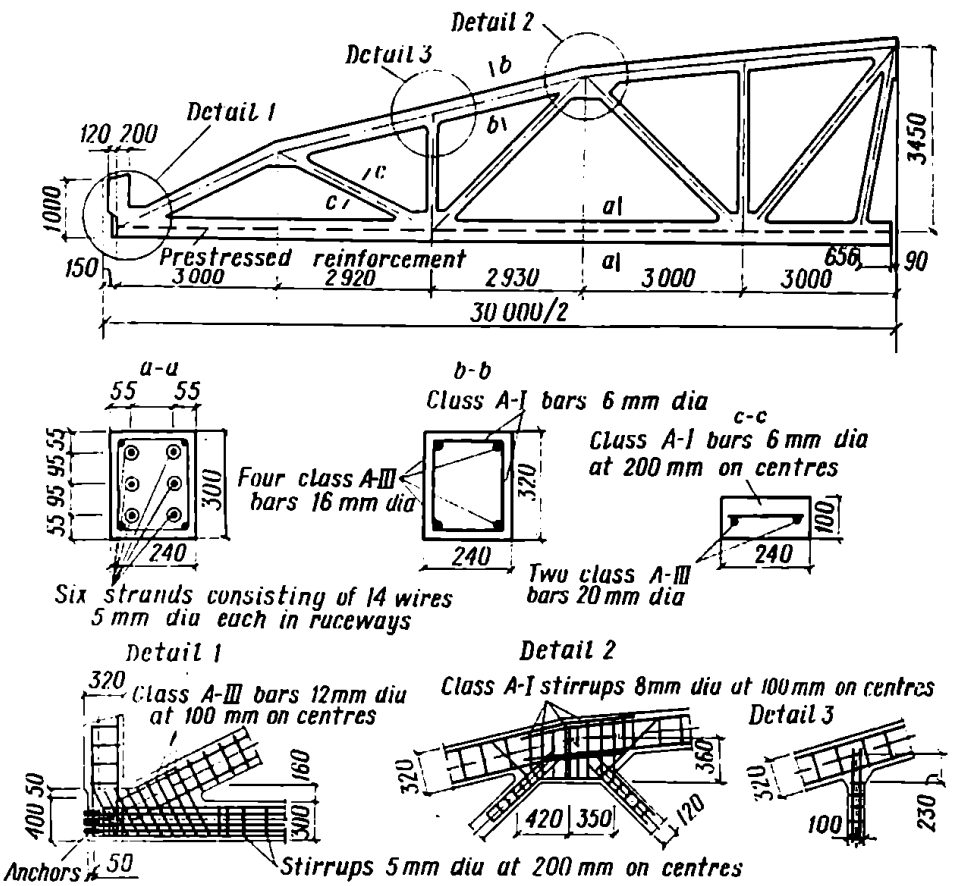



Fig. X.29. Camel-back truss with a posttensioned bottom chord

In posttensioning, the prestressed steel is placed in raceways which, after the steel had been tensioned and anchored, are filled with grout applied under pressure. If they are to bridge spans up to 24 m, trusses are fabricated in one piece. Otherwise, they are assembled from half-trusses joined at the job by welding together embedded steel parts.

The construction of a camel-back reinforced concrete truss (with posttensioned cable reinforcement) is shown in Fig. X.29.

*Roofs use ribbed reinforced concrete slabs* 3 × 12 m and 3 × 6 m or 1.5 × 12 m and 1.5 × 6 m in plan, installed on girders. In construction and design, they are similar to ribbed floor slabs (see Sec. X.2).

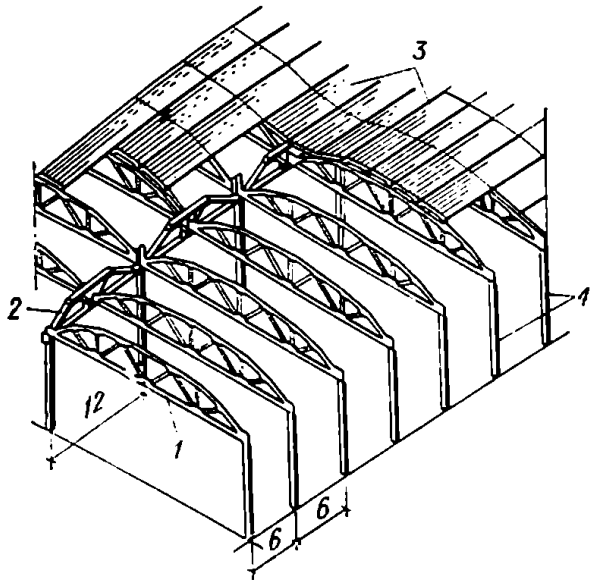



Fig. X.30. Roof of a one-storey industrial building with intermediate trusses
*1*—main truss; *2*—intermediate truss; *3*—roof slabs; *4*—columns

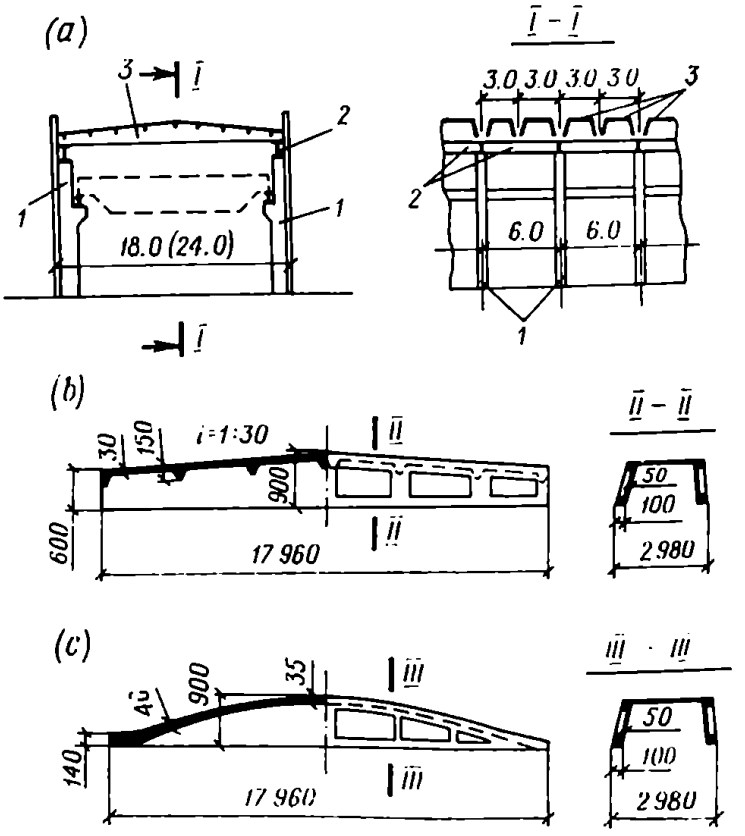



Fig. X.31. One-storey industrial building with large-size roof slabs
*1*—column; *2*—longitudinal beam; *3*—large-size roof slab

When the bay of a building is 12 m, use may be made of girders and slabs intended for the 6-m bay, and the necessary intermediate supports are provided by beams or trusses (Fig. X.30).

For some time past, one-storey industrial buildings have been using large-size slabs 3 × 18 m and 3 × 24 m in plan, laid on longitudinal beams resting on columns, or on longitudinal bearing walls (Fig. X.31*a*). This reduces the number of members to be erected, and cuts down the material and labour requirements.

In the USSR, two basic types of large-size prestressed concrete slabs are low-pitched ribbed slabs (Fig. X.31*b*) and type КЖС barrel slabs (Fig. X.31*c*).

*Columns for one-storey industrial buildings* (Fig. X.32) may be solid single-piece (rectangular in cross section) or open-work two-piece consisting of two side members connected by spacers. The width of a column, $b$, usually ranges between 40 and 60 cm. In the plane of a transverse frame (if a building is equipped with an overhead travelling crane) the column side is $h_t$ = 38 to 60 cm above the crane beam (to provide good support for the girder), and $h_b = H_b/10$ to $H_b/14$ (where $H_b$ is the length of the column as reckoned from the top of the footing to the bottom of the crane beam) below the crane beam (to ensure the necessary rigidity). In solid single-piece columns, $h_b$ usually ranges between 60 and 80 cm, and in two-piece columns, $h_b$ is from 120 to 160 cm. The width of a side member is $h_{s\text{-}m}$ = 25 to 35 cm. The centre-to-centre distance between the spacers is $s$ = 2 to 3 m.

The columns resist a normal compressive force and bending moments in the plane of the transverse frame, so, they work in eccentrical compression. The longitudinal load-bearing reinforcement is distributed at the faces normal to the plane of flexure.

The bending moments acting in the columns are determined from the analysis of the transverse frame. If the girders in all frame bays are at the same level, the most suitable approach is the method of displacements, because the transverse frame has only one unknown, namely the linear displacement of the top end of the columns, irrespective of the number of bays.

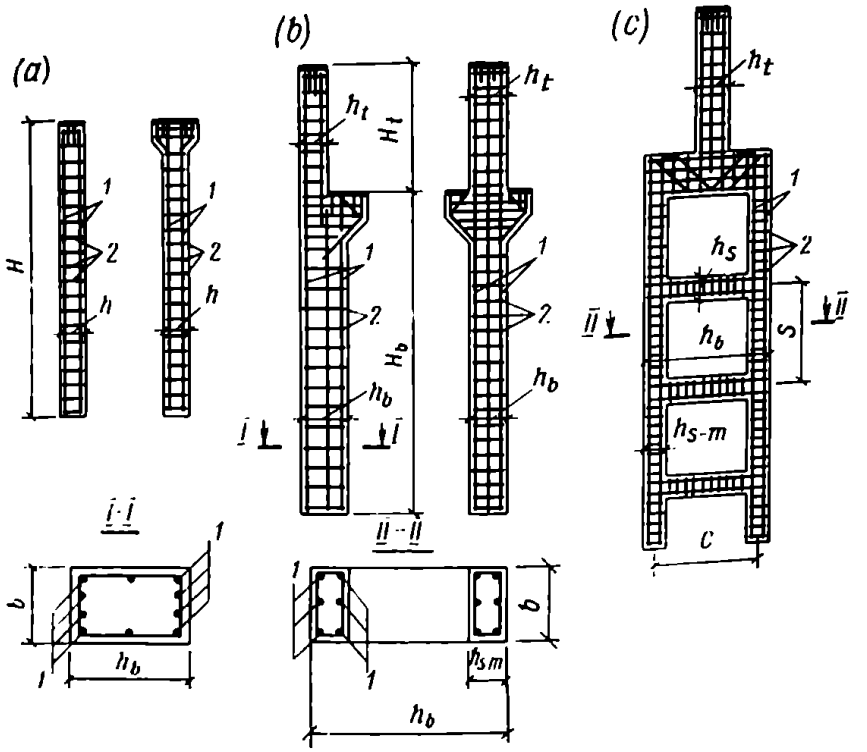



Fig. X.32. Columns of one-storey industrial buildings
(*a*) in buildings having no overhead travelling cranes; (*b*) in buildings having overhead travelling cranes; (*c*) two-piece column in buildings equipped with overhead travelling cranes; *1*— longitudinal load-bearing reinforcement; *2*— stirrups

The basic frame is formed by introducing a horizontal linear constraint preventing the top end of the columns from horizontal displacement (Fig. X.32A). Then, all the columns (struts) are fixed at one end in the footings and have a hinged immovable support at the other. The reactions appearing in such struts under various load combinations, as calculated by the methods of structural mechanics, are given in Appendix XVII. The unit displacement $\Delta = 1$ gives rise to bending moments in the struts and a reaction $B_\Delta$ in the constraint. External loads likewise induce bending moments in the loaded struts and a reaction $B_p$ in the constraint. For each load combination, we may set up a canonical equation equating all the reac-

tions in the horizontal linear constraint to zero, because real structures have no constraints. Let $r_{11}$ be the reaction in the constraint, induced by the displacement of all struts by $\Delta = 1$. It is equal to the sum of the reactions in all the the struts of the frame due to the unit displacement:

$$r_{11} = \sum B_{\Delta}$$

When the displacement $\Delta$ takes on a certain definite value, the reaction in the constraint is $r_{11}\Delta$. Now let $r_{1p}$ be the reaction in the constraint, induced by all the loads applied to the struts. It is equal to the sum of the reactions in all the struts of the frame:

$$r_{1p} = \sum B_p$$

Consequently, the canonical equation may be written as follows

$$r_{11}\Delta + r_{1p} = 0$$

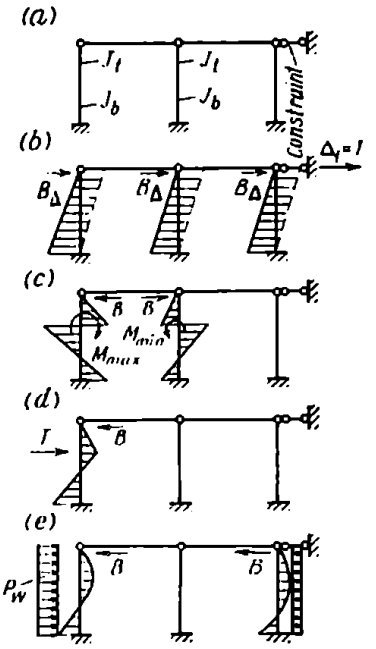



Fig. X.32A. To the analysis of the transverse frame of a one-storey industrial building

(a) basic system; (b) reactions and strut bending moment diagrams due to displacement $\Delta = 1$; (c) reactions and bending moment diagrams due to vertical crane load; (d) reactions and bending moment diagrams due to crane braking; (e) reactions and bending moment diagrams due to wind load

The coefficient $r_{11} = \sum B_{\Delta}$ is the same for all load combinations. The coefficient $r_{1p} = \sum B_p$ is determined for each load combination (taking into consideration the sign of $B_p$), and, for each load combination, the displacement is given by

$$\Delta = -r_{1p}/r_{11}$$

Once $\Delta$ is known, the elastic reaction is calculated for each strut as follows

$$B_{el} = B_{\Delta}\Delta + B_p$$

If a given strut is unloaded, $B_p = 0$.

To plot the bending moment diagram, each strut is regarded as a cantilever fixed at the footing and subjected to the force $B_{el}$ applied at its end and to external loads.

When a frame is designed to carry a crane load, we have to take into consideration the spatial behaviour of the entire skeleton, because the unloaded transverse frames are engaged by the rigid body of the roof and assist the loaded one in carrying the load. For this purpose, the canonical equation is extended to include the spatial behaviour factor $c_{sp} = 4$ (when the centre-to-centre distance between the frames is 6 m) or $c_{sp} = 3.4$ (when the bay is 12 m):

$$c_{sp} r_{11} \Delta + r_{1p} = 0$$

With crane loads, $\Delta$ may approximately be taken as zero. After an appropriate bending moment diagram has been plotted for each load combination, it is necessary to tabulate the design forces at the characteristic sections of the columns, namely above the crane cantilever, directly below the cantilever, and at the top of the footing. For constant-section columns (those carrying no crane load), the design section is the bottom section. In the table, the bending moment and normal force (and the shearing force for two-piece columns) caused by each type of load are entered for each design section, and the worst load combination is then chosen.

The selection of the cross-sectional area of a column consists in determining the steel area $F_s$ and $F'_s$ at the design sections, using the expressions for eccentrical compression (see Chap. VII). If no overhead travelling cranes are installed, the effective length of columns (necessary to allow for the effect of buckling) is $l_0 = 1.5H$ for single-bay buildings and $l_0 = 1.2H$ for buildings having two or more bays. For buildings with overhead travelling cranes resting on single-span crane beams, $l_0 = 1.5H_b$ for sections below the crane beam, and $l_0 = 2.5H_t$ for sections above the crane beam (if the crane load is taken into consideration, $l_0 = 2H_t$).

For a two-piece column the first thing to do is to determine the force acting in the side members. The longitudi-

nal compressive force $N$ is equally shared by the side members, whereas the total bending moment $M$ adds load to one side member and unloads the other with a force $M/c$, where $c$ is the centre-to-centre distance between the side members. With allowance for buckling, the moment is taken as $M\eta$ [$\eta$ is given by Eq. (VII.9)]. Thus,

$$N_{s\text{-}m} = N/2 \pm M\eta/c$$

If $M\eta/c > N/2$, one of the side members is in tension.

The bending moments in the side members are functions of the shearing force $Q$. They are found assuming that the zero points of the moment diagram lie in the middle of the panel $s$:

$$M_{s\text{-}m} = (Q/2)\,(s/2) = Qs/4$$

The necessary steel area for a side member is found as for eccentrical compression (or eccentrical tension) caused by the simultaneous action of $N_{s\text{-}m}$ and $M_{s\text{-}m}$.

The spacers in a two-piece column work in bending. The forces in a spacer are

$$M_s = Qs/2$$

$$Q_s = 2M_s/c = Qs/c$$

The columns are checked for stability across the plane of a transverse frame, assuming that the force $N$ is applied with an accidental eccentricity.

## X.5. Buildings for Water Supply and Sewage Disposal Systems and Boilers

Buildings for water supply systems include first-level, second-level and circulation pumping stations, and buildings for filters, clarifiers and water softening plants. Sewage disposal systems require buildings for pump and blower houses and sludge pumping stations.

Plants differing in their purpose should preferably be arranged in the same building. This will cut down the space occupied by the plants, the size of the construction site,

and the length of the services, and, as a consequence, reduce the cost and time of construction.

The above buildings can be erected on the surface, half-buried (with the floor level slightly below the grade), buried (with the floor level considerably below the grade), and underground.

As a rule, buildings are equipped with overhead monorail conveyers, overhead-track hoists, or overhead travelling cranes to facilitate erection and attendance.

Buildings for water supply and sewage disposal systems may have brickwork bearing walls and reinforced concrete floors. The substructure of a building consists of heavy masonry or concrete walls. If the depth of a building is considerable and the ground water level is high, the substructure is preferably built as a caisson (Fig. X.33).

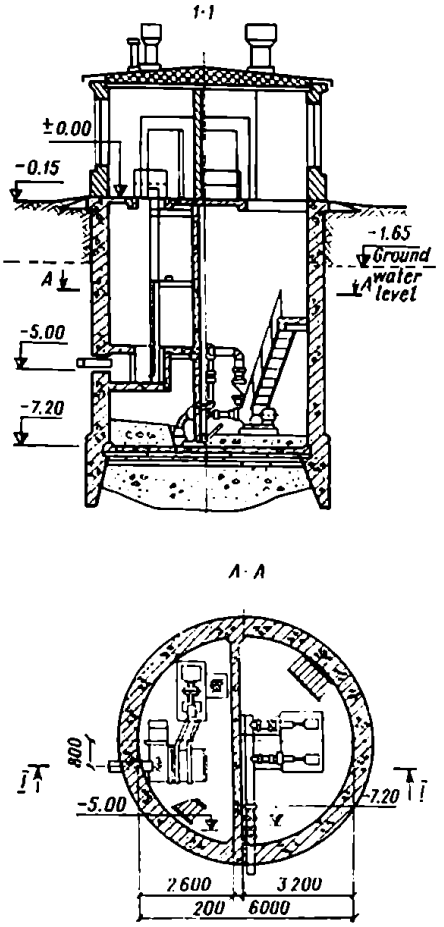



Fig. X.33. Pumping station in a sewage disposal system

In the USSR, standard type-design buildings for water supply and sewage disposal systems are single-bay buildings with a bay of 6, 9, 12, and 18 m. The height of the superstructure (as reckoned from the grade to the bottom of the roof) in buildings equipped with overhead-track hoists ranges between 3.6 and 9.6 m at 1.2-m intervals, that in buildings having overhead travelling cranes is 12.6 m. The depth of the substructure is 1.2 m in half-buried buildings, from 2.4 to 6 m at 1.2-m intervals in buried buildings, and 6 m in underground buildings. The superstructure is similar in design to single-storey industrial buildings (see Sec.

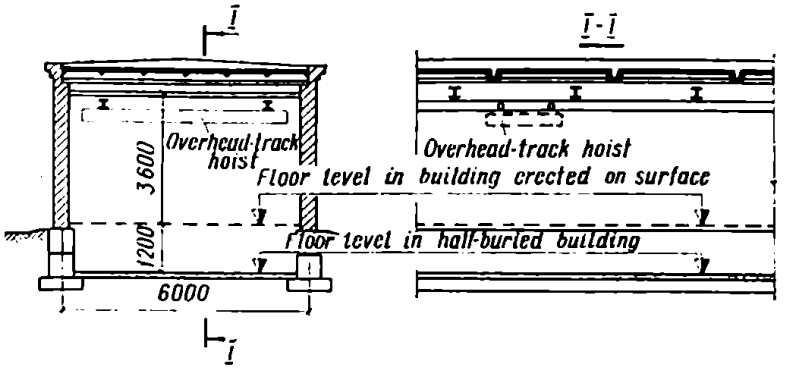



Fig. X.34. Standard 6-m bay surface-type and half-buried buildings for water supply and sewage disposal systems

X.4). It is assembled from type-design prefab reinforced concrete columns, beams, roof slabs, and wall panels. The substructure is made of prefab reinforced concrete panels used in rectangular tanks and water treatment structures (see Sec. XI.4). If a building extends below the ground water level, the bottom of the building and, sometimes, the entire substructure is made of in-situ reinforced concrete.

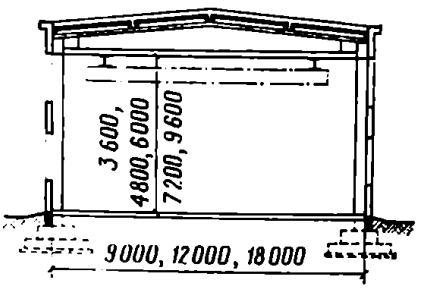



Fig. X.35. Standard 9-, 12-, and 18-m bay buildings for water supply and sewage disposal systems

The superstructure of a building having a bay of 6 m consists of exterior (usually brickwork) bearing walls carrying reinforced concrete roof slabs 3 × 6 m or 1.5 × 6 m in size (Fig. X.34). When the bay of a building is 9, 12, or 18 m, the superstructure is of the skeleton construction: columns spaced 6 m apart give support to single-span roof beams which, in turn, carry roof slabs 3 × 6 m or 1.5 × × 6 m in size. Wall panels are installed outside the columns and overhead-track hoists are mounted on the roof beams (Fig. X.35).

Buildings for pumping stations having a bay of 18 m and equipped with overhead travelling cranes use two-piece columns.

Substructure walls are made of prefabricated reinforced concrete panels with a nominal length (along a building) of 3 m and a height corresponding to the depth of the substructure, namely 2.4, 3.6, 4.8, or 6 m. The panels are installed in a longitudinal groove provided in a precast reinforced concrete footing, and the gaps are filled with in-situ concrete using fine gravel as the aggregate. Owing to this, the panels are safely fixed in the footing and can resist lateral earth pressure, working as cantilevers (Fig. X.36). Panels 2.4 m high are 15 cm thick, and panels 3.6, 4.8, and 6 m high have a variable thickness of 20, 28, and 36 cm respectively at the bottom and 14 cm at the top*.

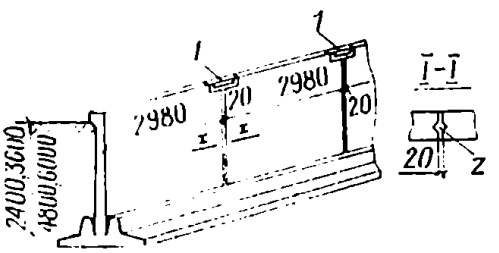



Fig. X.36. Substructure of a building protected by precast panels
*1*—joint between panels: welded stick-outs, concreting in-situ; *2*—key joint filled with grout applied under pressure

The construction of standard buried buildings intended for localities free from ground water are shown in Fig. X.37*a* and *b*.

The columns supporting the roof are installed in reinforced concrete sockets provided in the gaps of the strip footing carrying the wall panels (Fig. X.37, Detail *A*). A gap of 50 mm is left between the wall panels and the columns to prevent the columns from resisting earth pressure. The bearing walls of the superstructure rest on the substructure panels.

If the ground water level is within the substructure of a building, the bottom should preferably be made as a precast reinforced concrete slab with a groove for wall panels (Fig. X.38*a* and *b*). With a considerable ground water upthrust (2 to 3 m), the entire substructure (bottom and walls) of buildings having a bay of 12 and 18 m is fabricated as an in-situ reinforced concrete box (Fig. X.38*c*).

If the site is free from ground water, the outer surface of the substructure walls is water-proofed by hot bitumen. In

* In more detail, the construction of these panels is discussed in Sec. XI.4.

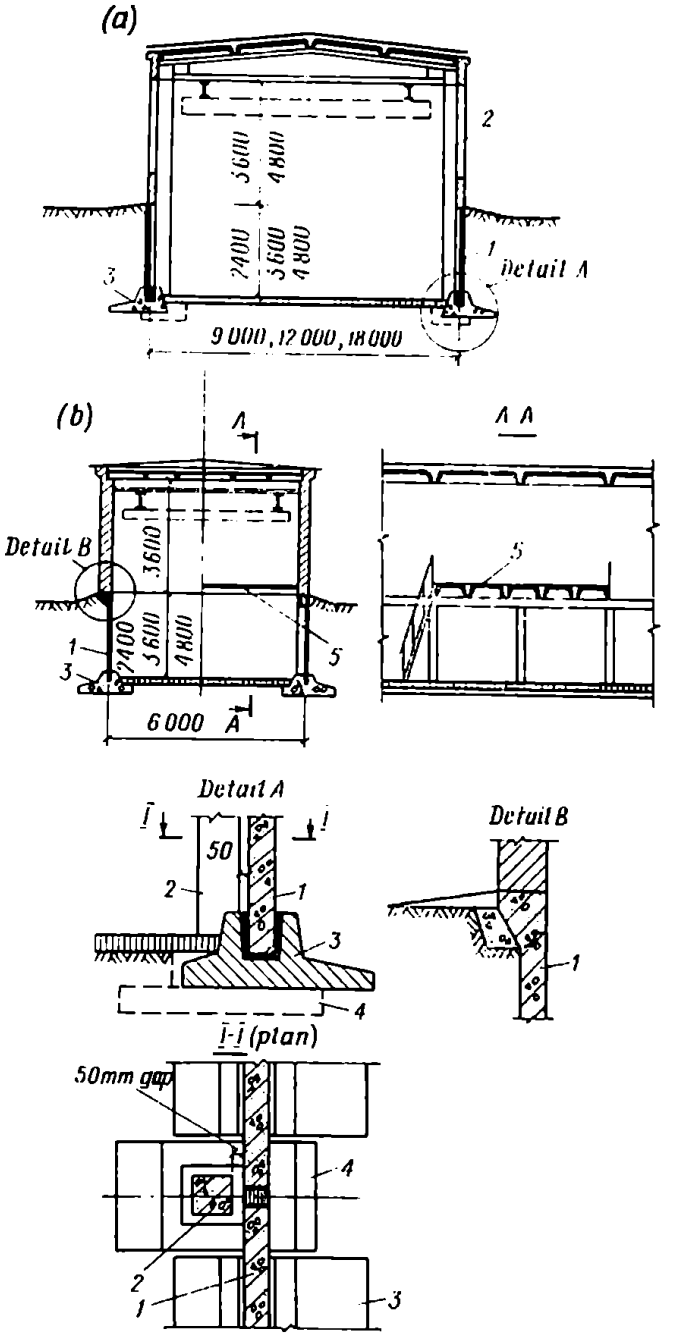



Fig. X.37. Buried buildings on a site free from ground water

(*a*) 9- to 18-m bay buildings; (*b*) 6-m bay building; *1*—substructure wall panels; *2*—column; *3*—reinforced concrete strip footing; *4*—column footing; *5*—maintenance platform assembled from precast slabs

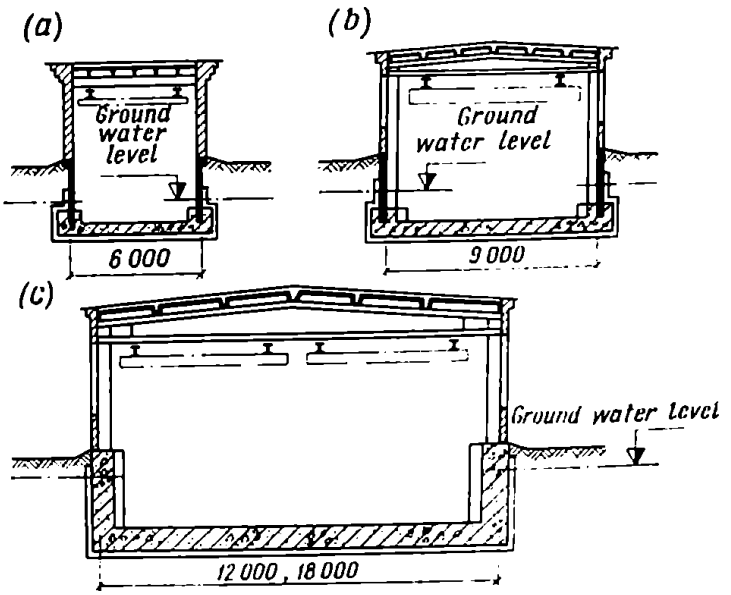



Fig. X.38. Buried buildings in the presence of ground water

the presence of ground water, water proofing is done with pasted materials up to an elevation at least 0.5 m above the maximum ground water level. On the side surfaces, the pasted insulation is protected by a brick wall half a brick thick.

Buildings for underground pumping stations have a bay of 6 m and a depth of 6 m. Physically, they are assembled in the same way: wall panels 6 m (5.4 m) high are installed in the grooves of a strip footing or, if there is ground water on the site, in the grooves of an in-situ bottom slab (Fig. X.39). Roof slabs are laid directly on the wall panels. Transverse steel beams carrying an overhead monorail conveyer are fastened to flanges welded to the embedded parts of the wall panels.

The superstructure of buildings for water supply and sewage disposal systems is designed as prescribed in Sec. X.4.

The substructure walls of a building resist a horizontal load due to the lateral earth pressure $p_e$ and a horizontal load $p_l$ due to a live load applied at the earth surface. In the presence of ground water, they also have to carry the load due to the water pressure $p_w$ (Fig. X.40). How to determine these loads is discussed in Sec. XI.3.

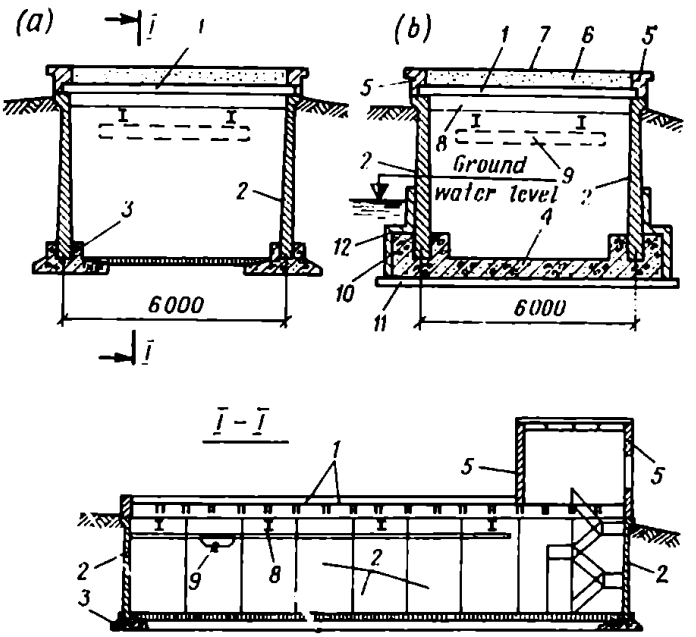



Fig. X.39. (*a*) Underground pumping station on a site free from ground water and (*b*) underground pumping station in the presence of ground water

*1*—roof panels; *2*—wall panels; *3*—strip footings; *4*—reinforced concrete bottom resisting ground water upthrust; *5*—brick wall; *6*—warmth-keeping jacket; *7*—water-proofing; *8*—transverse steel beams; *9*—overhead-track hoist; *10*—water-proofing; *11*—concrete layer; *12*—protective wall

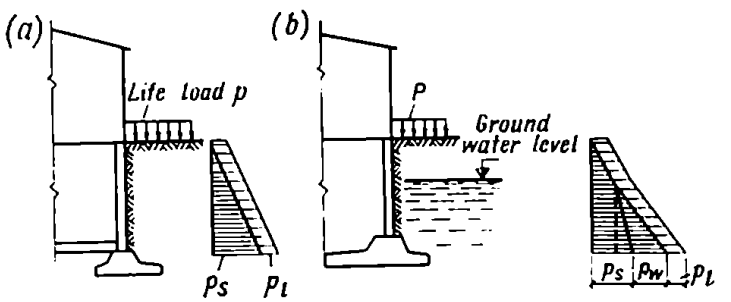



Fig. X.40. Horizontal loads imposed on a substructure
(*a*) without ground water; (*b*) with ground water

Bending moments in wall panels are found as in a vertical cantilever 1 m wide fixed in the footing. From the moments thus calculated, one determines the cross-sectional area of the vertical tensile reinforcement near the outer face of the panel. If the wall panels are subjected to a vertical load caused by the superstructure wall, they resist the simultaneous action of the bending moment and the normal force. In this case, the steel area required at each face of the panel is determined as for eccentrical compression (see Sec. VII.2).

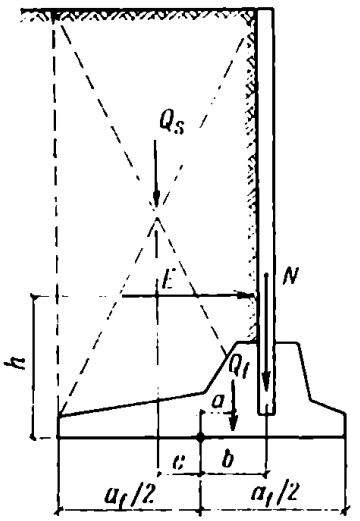



Fig. X.41. Loads imposed on a strip footing in buried buildings

The footing of the wall panels is loaded eccentrically. It carries the resultant $E$ of the lateral earth pressure, the load $N$ due to the weight of the wall panels and the structures resting on them, the load $Q_f$ due to the self-weight of the footing, and the load $Q_s$ due to the weight of the backfill over its steps (Fig. X.41).

The footing is usually extended outside the building in order that the force $Q_s$ can produce a moment opposite to the moments due to the forces $E$, $N$, and $Q_f$. Because of this, the pressure beneath the bottom of the footing is distributed more uniformly.

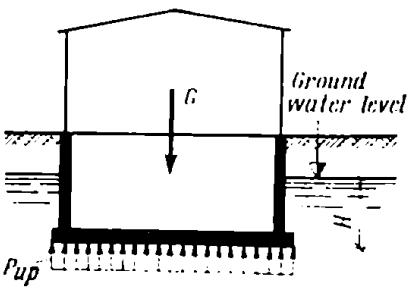



Fig. X.42. To the check for resistance to floating-up

In the presence of ground water (Fig. X.42), the bottom has to resist the water upthrust. The water upthrust per unit area of the bottom is $p_{up} = \gamma H$ where $\gamma$ is the density of water and $H$ is the distance from the bottom to the maximum ground water level with allowance for seasonal variations.

Because $\gamma = 1$, the water upthrust is numerically equal to $H$. The bottom slab should be designed for bending caused by the water upthrust $p_{up}$, and the building as a whole should be checked for resistance to floating-up subject to

$$G \leqslant P$$

where $G$ is the weight of the building multiplied by 0.9, and $P$ is the resultant of the water upthrust taken with the overload factor 1.1 applied:

$$P = 1.1 p_{up} F_f$$

Here, $F_f$ is the bottom slab area.

In calculating the weight of the building that will resist floating-up, only those members may be taken into consideration which will be erected with the ground water level lowered for the duration of construction. In view of this, the bottom slab is made appreciably thick.

Buildings for boilers are assembled from type-design prefabricated reinforced concrete elements similar to those used in one-storey industrial buildings (see Sec. X.4). The bay of such buildings ranges between 12 and 24 m, and the height, between 6 and 14.4 m. Access to the boilers for servicing is from built-in footwalks. Figure X.43 shows the layout of a building for Soviet-made type ДКВР-20-13 boilers.

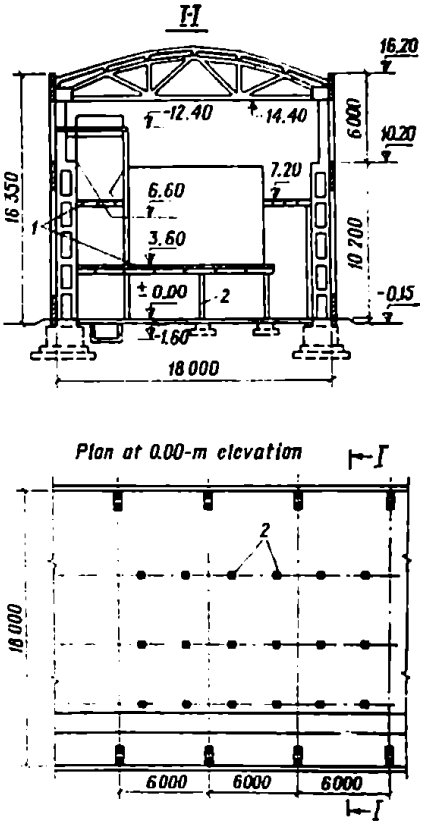



Fig. X.43. Layout of a boiler house

*1*—maintenance platforms; *2*—columns supporting maintenance platforms

CHAPTER ELEVEN

# SPECIAL STRUCTURES FOR WATER SUPPLY AND SEWAGE DISPOSAL SYSTEMS

## XI.1. General

Water supply and sewage disposal projects incorporate various reservoirs for transportation, storage, and treatment of waste water and other liquid effluents. These structures are usually made of reinforced concrete.

Tanks may be cylindrical or prismatic (that is, circular or rectangular in plan), buried or installed on the ground surface, and covered or open-top, with flat or conical floors.

The shape and size of a tank depend on the purpose it is to serve. When only capacity is critical (as in the case of a water tank), the shape and size are chosen from several design alternatives on the basis of technical and economic advantages.

When choosing the shape of a tank, it should be remembered that the perimeter of the walls of a circular tank is smaller than that of a rectangular tank of the same capacity, and that the pressure of the water the tanks contain causes the cylindrical walls to work in axial tension and the flat walls in bending (or eccentrical tension). Because of this, steel water tanks are always circular with walls of thin sheets. For reinforced concrete structures, circular shape is likewise preferable because the walls are easier to prestress (by tensioning ring or hoop reinforcement). This improves their cracking resistance and cuts down concrete and steel consumption (because the walls are thinner and high-strength reinforcing steel is used).

In the walls of circular tanks, the tensile forces are directly proportional to the diameter, therefore the wall thickness and steel area should be increased with increasing diameter, even if the depth of a tank remains the same.

In rectangular tanks, the bending moments in the walls only depend on the depth, so tanks of different dimensions in plan may be built with the same wall thickness and load-bearing steel area.

Experience has shown that for a capacity of two to three thousand cubic metres circular tanks are more economical than rectangular tanks, whereas for capacities of five to six thousand cubic metres, the latter are preferable. If the capacity of a tank is relatively small, rectangular tanks may sometimes prove more economical than their circular counterparts. For example, rectangular settling tanks to be installed in buildings for water treatment stations are preferable, because they will occupy less space than circular tanks, and the size of the building may, therefore, be reduced. A compact arrangement of structures is always more attractive economically as it helps to reduce the site size and to cut down the cost of fencing, services, etc.

The technical and economic particulars of buried structures for water supply and sewage disposal systems largely depend on their depth: deeper tanks have to withstand a greater water pressure exerted on the walls, call for deeper pipelines, and are more difficult to erect. If the depth of a buried structure is not specified in advance to suit process requirements, it is usually taken as 4.8 m, or 3.6 m if the tank capacity is small.

Reinforced concrete tanks may be of in-situ, precast, or precast/in-situ construction. In precast/in-situ tanks, the floors may be cast in-situ and the walls and roof may be assembled from prefabricated members; also, precast roofs may be installed on in-situ walls. Prefab members cut down labour requirements, material consumption, and construction time.

In Soviet practice, tanks and similar structures are extensively standardized in terms of both construction types and member sizes. The prefab members come in complete kits which can be combined with those for industrial buildings and used in the erection of any structures for water supply and waste disposal systems.

To meet watertightness requirements, the walls and roofs of reinforced concrete tanks are made of dense heavy concrete. The concrete brands in terms of compressive strength

are at least M-200, in terms of watertightness B4 to B8, and in terms of frost resistance Mpa 100 to Mpa 150. In the joints between precast members, it is advisable to use concrete with expanding cement as the binder.

For a better watertightness, the inner surface of tanks is given a coat of cement plaster, shotcrete, or ethynol lacquer or other polymer compounds. If the liquid contained within a tank is likely to attack concrete (for example, acid or slightly acid waters), the concrete is protected with water glass, epoxy-resin-based polymer compounds, and the like. In the absence of ground water, buried tanks erected in draining soils are waterproofed by two layers of bitumen putty applied from the outside. In some aeration and settling tanks, a layer of asphalt is applied or two layers of roll insulation are pasted with bitumen putty between the concrete baseplate and the reinforced concrete bottom. If the ground water level is higher than the tank floor, the bottom should be given a coat of at least three layers of pasted insulation, and at least two layers of pasted insulation should be installed on the walls to a height not less than 0.5 m above the maximum ground water level. Vertical insulation is protected by a hold-down brick wall half a brick thick. Above the pasted insulation, the walls are given a coat of bitumen putty.

All metal structures and embedded steel parts and fasteners are protected against corrosion with PVC lacquer, a mixture of ethynol lacquer and asbestos, or coal-tar varnish. The parts difficult of access for renewal of paintwork should be galvanized. In covered structures, reinforced concrete roof members should be painted too, because they are exposed to an elevated humidity.

When choosing a site, it should be remembered that tanks cannot be erected on settlement soils which are rather sensitive to water. Also, tanks should be set up above the ground water level, because otherwise, it would be necessary to lower the ground water level during construction, to install a heavier floor to resist water pressure from beneath, to use pasted multi-layer insulation, and to take measures to prevent tanks from floating-up. To cope with the last-mentioned problem, the amount of backfill over the roof would have to be increased, which would call for a heavier

roof, or an additional layer of concrete would have to be laid on the floor. Furthermore, open-top structures would have to he filled with water during high ground-water periods, which might interfere with their normal service.

## XI.2. Construction Types of Circular Reinforced Concrete Tanks

Circular tanks include water tanks, various settling tanks, clarification tanks, methane tanks, aeration filters, and so on.

**Water Tanks.** These are intended as regulating or stand-by reservoirs in the water supply systems of populated areas or industrial projects. As a rule, water tanks are of the buried covered construction with a flat floor. For heat insulation, the backfill over a tank roof is chosen 0.5 to 1 m deep, depending on the expected temperature in winter. Sometimes, more effective cellular concrete or ceramsite is used as heat insulation. Tanks installed on water towers or floors in building are usually of the open-top construction*.

In the Soviet Union, water tanks are built in the following standard capacities, diameters, and heights.

| Capacity, $m^3$ | 50 | 100 | 150 | 250 | 500 | 1 000 | 2 000 | 3000 | 6 000 |
|---|---|---|---|---|---|---|---|---|---|
| Diameter, m | 4.5 | 6.5 | 8 | 10 | 12 | 18 | 24 | 30 | 42 |
| Height, m | 3.6 | 3.6 | 3.6 | 3.6 | 4.8 | 4.8 | 4.8 | 4.8 | 4.8 |

As a rule, the floors of water tanks are cast in-situ and the walls and roofs are assembled from prefabricated members. If the capacity of a tank does not exceed 2 000 $m^3$, the entire structure may be cast in-situ. In this case, the walls are prestressed by tensioning the hoop reinforcement wrapped around the tank. If the tank capacity is below 500 $m^3$, the walls may be left non-prestressed.

The in-situ roofs of buried tanks are usually of the flat-slab construction. They rest on walls and columns having capitals (Fig. XI.1). The flat-slab roof is made rather thin, and its underside is made smooth to improve ventilation above the water surface.

* Such tanks may have a light roof which does not affect their design layout.

The precast walls of circular tanks consist of prefabricated panels of a length equal to the tank height. The panels are vertically installed in a slot provided in the in-situ floor, and the joints between the panels are grouted. After the grout has attained an adequate strength, the wall is compressed by prestressed reinforcement wrapped around the tank. After the steel has been tensioned, it is shotcreted

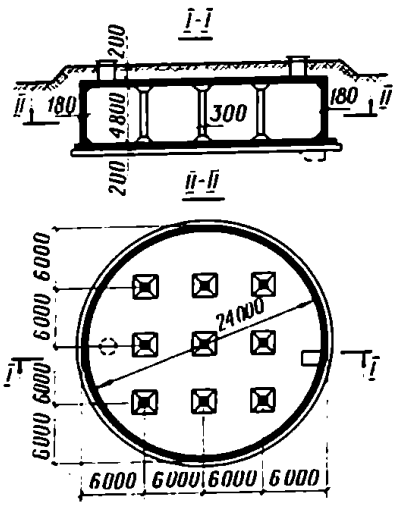



Fig. XI.1. Circular in-situ reinforced concrete buried tank with a flat-slab roof

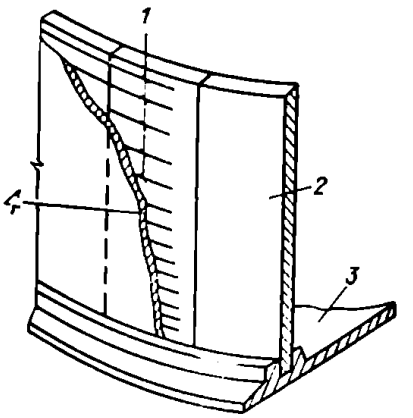



Fig. XI.2. Prestressed wall of a precast circular tank
*1*—prestressed hoop reinforcement; *2*—wall panel; *3*—floor; *4*—shotcrete

for protection (Fig. XI.2). The internal columns supporting the roof are installed on a footing slab laid on the floor portion having an increased thickness.

Precast roofs are generally of the slab and girder construction. Here, the columns carry girders which, in turn, give support to ribbed slabs (Fig. XI.3*a*). Precast roofs may also be assembled from rectangular rib-rimmed slabs laid directly on columns (Fig. XI.3*b*), or from trapezoidal ribbed slabs installed on ring beams supported on columns (Fig. XI.3*c*).

In Soviet practice, the wall panels for precast water tanks shown in Fig. XI.4 are chosen from specialized kits of parts for water supply and sewage disposal structures, whereas the columns, girders, and roof slabs, from kits of standardized members for multistorey industrial buildings.

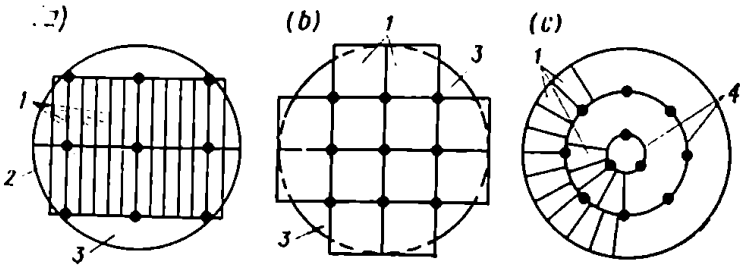



Fig. XI.3. Precast roofs of circular tanks
*1*—panels; *2*—girders; *3*—in-situ portions; *4*—ring beams

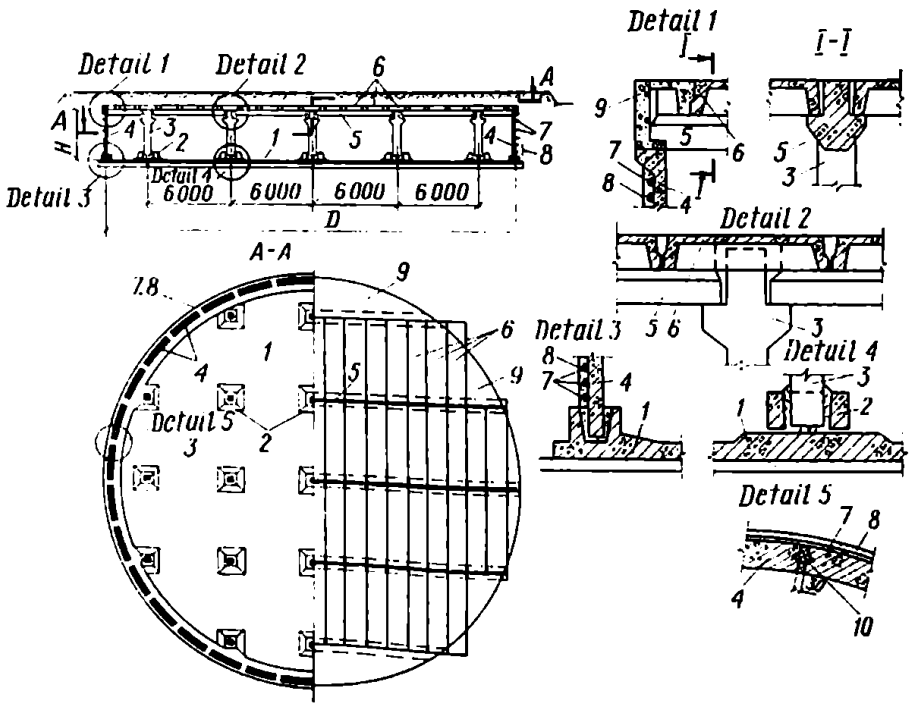



Fig. XI.4. Circular tank assembled from prefab members
*1*—in-situ floor; *2*—footing block; *3*—column; *4*—wall panel; *5*—girder; *6*—roof panel; *7*—prestressed hoop reinforcement; *8*—shotcrete; *9*—in-situ roof portions; *10*—cement grout

**Vertical Settling Tanks.** These are open-top circular tanks with a conical floor (Fig. XI.5). The clarified water is withdrawn by an upper circular launder. A gangway is provided above the settling tank. The conical floor of vertical settlers is cast in-situ, and the circular wall is either

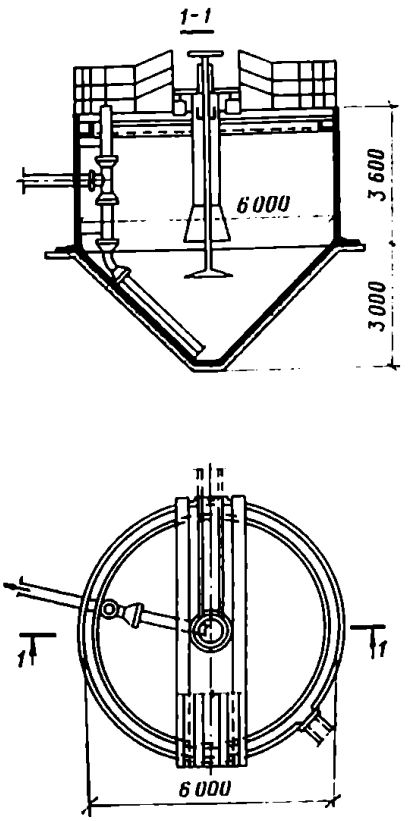



Fig. XI.5. In-situ vertical settling tank

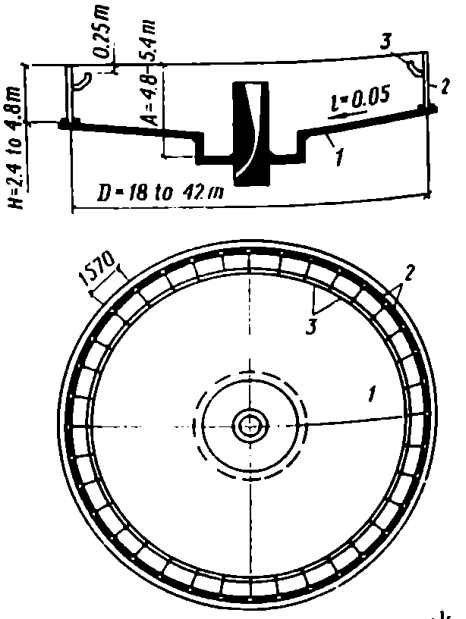



Fig. XI.6. Radial settling tank assembled from prefab members *1*—in-situ floor; *2*—wall panels; *3*—circular launders

assembled from prefab members or cast in-situ without prestressing, because standardized tanks have a relatively small capacity with a diameter of 4 to 9 m and a height of the cylindrical section of 3.6 and 4.2 m.

A similar construction applies to biocoagulators, contact tanks, and silt pits.

**Radial Settling Tanks.** Radial settlers are open-top circular tanks of large diameter and low height. As a rule, they have in-situ floors slightly sloping toward the centre. The walls may either be cast in-situ or assembled from prefabri-

cated members. The clarified water is withdrawn by a circular launder. Standardized circular settlers (Fig. XI.6) are from 18 to 42 m in diameter and have a height of 3, 3.6, and 4.2 m. If the height is 3.6 or 4.2 m, the walls should be prestressed.

A similar construction applies to *thickeners.*

**Clarification Tanks.** In construction, clarification tanks are similar to vertical settling tanks, except that they have

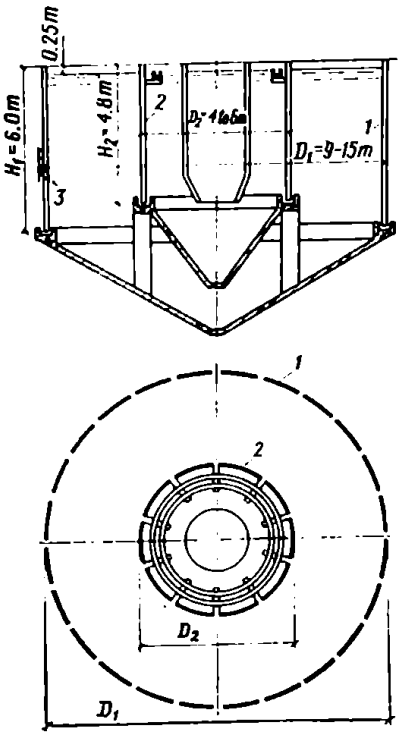



Fig. XI.7. Clarification tank
*1*—prefab exterior wall panels; *2*—prefab interior wall panels; *3*—prestressed hoop reinforcement

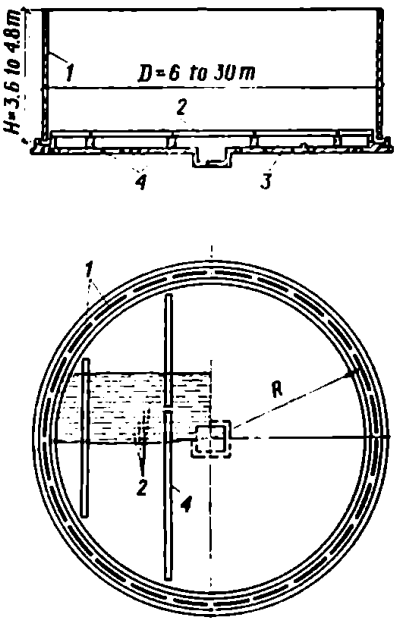



Fig. XI.8. Circular aeration filter assembled from prefab members
*1*—prefab wall panels; *2*—floor bars; *3*—in-situ floor; *4*—beams

an additional internal cylindrical chamber resting on suitable posts and having a conical floor (Fig. XI.7). The necessary standardized dimensions are shown in the figure.

**Circular Aeration Filters.** These have an in-situ flat floor with a pit at the centre, precast walls, and an additional perforated floor (Fig. XI.8). Standard aeration filters ran-

ge between 6 and 30 m in diameter and have a height of 3 or 4.8 m. The perforated floor is assembled from standard bars laid on precast beams.

**Methane Tanks.** These are covered circular tanks having a conical roof. The capacity of methane tanks may reach five thousand cubic metres. Up to now, they have been built of in-situ reinforced concrete. Prestressing makes it possible to erect methane tanks from prefabricated members.

The in-situ walls of circular tanks are reinforced with welded-wire fabric placed in two layers. To facilitate concreting, the walls are chosen to be at least 15 cm thick. The joint between the wall and floor has an arch brace to improve the bond. For all types of tanks, the thickness of the in-situ floor is taken as not less than 12 cm for dry soils and not less than 16 cm for wet soils.

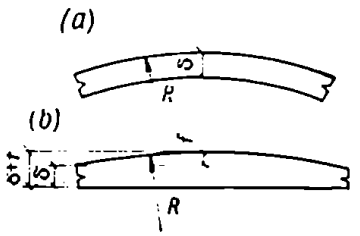



Fig. XI.9. Horizontal cross section of prefab wall panels in circular tanks

As previously noted, precast walls are assembled from panels vertically installed in a floor slot whose depth should be at least half as large again as the panel thickness.

The panel height is equal to the height of the structure. For standardized water supply and sewage disposal structures, it ranges between 3 and 4.8 m at 0.6-m intervals. The panel width is taken as a multiple of $\pi = 3.14$ so that an integer number of panels may be used to make up a wall: $b = \pi/2 = 1.57$ m or $b = 3\pi/4 = 2.35$ m. The panel thickness $\delta$ is determined by an incipient-cracking analysis. Standard-size panels are 12, 14, and 16 cm thick.

In their horizontal cross section, the panels may be curvilinear (such as shell-type panels, Fig. XI.9*a*) or have a curvilinear outer surface and a flat inner surface (such as flat-convex panels, Fig. XI.9*b*). Shell-type panels require less concrete than their flat-convex counterparts (especially with small tank diameters), but are more difficult to manufacture.

The radius of standard shell-type panels is 3.6 and 9 m. Such panels are used in tanks up to 18 m in diameter, and

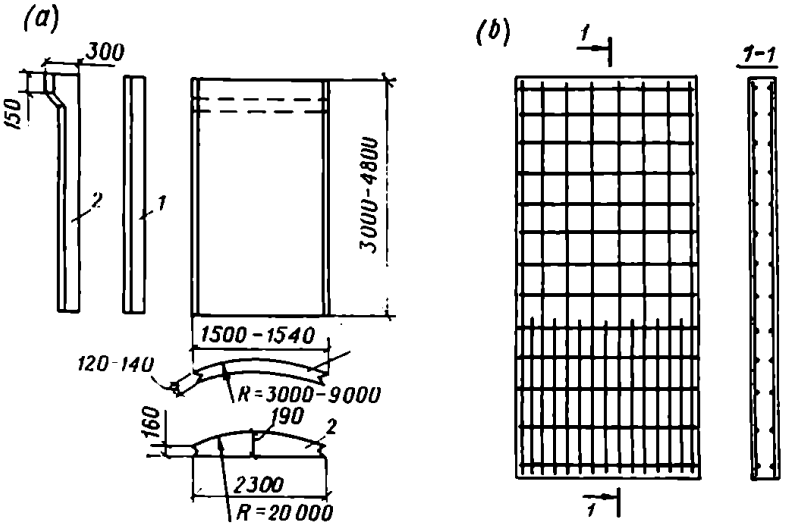



Fig. XI.10. Prefab wall panels of standard circular tanks
(*a*) general view; (*b*) distribution of reinforcement; *1*—shell-type panels; *2*—flat-convex panels

flat-convex panels whose radius on the curvilinear side is 20 m are intended for tanks from 24 to 42 m in diameter.

The vertical side faces of wall panels have triangular slots. After the joint between two panels has been grouted under pressure, a rhombic key is formed at the joint, which is compressed when the hoop steel is tensioned. The construction of wall panels is shown in Fig. XI.10.

Wall panels are equipped with embedded steel parts for attachment of circular overflow launders (Fig. XI.11) and roof members (Fig. XI.4, Detail *1*).

The precast wall panels may be connected to the floor rigidly so as to prevent the wall from radial displacement in the circular slot (Fig. XI.12*a*), or they may be hinged (Fig. XI.12*b*). In the former case, the gap between the panels and the floor is filled with concrete using fine broken stone as the aggregate; in the latter case, the gap is filled with cold asphalt putty which can deform during wall displacement without affecting the watertightness of the joint. Rigid joints are generally made in water tanks, and hinged joints, in sewage-disposal settlers, clarification tanks, and the like.

During the erection and alignment of the wall panels that are to be rigidly connected to the floor, the gap be-

ween the panels and the slot faces is temporarily filled with clean dry sand. This keeps dirt out of the gap and enables the wall to move in the radial direction when the hoop reinforcement is tensioned. Because of this, no bending moments appear in the wall in the vertical plane during the transfer of the prestress. After the steel has been tensioned, the sand is blown out and the gap is filled with concrete.

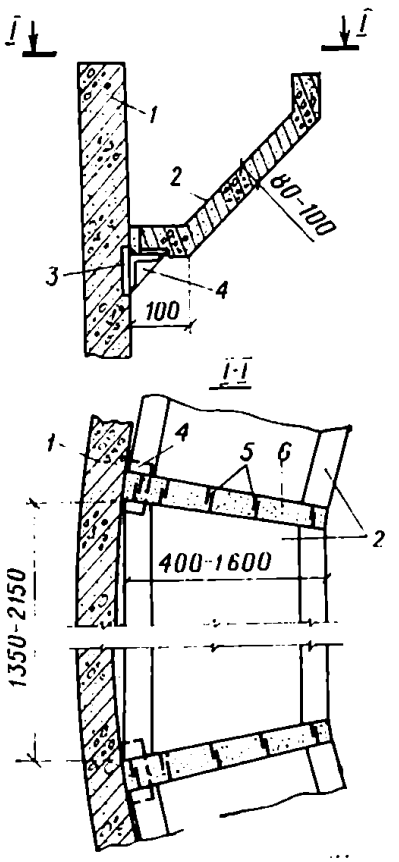



Fig. XI.11. Circular launders in circular tanks

*1*—wall panel; *2*—prefab member of a launder; *3*—embedded steel parts; *4*—supporting table; *5*—launder reinforcement stick-outs to be welded during erection; *6*—in-situ concrete using fine broken stone as aggregate

The hoop reinforcement from high-strength wire is tensioned on the walls by wrappers (Fig. XI.13).

The bar reinforcement is usually tensioned by electrical heating. The hoop bar is assembled from submultiple lengths; to the end of each portion are welded short bars: one screw-threaded, and the other plain-welded to an anchor stop. At the latter, the bars are joined together by nuts (Fig. XI.14). After the steel ring has been installed in the design position, the nuts are tightened to take up the slack of the bars, and electrical heat is applied. When heated, the bars elongate, and gaps appear between the nuts and stops. The nuts are then screwed as far as they will go to fix this elongation. As the bars cool, the reinforcing ring shortens, the tank wall is compressed, and the reinforcement remains in tension.

There also exist other methods of tensioning the reinforcement of circular tank walls, namely by jacks with the bars guyed up from the concrete surface to reduce friction,

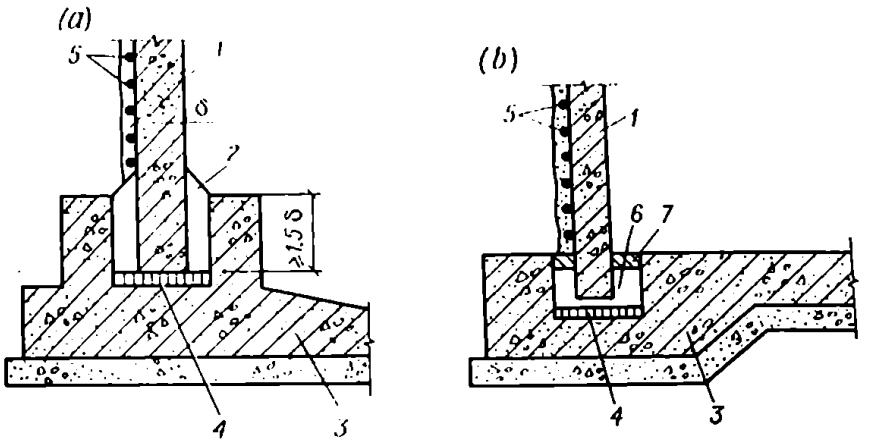



Fig. XI.12. Joint between a cylindrical wall and a floor
(a) rigid joint; (b) movable joint; 1—wall; 2—concrete using fine broken stone as aggregate; 3—floor; 4—levelling grout; 5—prestressed hoop reinforcement; 6—asphalt putty; 7—asbestos-cement grout

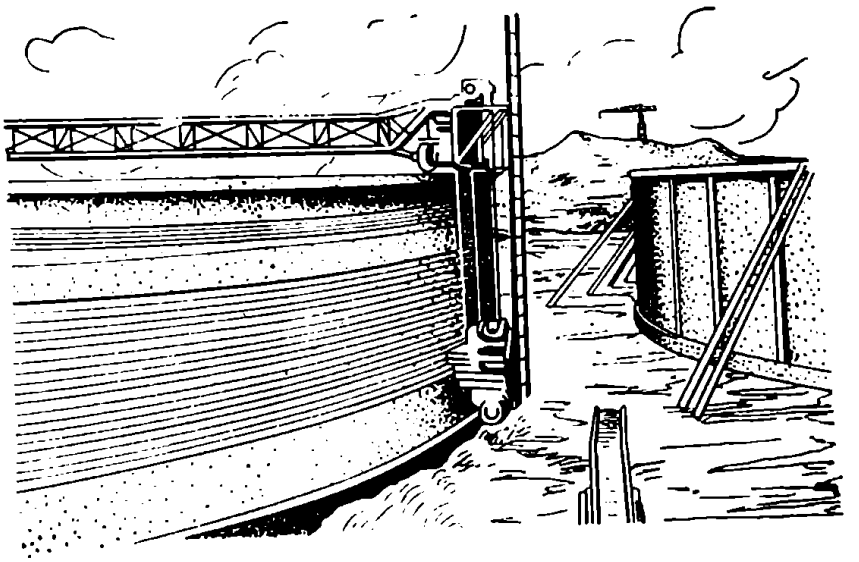


Fig. XI.13. High-strength wire tensioned by a wrapper

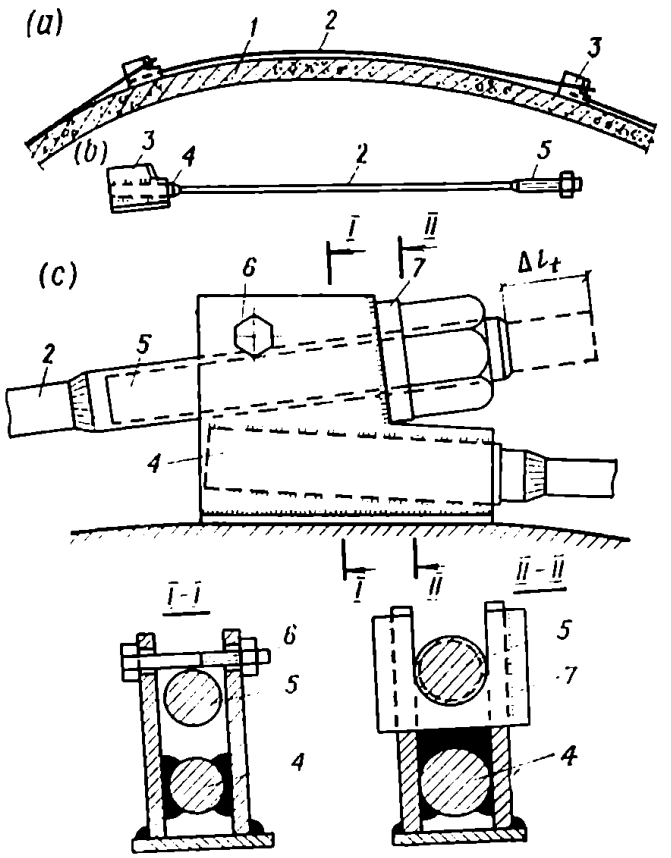



Fig. XI.14. Prestressed bars tensioned by electrical heat
(*a*) distribution of reinforcement over cylindrical wall; (*b*) reinforcing member; (*c*) anchor stop; *1*—wall; *2*—prestressed reinforcing bar; *3*—anchor stop; *4*—large-diameter short bar; *5*—screw-threaded large-diameter short bar; *6*—tie bolt; *7*—stop plate

and by slipping the reinforcement down the conical surface of the wall.

If the capacity of a tank is small, the reinforcing steel may be tensioned mechanically (Fig. XI.15). Here, a reinforcement hoop is made up of several bars having threaded short bars welded to their ends. The bars are joined together at vertical metal posts and tensioned by nuts screwed with torque wrenches or by jacks. The reinforcing steel is tensioned after the grout in the vertical joints has attained at least 70% of its design strength.

The distance between the prestressed hoops may vary from 15 or 20 cm to 10 mm (for wire reinforcement).

After the hoop reinforcement has been prestressed, it is given several coats of shotcrete to protect it against corrosion, so that the concrete cover is at least 25 mm. The first layer of shotcrete is applied immediately after tensioning, and all the other layers, when the tank has been filled with water, so that no cracking occurs in the shotcrete when the tank is refilled. The inner surface of the tank is plastered or shotcreted prior to tensioning the reinforcement so as to compact the plaster to the wall panels.

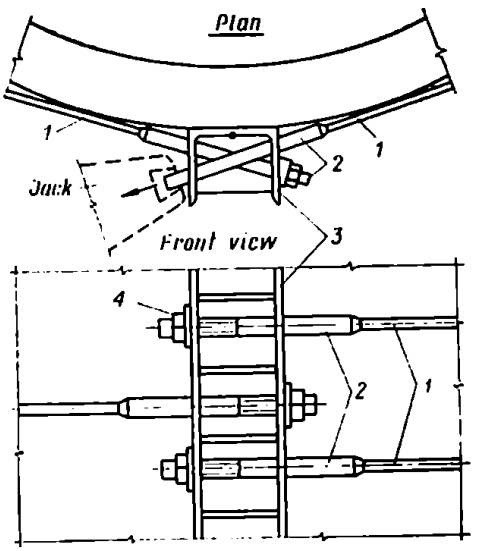



Fig. XI.15. Hoop reinforcement tensioned mechanically

*1*—prestressed hoop reinforcement; *2*—screw-threaded short bar welded to the hoop; *3*—metal post; *4*—nut

The cracking resistance of the floor can be increased by applying prestressed ring reinforcement. In the joint of Fig. XI.16, the floor is compressed by the bottom turns of the circular wall reinforcement. Such joints are used in tanks intended for fuel oil and petroleum products. The gap between the floor and the supporting concrete layer is filled with sand to reduce friction during the transfer of the prestress. In addition, two layers of asphalt sheathing paper are laid over the sand bed to prevent laitance leakage when concreting the in-situ floor.

With progress in technology and experience, prefabricated reinforced concrete members for buildings and engineering structures are continually improved. In a new Soviet type-design series for water supply and sewage disposal structures, shell-type wall panels are recommended for circular tanks having a diameter $D$ of 4.5 to 9 m (the radius of the curvilinear surface is $R = 3$ m). In larger tanks, use is made of flat-

convex panels with $R = 7.5$ m (for a tank diameter of 9 to 18 m) and $R = 15$ m (for a tank diameter of 24 to 50 m).

The panel width is 1.5 m (for $D \leqslant 18$ m) and 2 m (for $D > 18$ m). At the joints, the panels are welded together via embedded parts and the joints are filled with an M-300 mixture of cement grout and sand applied under pressure. Shell-type panels are rigidly connected to one another, and flat-convex panels have movable joints using asphalt or Thiokol sealing compound.

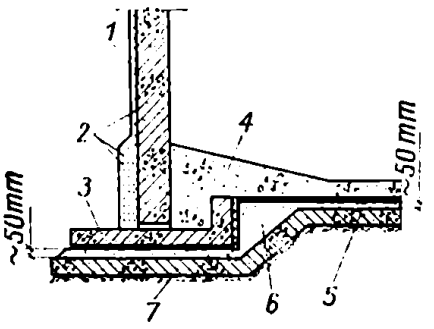



Fig. XI.16. Floor compressed by the prestressed reinforcement of a wall

*1*—wall panel; *2*—prestressed hoop reinforcement; *3*—prefab floor member; *4*—in-situ floor; *5*—asphalt sheathing paper; *6*—sand; *7*—concrete layer

## XI.3. Design of Circular Tanks

The liquid contained in a tank exerts a hydrostatic pressure on the tank walls, which increases linearly with depth (Fig. XI.17*a*).

The characteristic pressure at the depth $h$ below the liquid level is

$$p^{ch} = \gamma h \qquad \text{(XI.1)}$$

where $\gamma$ is the unit weight of the liquid.

The overload factor for the hydrostatic pressure is $n = 1.1$, so the design pressure is

$$p = 1.1\gamma h$$

For water, $\gamma = 1$, so

$$p = 1.1h \qquad \text{(XI.2)}$$

The internal uniformly distributed pressure causes the wall to work in axial tension. The circular tensile force $T_0$ acting in the wall can be determined considering a half-ring of radius $r$ in the state of equilibrium (Fig. XI.17*b*). The resultant of the hydrostatic pressure $p \times 2r$ counterbalances the force in the wall $2T_0$, whence

$$T_0 = pr = 1.1\,hr \qquad \text{(XI.3)}$$

Thus, for a wall separated from the floor, the circular tensile force $T_0$ increases linearly with increasing hydrostatic pressure. The radial deflection of the wall rises with rising circular force (Fig. XI.18*a*).

When the wall is rigidly connected to the floor, the radial deflections in the area adjoining the floor are restrained and are practically zero at floor level because of negligibly small in-plane deformability of the floor. As a result, the wall bends (Fig. XI.18*b*) and gives rise to a bending moment $M$ acting along the generatrix (in the plane of the meridional cross section), and to a shearing force $Q$. The moment $M$ and the force $Q$ are a maximum at floor level and rapidly decrease towards the top.

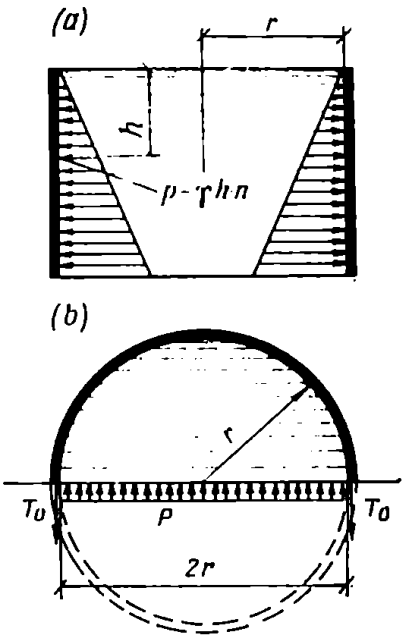



Fig. XI.17. To determining circular tensile forces in a cylindrical wall

The circular tensile force diagram is similar to the hydrostatic pressure diagram, that is, triangular in shape, starting from zero at the top and reaching a maximum at the floor. If the joint between the wall and floor is rigid, the circular force diagram is a curve having zero points at the top and bottom (Fig. XI.18*b*).

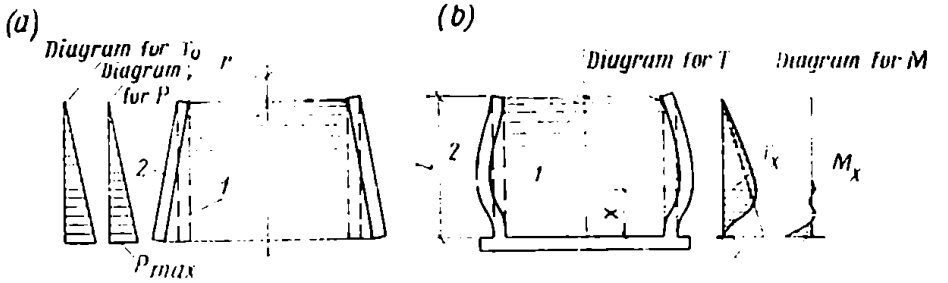



Fig. XI.18. Deflections and forces in the walls of open-top circular tanks, induced by internal hydrostatic pressure
*1*—initial state; *2*—deformed state

The wall is an axisymmetric thin shell. A strip of width $b = 1$ cut from the shell may be regarded as a beam fixed at the bottom and supported along the entire length by elastic forces which are the radial components of the circular forces $T$ (Fig. XI.19). The sag of this beam is proportional to the elastic reaction, so for design purposes, it may be regarded as a beam on an elastic foundation characterized by a modulus of subgrade reaction.

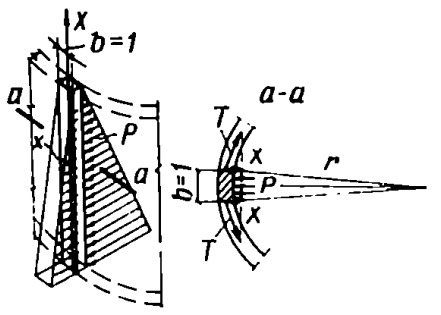



Fig. XI.19. To the design of a cylindrical wall

Design expressions are obtained by solving the differential equation describing the bending of the beam on an elastic foundation subject to appropriate boundary conditions. At any section within the distance $x$ of the floor, the circular tensile force $T$ is given by

$$T = T_0 - p_{max} r [\eta_1 + \eta_2 (1 - s/l)] \qquad (XI.4)$$

where $T_0$ is the circular force at the section in question deduced from Eq. (XI.3), that is, as for a wall separated from the floor; $r$ is the tank radius; $l$ is the wall height; $p_{max}$ is the hydrostatic pressure at the wall bottom; $s$ is the characteristic of wall rigidity:

$$s = 0.76 \sqrt{r\delta} \qquad (XI.5)$$

$\delta$ is the wall thickness; and $\eta_1$ and $\eta_2$ are the coefficients for the design of beams on an elastic foundation:

$$\eta_1 = \exp(-\varphi) \cos \varphi, \qquad \eta_2 = \exp(-\varphi) \sin \varphi$$

looked up in Table XI.1 depending on the factor $\varphi = x/s$ (where $x$ is the distance from the wall bottom to the section in question).

To a first approximation, the wall thickness may be taken as

$$\delta \approx 0.5 rl \geqslant 12 \text{ to } 15 \text{ cm} \qquad (XI.6)$$

Here, if $r$ and $l$ are in metres, $\delta$ is in centimetres.

The bending moment in the wall is a maximum at floor level

$$M = 0.5\, p_{max} s^2 (1 - s/l) \qquad \text{(XI.7)}$$

When the joint between the floor and wall is movable (the circular slot is filled with asphalt putty), the wall is pre-

**Table XI.1. Coefficients $\eta_1$ and $\eta_2$**

| $\varphi$ | 0 | 0.1 | 0.3 | 0.5 | 0.8 | 1 | 1.2 | 1.4 |
|---|---|---|---|---|---|---|---|---|
| $\eta_1$ | 1 | 0.900 4 | 0.707 8 | 0.532 3 | 0.313 | 0.198 8 | 0.109 2 | 0.041 9 |
| $\eta_2$ | 0 | 0.090 3 | 0.218 9 | 0.290 8 | 0.322 3 | 0.309 6 | 0.280 7 | 0.243 |

| $\varphi$ | 1.6 | 1.8 | 2 | 2.2 | 2.7 | 3 |
|---|---|---|---|---|---|---|
| $\eta_1$ | −0.005 9 | −0.037 6 | −0.056 4 | −0.065 2 | −0.060 8 | −0.049 3 |
| $\eta_2$ | 0.201 8 | 0.161 | 0.123 1 | 0.089 6 | 0.028 7 | 0.007 |

| $\varphi$ | 3.5 | 4 | 4.5 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| $\eta_1$ | −0.028 3 | −0.012 | −0.002 4 | 0.002 | 0.002 4 | 0.000 7 |
| $\eta_2$ | 0.010 6 | 0.013 9 | 0.010 9 | −0.006 5 | −0.000 7 | −0.000 6 |

vented from radial deflection by the frictional force $Q_{fr}$ acting at the bottom of the wall

$$Q_{fr} = Nf \qquad \text{(XI.8)}$$

where $N$ is the normal force in kgf $m^{-1}$ (N $m^{-1}$), and $f$ is the coefficient of friction taken as 0.5.

In open-top tanks, the frictional force owes its existence solely to the weight of the wall, whereas in covered structu-

res, it is also contributed by the weight of the roof with the backfill over the top (the load is taken from an appropriate load strip). The frictional force is calculated for two design cases: (1) without backfill (a hydrostatic test of the tank), and (2) with backfill (service conditions).

The circular tensile forces in the wall are given by

$$T = T_0 - 2rQ_{fr}\eta_1/s \qquad \text{(XI.9)}$$

If Eq. (XI.8) yields

$$Q_{fr} > p_{max}\, s/2$$

then the frictional force in Eq. (XI.9) should be taken as*

$$Q_{fr} = p_{max}\, s/2 \qquad \text{(XI.10)}$$

The maximum moment induced by $Q_{fr}$ is determined as

$$M = Q_{fr}s\eta_2 \qquad \text{(XI.11)}$$

It is applied at a distance

$$x = 0.6\sqrt{r\delta} \qquad \text{(XI.12)}$$

from the floor.

In low-capacity tanks (up to 100 m³), the circular tensile forces in the wall may be determined without taking into consideration the connection between the wall and floor [see Eq. (XI.3)].

The design expressions for circular tensile forces and bending moments induced in a circular wall by the internal hydrostatic pressure describe the state of stress in service for tanks installed on the ground surface, and during a hydrostatic test (in the absence of backfill) for buried tanks.

The backfill gives rise to circular compressive forces in a tank wall. As a rule, they are not taken into account in design calculations because they act against the circular tensile forces in a tank filled with liquid, and are taken up by the concrete wall in an empty tank.

To allow for the effect of the backfill (in an evacuated tank), we determine the respective bending moment in

* This is because for $Q_{fr} = p_{max}s/2$ at the floor level, Eq. (XI.9) yields $T = 0$. If $Q_{fr}$ were greater than $p_{max}s/2$, the force $T$ would change the sign (compression would take place instead of tension), which is unfeasible physically.

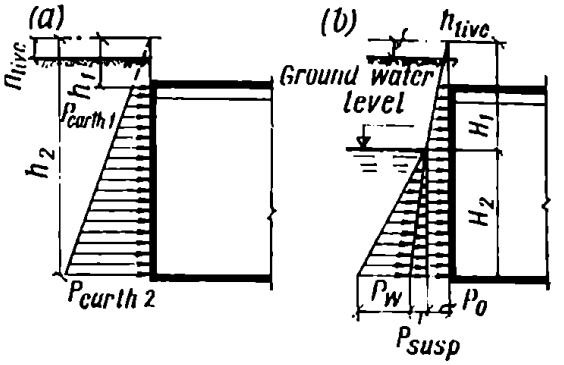



Fig. XI.20. Earth pressure exerted on a buried structure

a meridional plane and choose the cross-sectional area of vertical reinforcement necessary to resist this moment. Like the water pressure, the lateral earth pressure increases with increasing depth linearly. The design pressure at the depth $h$ below the ground surface is

$$p_{earth} = \gamma_{earth} hkn \qquad \text{(XI.13)}$$

where $\gamma_{earth}$ is the bulk unit weight of the soil, $n$ is the overload factor ($n=1.2$ for backfill), and $k$ is the coefficient accounting for soil cohesion

$$k = tg^2 (45° - \varphi/2) \qquad \text{(XI.14)}$$

($\varphi$ is the angle of internal friction).

The live load at the ground surface $p_{live}$ ($p_{live}$= 1 to 2.5 tf $m^{-2}$ or 10 to 25 kN $m^{-2}$) is replaced by an equivalent soil layer of depth $h_{live} = p_{live}/\gamma_{earth}$. If, with allowance for $h_{live}$, we designate the distance from the ground surface to the top and bottom of the wall as $h_1$ and $h_2$, respectively (Fig. XI.20*a*), then the lateral earth pressure at the top of the wall will be given by

$$p_{earth1} = \gamma_{earth} h_1 kn$$

and at the bottom of the wall by

$$p_{earth2} = \gamma_{earth} h_2 kn$$

If the wall is rigidly connected to the floor, the maximum bending moment induced by this load is

$$M_{earth} = 0.5 p_{earth2} s^2 [1 - (1 - p_{earth1}/p_{earth2})\, s/l] \qquad \text{(XI.15)}$$

If the bottom of a tank lies below the ground water level (Fig. XI.20*b*), the lateral earth pressure is the sum of the pressure $p_0$ exerted by the soil layer of depth $H_1$ (from the ground surface to the ground water level), the pressure $p_{susp}$ due to the soil particles suspended in water (within the depth $H_2$), and the water pressure $p_w$:

$$p_0 = \gamma_{earth} H_1 nk \qquad \text{(XI.16)}$$

$$p_{susp} = \gamma_{susp} H_2 nk \qquad \text{(XI.17)}$$

where $\gamma_{susp}$ is the bulk unit weight of soil suspended in water

$$\gamma_{susp} = (\gamma'_{earth} - \gamma_w)(1 - m)$$

Here, $\gamma'_{earth}$ is the bulk unit weight of the soil particles, $\gamma_w = 1$ is the unit weight of water, and $m$ is the soil porosity in fractions of unity;

$$p_w = \gamma_w n H_2 = 1.1 H_2 \text{ tf m}^{-2} \ (11 H_2 \text{ kN m}^{-2}) \qquad \text{(XI.18)}$$

$$p_{earth2} = p_0 + p_{susp} + p_w \qquad \text{(XI.19)}$$

The values of $\gamma'_{earth}$, $m$, and $\varphi$ are determined during the geological survey of the site. For sands, we may approximately adopt $\gamma_{earth} = 1.8$ t m$^{-3}$, $\gamma_{susp} = 1.1$ t m$^{-3}$ (at $\gamma'_{earth} = 2.7$ t m$^{-3}$ and $m = 0.35$), and $\varphi = 40°$.

The resultant of the ground water upthrust applied to the bottom of area $F_b$ is

$$P^{ch} = \gamma_w H_2 F_b = H_2 F_b \qquad \text{(XI.20)}$$

The weight of the evacuated tank $G^{ch}$ (including the backfill over its top) multiplied by $n = 0.9$ should exceed the resultant of the upthrust taken with the overload factor $n = 1.1$ applied:

$$0.9 G^{ch} \geqslant 1.1 P^{ch} \qquad \text{(XI.21)}$$

As previously noted in Sec. XI.1, this condition cannot be met in some open-top structures. If so, the tank should always be kept filled during the high ground water level period.

The circular tensile forces acting in the walls of circular tanks are determined at every metre (or a half-metre) of the height. For each zone 1 m high, the necessary cross-sec-

tional area of the hoop reinforcement is found from the maximum design circular force (taken with a margin of safety):

$$F_s = T/R_s \qquad \text{(XI.22)}$$

In non-prestressed in-situ walls, the hoop reinforcing steel is placed on either side, keeping the concrete cover at a required value.

In prestressed walls, the hoop reinforcement is tensioned and placed on the outside.

The incipient-cracking and crack-opening analysis of a wall is carried out for the most heavily loaded wall tier as prescribed in Chap. V. Here, the tensile force $T$ should be regarded as a long-time load.

Prestressed walls* should meet the cracking-resistance requirements of the first category. The incipient-cracking analysis should be done for the design circular force $T$, subject to the condition $T \leqslant N_{cr}$ (where $N_{cr}$ is the force at which the wall develops cracks). In in-situ walls, $N_{cr}$ is found from Eq. (V.27), and in precast walls, $N_{cr}$ should be determined, disregarding the contribution from the non-prestressed hoop reinforcement (that is, the horizontal reinforcement of the wall panels) because it is interrupted at joints, and assuming that the grout in the joints between the wall panels does not carry any load. Therefore, the design expression may be written as follows

$$T \leqslant N_0 \text{ or } T \leqslant m_{ac}\sigma_0 F_{pr} \qquad \text{(XI.23)}$$

where $N_0$ is the prestressing force, $\sigma_0$ is the prestress in the hoop reinforcement with allowance for all losses, $F_{pr}$ is the cross-sectional area of the prestressed steel, and $m_{ac}$ is the tensioning accuracy factor ($m_{ac} = 0.9$).

At a low percentage of reinforcement, $\mu = F_{pr}/F_c < 0.25\%$, the adopted values of $F_{pr}$ and $\sigma_0$ should additionally be checked to see if the wall of the filled tank (without backfill) can withstand a compressive stress of at least 8 kgf cm$^{-2}$ (0.8 MPa).

---

* Assuming that prestressed hoop reinforcement of class A-IV and higher is used.

Dividing either side of Eq. (XI.23) by $F_c = 100\delta$ cm$^2$ gives the design formula

$$\sigma_c = m_{ac}\sigma_{0\mu} \geqslant 8 \text{ kgf cm}^{-2} \text{ (0.8 MPa)}$$

Non-prestressed walls reinforced with class A-I, A-II, and A-III hoop reinforcement should meet the cracking-resistance requirements of the third category.

The width of a crack due to the circular force brought about by the characteristic load $T^{ch} = T/1.1$ should not exceed 0.1 mm ($a_{cr,\ l\text{-}t} \leqslant 0.1$ mm).

How to determine $a_{cr}$ for members in axial tension has been discussed in Chap. V.

The vertical reinforcement of a wall is chosen to carry the maximum bending moment. The inside reinforcement resists tensile forces due to the moment $M$ brought about by the internal liquid pressure (in the absence of backfill), and the outside reinforcement takes up tensile forces induced by the moment $M_{earth}$ exerted by the backfill (in an evacuated tank).

The necessary steel area is found as for a rectangular sec-

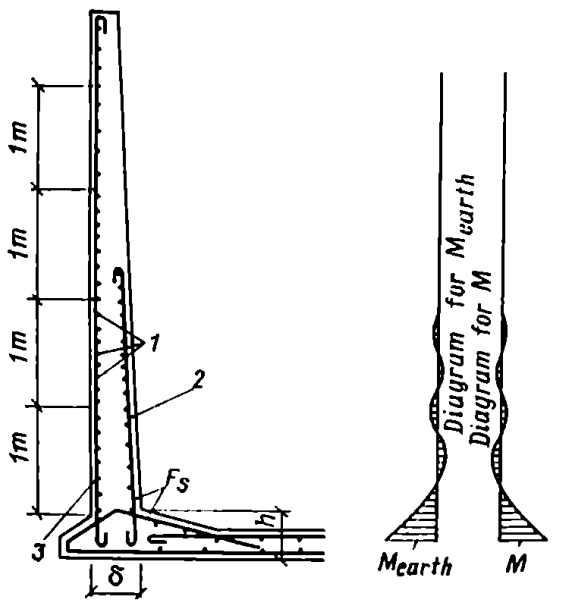



Fig. XI.21. Distribution of reinforcement in an in situ circular tank
*1*—hoop reinforcement designed to carry the force $T$; *2*—vertical reinforcement designed to resist the moment $M$ induced by water pressure; *3*—vertical reinforcement designed to carry the moment $M_{earth}$ induced by lateral earth pressure

tion of width $b = 100$ cm. The distribution of reinforcement in an in-situ wall is shown in Fig. XI.21.

In precast wall panels, the vertical reinforcing steel should additionally be checked for capacity to carry bending moments likely to arise in transit and erection. If the vertical reinforcement has a cross-sectional area sufficient to carry erection loads, but fails to resist bending moments in service, the bottom of the panel should be reinforced with an additional piece of fabric (see Fig. XI.10*b*).

High panels are additionally reinforced with longitudinal prestressed bars to improve their cracking resistance in transit and erection.

The floor of a circular tank is a flat or conical slab resting on an elastic foundation and connected to the tank wall around its circumference either rigidly or by means of hinges.

In the absence of ground water, the self-weight of the floor and the weight of the liquid are balanced out by the contact soil pressure. Bending moments arise in the floor where it joins the cylindrical wall or the column footings.

The expressions for the forces in a circular wall have been derived assuming that the wall is completely fixed at the floor (or has a movable support). In real structures, however, the wall, the support ring built where the wall joins the floor, and the floor deform together, so that the forces in all these elements should be found considering the assembly as a whole (Fig. XI.22). Similarly, the behaviour of

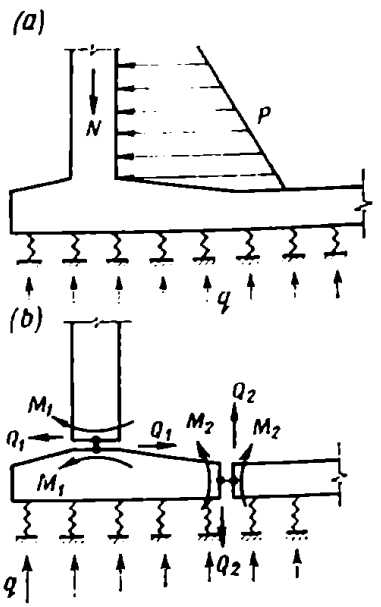



Fig. XI.22. Design system for the joint between the wall, floor, and supporting ring

(*a*) actual system; (*b*) basic design system; *p*—water pressure; *N*—force exerted on the joint by the wall; *q*—contact soil pressure; $M_1$—bending moment in the wall at the joint face; $Q_1$—shearing force in the wall at the joint face; $M_2$—bending moment in the floor at the joint face; $Q_2$—shearing force in the floor at the joint face

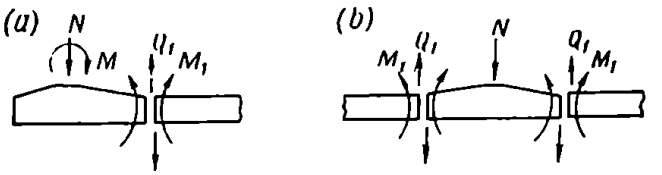



Fig. XI.23. To the approximate design of the floor

the floor is inseparable from that of the thickened areas beneath the columns. Since accurate calculations are tedious, approximate methods serve practical purposes as well. For example, at the joint between the wall and the floor, only the support ring and the floor are assumed to work together in resisting the forces $N$ and $M$ transferred to the ring by the wall (Fig. XI.23*a*). Here, the floor is regarded as a slender beam of unit width cut by two sections parallel to the diameter, and resting on an elastic foundation. A similar design system is adopted for the joints between the floor and the column footings (Fig. XI.23*b*). The portion cut from the support ring (or the thick floor section beneath the column footing) is usually regarded as a rigid beam. A slender beam on an elastic foundation is that in which $l/s \geqslant 3$, and a rigid beam is that in which $l_1/s_1 \leqslant 0.75$, where $l$ ($l_1$) is the length of the beam and $s$ ($s_1$) is the rigidity characteristic* given by

$$s = \sqrt[4]{4EJ/bc}$$

Here, $E$ is the modulus of elasticity of the beam material, $J$ is the moment of inertia of the section, $b$ is the section width (in our case, $b = 1$), and $c$ is the modulus of subgrade reaction taken as 1 to 3 kgf cm$^{-3}$ for weak soils and as 3 to 8 kgf cm$^{-3}$ for medium-density soils.

At the joint between two portions of the floor, the forces $M_1$ and $Q_1$ are determined by equating their mutual displacements to zero, which can be expressed by the following canonical equations:

$$a_{11}M_1 + a_{12}Q_1 + a_{1p} = 0$$
$$a_{21}M_1 + a_{22}Q_1 + a_{2p} = 0$$

* The characteristic of rigidity for a circular wall given by Eq. (XI.5) is derived from the expression $s = \sqrt[4]{4EJ/bc}$ for $c = E\delta/r^2$ and $J = b\delta^3/12$.

The rotation and sag of a slender beam (multiplied by $EJ$) due to the moment $M_1 = 1$ applied at the end are respectively as follows

$$a_{11}^{sl} = s$$
$$a_{21}^{sl} = -s^2/2$$

The rotation and sag due to the force $Q_1 = 1$ applied at the end are respectively as follows

$$a_{12}^{sl} = a_{21}^{sl}$$
$$a_{22}^{sl} = s^3/2$$

For a rigid beam, we have respectively

$$a_{11}^{r} = 3s_1^4/l_{11}^3, \quad a_{12}^{r} = a_{21}^{r} = -1.5s_1^4/l_1^2, \quad a_{22}^{r} = s_1^4/l_1$$

The total displacements are

$$a_{11} = a_{11}^{sl} + a_{11}^{r}, \quad a_{12} = a_{21} = a_{12}^{sl} + a_{12}^{r},$$
$$a_{22} = a_{22}^{sl} + a_{22}^{r}$$

The load terms due to the moment $M$ and the force $N$ applied to the support ring (Fig. IX.23*a*) are given by

$$a_{1p} = -M \cdot 3s_1^3/l_1^2, \quad a_{2p} = -M \cdot 1.5s_1^4/l_1^2 - Ns_1^4/4l_1$$

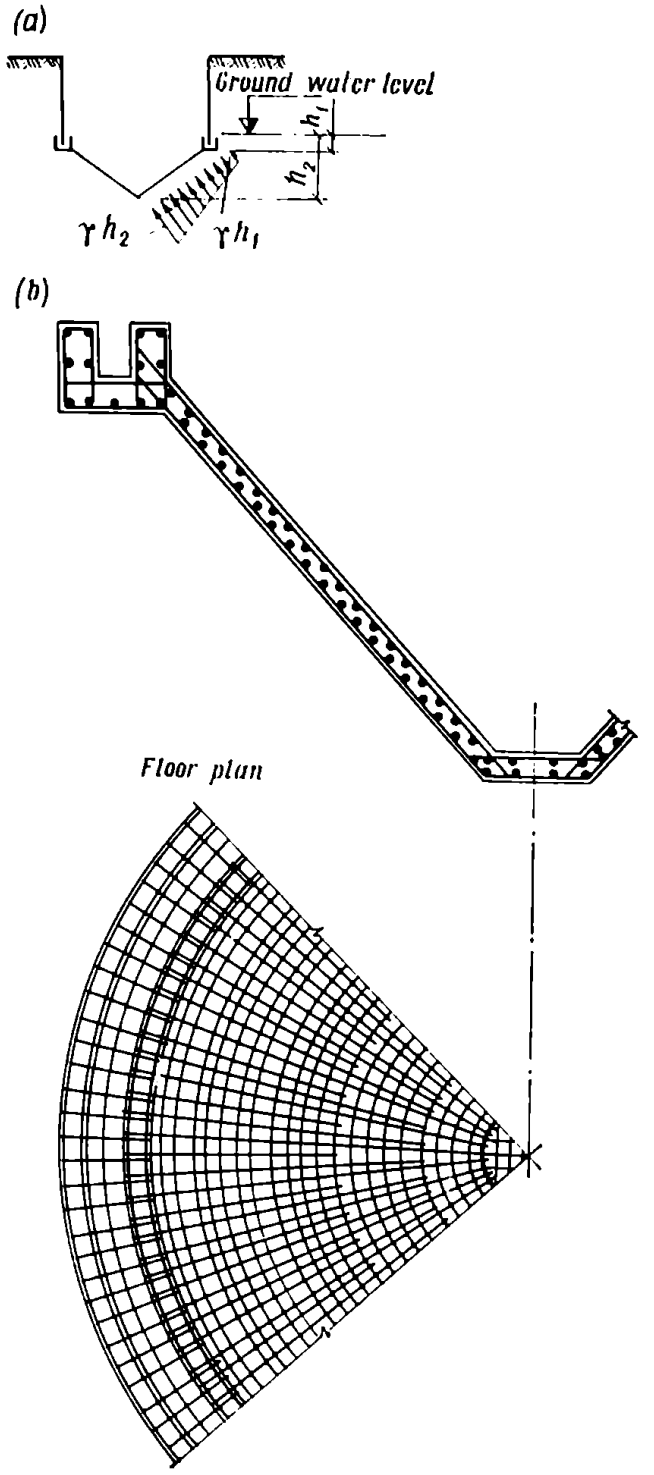
Owing to the symmetry of the joint between the floor and the thick column footing, the rotation is $a_{1p} = 0$ and the sag induced by the force $N - 2Q_1$ is

$$a_{2p} = (N - 2Q_1)\, EJ/l_1bc$$

In the floor, reinforcing steel is placed both at the top and at the bottom. Where the floor adjoins the bank walls (or the support rings) and beneath the columns, the amount of reinforcement is found by analysis for the respective bending moment. Elsewhere, the top and bottom steel consists of bars 8 to 10 mm in diameter spaced 15 to 20 cm apart (in either direction), chosen from structural considerations.

In the presence of ground water, a flat floor is subjected to a uniformly distributed upward-directed design pressure due to the water upthrust

$$p = 1.1\ H_2 \text{ tf m}^{-2} \text{ (kN m}^{-2}\text{)}$$




24. Conical floor

As a first approximation, the bending moments induced by this pressure in the tank floor may be determined as in the slab of a flat-slab floor put upside down and resting on the internal columns whose capitals are the footings.

In a conical floor, the water upthrust, which increases linearly on moving away from the support ring towards the apex (Fig. XI.24*a*), gives rise mostly to compressive forces*. The radial and ring reinforcement of the floor is placed on either side (Fig. XI.24*b*). As a rule, it is made of class A-II bars 10 mm in diameter and spaced about 15 cm apart.

The roof members (slabs and girders) of covered tanks are designed as prescribed in Chap. X. The internal columns are designed to resist compression as discussed in Chap. IV.

Worked examples for the members of circular tanks will be found in Chap. XII.

## XI.4. Construction Types of Reinforced Concrete Rectangular Tanks

Rectangular tanks include water tanks, various settlers, clarification tanks, aeration tanks, mixers, oil and sand traps, filters, silt compactors, etc. Most of them are open-top structures, except water tanks and some settlers and filters which are preferably built covered. Prefabricated reinforced concrete members in rectangular tanks are used in walls, columns, roofs, and launders; the floor is usually cast in-situ.

**Water Tanks.** These have a flat floor and a roof supported on internal columns. The column grid size usually ranges between 4 and 6 m square. If their height does not exceed 4 m, the walls of in-situ rectangular tanks are made smooth and rigidly connected to the roof and floor (Fig. XI.25*a*). For this purpose, the wall panels have arch braces in the corners. If a tank is higher than 4 m, its walls have ribs facing outwards (Fig. XI.25*b*). To reduce the wall height, the floor may have sloping portions as shown in Fig. XI.25*c*. In-situ roofs are usually of the flat-slab construction.

---

* Meridional and circular forces are found as in a conical shell of revolution.

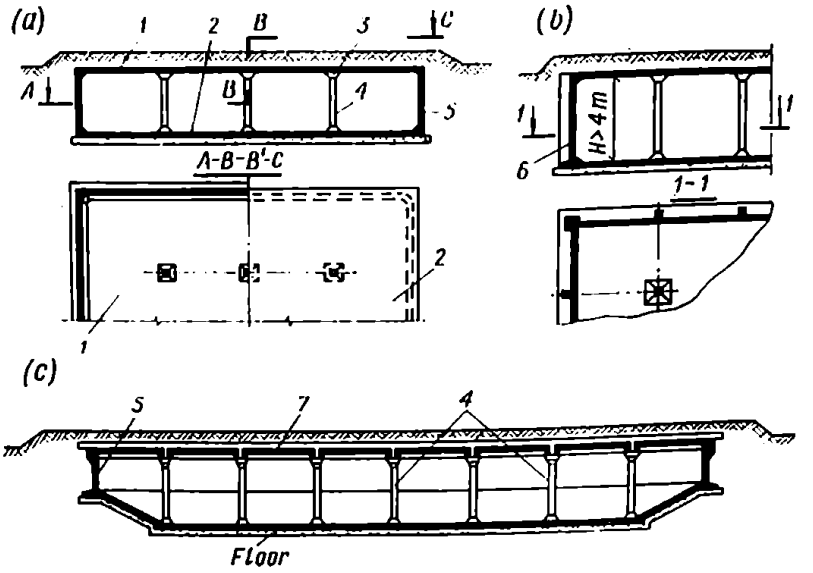



Fig. XI.25. Construction types of rectangular tanks
*1*—roof flat slab; *2*—floor flat slab; *3*—capital; *4*—column; *5*—wall; *6*—ribbed wall; *7*—prefab roof slabs

In the USSR, rectangular water tanks (Fig. XI.26) are standardized as regards capacity and size:

| Capacity, m³ | 50 | 100 | 250 | 500 | 1 000 |
|---|---|---|---|---|---|
| Plan dimensions, m | 3×6 | 6×6 | 6×12 | 12×12 | 12×18 |
| Capacity, m³ | 2 000 | 3 000 | 6 000 | 10 000 | 20 000 |
| Plan dimensions, m | 18×24 | 24×30 | 36×36 | 48×48 | 66×66 |

As is seen from the table, the plan dimensions of rectangular water tanks are multiples of a 6-m module. The standard height is 4.8 m, or 3.6 m if the tank capacity is not larger than 500 m³.

In-situ floors are usually made flat, at least 12 cm thick and have thicker areas where columns and wall panels are

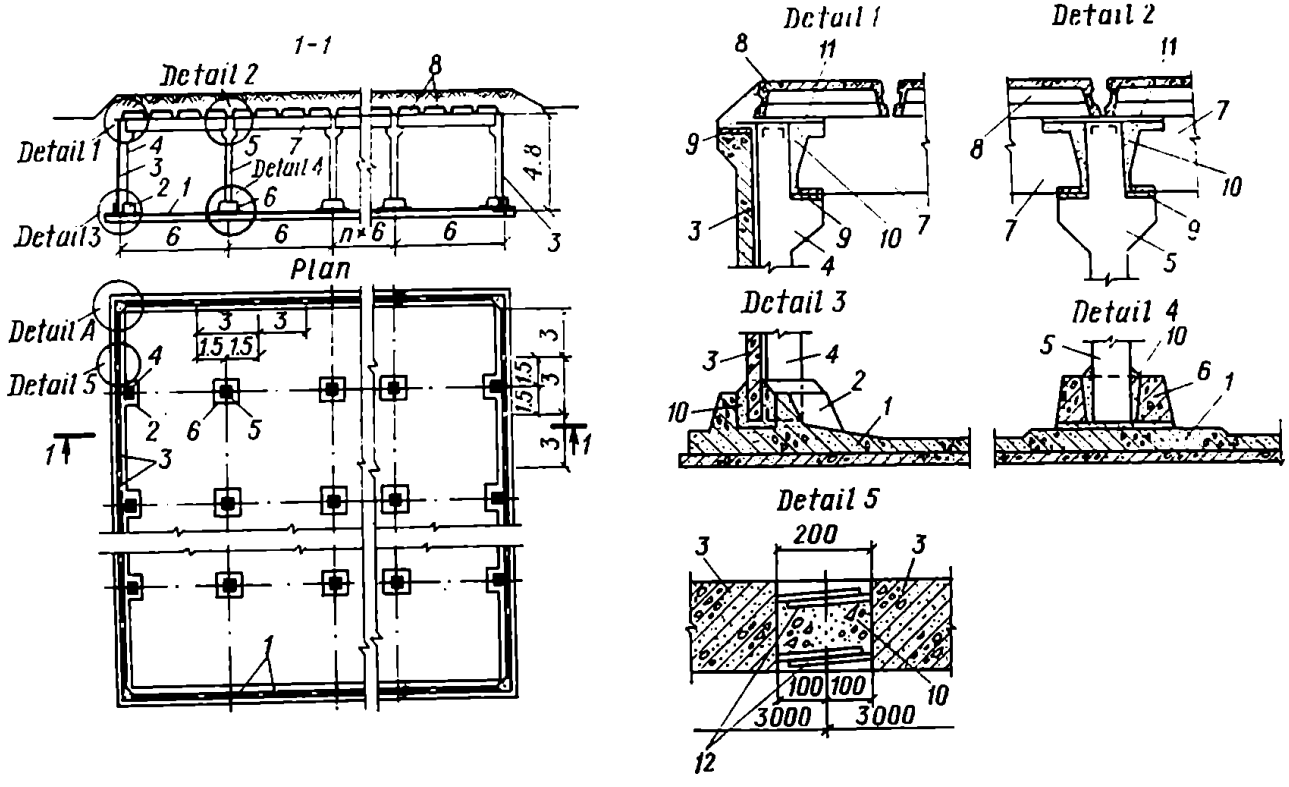



**Fig. XI.26. Precast rectangular tanks**

*1*—in-situ floor; *2*—outer column footing (boss in the floor); *3*—wall panels; *4*—outer column; *5*—intermediate column; *6*—footing block; *7*—girder; *8*—roof panels; *9*—embedded parts; *10*—in-situ concrete; *11*—to be welded in a pool of molten metal; *12*—reinforcement stick-outs to be welded during erection; Detail *A* is shown in Fig. XI.35

installed. The wall panels are 4.8 m (3.6 m) high and have a nominal width of 3 m.

Precast square-section columns are installed in footing sockets.

The roofs of rectangular tanks are assembled from prefabricated type-design girders and ribbed slabs 1.5 × 6 m in plan, ordinarily intended for multistorey industrial buildings (see Fig. XI.26). To keep the tank warm, the roof is usually covered with a layer of soil. Sometimes, use is made of more effective warmth-keeping jackets, such as foam concrete and foamglass, which markedly reduce the load imposed on the roof elements.

Since the temperature in water tanks equipped with warmth-keeping jackets is held constant, they usually have no expansion or contraction joints.

**Horizontal Settling Tanks.** These are long multi-section tanks rectangular in plan in which the liquid flows slowly so that the suspended solid particles have time to settle.

Horizontal settlers used in water supply systems are generally of the covered construction with a thermally insulated roof, whereas those employed in sewage disposal systems are open-top.

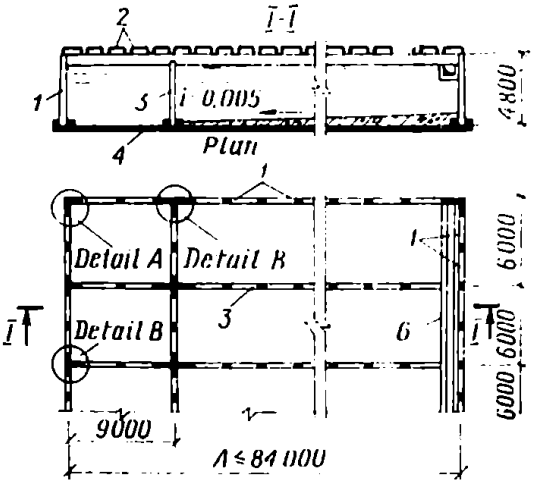



Fig. XI.27. Covered horizontal settler assembled from standard members

*1*—wall panels; *2*—roof panels; *3*—partition panels; *4*—in-situ floor; *5*—concrete layer; *6*—launder; Details *A* and *B* are shown in Fig. XI.35

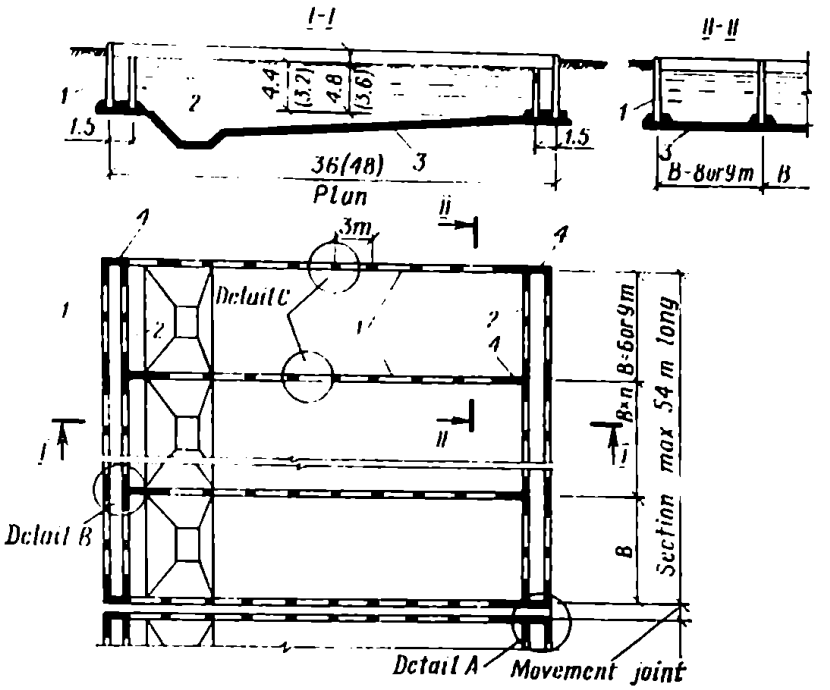



Fig. XI.28. Open-top horizontal settler assembled from standard members

*1*—wall panels; *2*—partition panels; *3*—in-situ floor; *4*—in-situ wall portions; Detail *C* is shown in Fig. XI.35

The height of covered horizontal settlers assembled from standardized prefab members (Fig. XI.27) is 4.8 m, the width of a cell is 6 m, and the length ranges between 36 and 75 m. The floor is cast in-situ, the walls are assembled from precast panels with a nominal width of 3 m. The roof slabs are installed directly on the longitudinal walls.

The height of open-top settlers assembled from standardized prefabricated members is either 3.6 or 4.8 m. They consist of cells 6 or 9 m wide and 36 or 48 m long (Fig. XI.28). Each section of a settler may be composed of several (from two to nine) individual cells, but the width of the section should not exceed 54 m, because otherwise movement joints would have to be made. At the top, the wall panels may be connected by spreaders installed at every 6 m.

**Oil Traps.** These are intended to catch petroleum products from sewage at oil refineries, oil fields, and oil tank farms.

Oil traps are multi-section structures rectangular in plan. Physically, they are similar to horizontal settling tanks.

Oil traps assembled from standardized precast members have a standard height of 2.4 m, a cell width of 2.3 or 6 m, and a length of 12 to 36 m (a multiple of 6 m). The walls are built of prefab panels and the floor is cast in-situ.

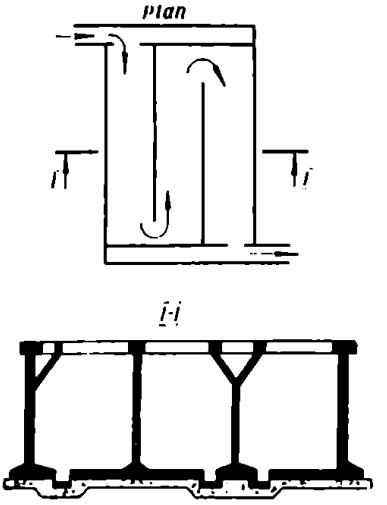



Fig. XI.29. In-situ aeration tank

**Aeration Tanks.** These are multi-section rectangular tanks intended for the biological purification of waste water (Fig. XI.29). Each section is divided into corridors by longitudinal partitions which stop short of the face walls on one side. Most commonly, use is made of three-corridor sections.

Aeration tanks are assembled from standardized sections to form two, three, or four corridors. The corridors are 4.5, 6, and 9 m wide, 4.8 and 6 m high, and 42 to 132 m long (the length is taken as a multiple of 6 m). The floor is cast in-situ, whereas the walls and partitions are made up of prefabricated panels 3 m wide (Fig. XI.30).

Because aeration tanks are long, they have movement joints installed at every 54 m, which accommodate steel expansion compensators or rubber keys (Fig. XI.31). To reduce friction arising when the strain blocks move with variations in temperature, a layer of sand is provided beneath the floor between the asphalt and concrete layers.

Other types of rectangular tanks (such as clarification tanks, oil separators, mixers, contact tanks, sand traps, biological filters, high-rate trickling filters, and the like) are of similar construction. They only differ in size, floor shape, and provision of launders or chutes.

Depending on the type of horizontal load they are to carry (water or lateral earth pressure), standardized wall panels for rectangular tanks can be divided into cantilever-

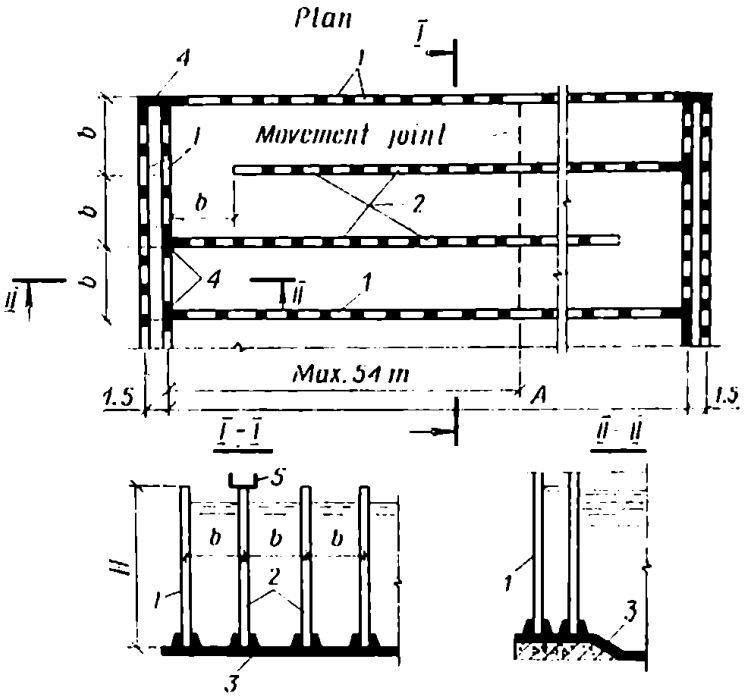



Fig. XI.30. Aeration tank assembled from standard prefab members
*1*—wall panels; *2*—partition panels; *3*—in-situ floor; *4*—in-situ wall portions; *5*—launder

type panels clamped in the floor (Fig. XI.32) and beam-type panels having a hinged support at the top in addition to the clamped bottom (Fig. XI.33). At the bottom, beam-type panels are embedded into a slot in the floor for at least $1.5\delta$ and cantilever-type panels for at least $2\delta$ (where $\delta$ is the panel thickness) (see Fig. XI.26, Detail *3*).

Beam-type panels are assumed to be supported at the top because the entire tank structure behaves as a closed system (Fig. XI.34*a*) where the roof joins together the walls and columns in covered structures, or reinforced concrete spreaders join together the walls and partitions in open-top structures (Fig. XI.34*b*). Beam-type panels have expanded top ends 300 × 200 mm in size, which, after the panels have been joined, work as a framing beam subjected to bending in a horizontal plane between the spreaders (or roof girders).

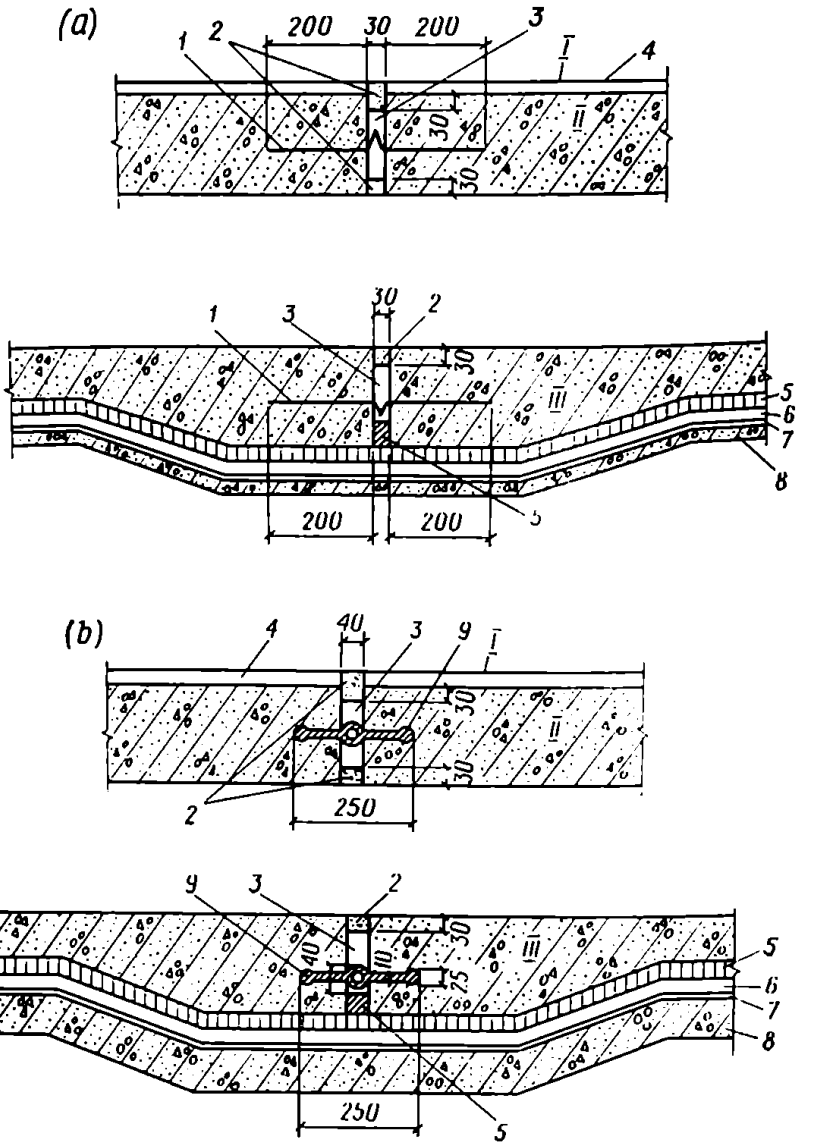



Fig. XI.31. (*a*) Movement jints with stee lexpansion compensators and (*b*) movemevt jints with rubber keus

*I*—inner face; *II*—wall; *III*—floor; *1*—expansion compensator; *2*—asbestos-cement grout; *3*—asbestos strand; *4*—shoterete plaster; *5*—asphalt; *6*—sand; *7*—prepared roofing paper; *8*—concrete layer; *9*—three-cam rubber key

Apart from wall panels, the standardized members for rectangular tanks include partition panels which do not have to resist water pressure because in service the water is on either side of the partition.

The width of cantilever-type wall panels and partition panels is 2.98 m. Their vertical side faces have a triangular groove, so that a rhombic key is formed at the joints after grouting. In addition, cantilever-type wall panels are joined by welding together their reinforcement stick-outs at the top and at mid-height, after which the joints are concreted.

The height of cantilever-type wall panels, $H$, ranges between 1.2 and 5.4 m in 0.6-m steps. At $H = 1.2$, 1.8, and 2.4 m, the panels have a constant thickness $\delta$ equal to 100, 130, and 150 mm, respectively. If $H \geqslant 3$ m, the panels have a varying thickness which is 140 mm at the top, whereas at the bottom it is 160 mm when $H = 3$ m, 200 mm when $H = 3.6$ m, 240 mm when $H = 4.2$ m, 280 mm when $H = 4.8$ m, and 340 mm when $H = 5.4$ m.

Partition panels have $H = 3.6$, 4.2, and 4.8 m and a constant thickness $\delta = 120$, 140, and 160 mm, respectively.

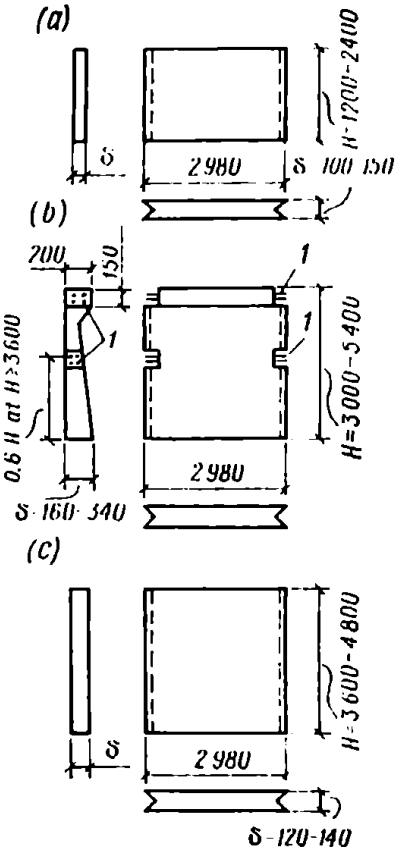



Fig. XI.32. Cantilever-type wall and partition panels for rectangular tanks

(*a*) wall panels, $H \leqslant 2.4$ m; (*b*) wall panels, $H > 2.4$ m; (*c*) partition panels; *1*—reinforcement exposed ends

The vertical joints in beam-type wall panels are made by welding together their horizontal reinforcement stick-outs, and the gap between the panels is concreted (see Fig. XI.26, Detail *5*). For ease of concreting, the gap is made 200 mm

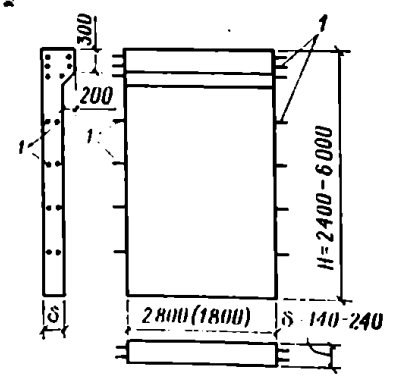



Fig. XI.33. Beam-type wall panels for rectangular tanks
*1*—reinforcement exposed ends

wide, so the panel width is 3 — 0.2 = 2.8 m. The necessary length in low-capacity tanks is obtained with make-up beam-type panels 1.8 m wide (2-m nominal width). The panels come in heights from 2.4 to 6 m in 0.6-m steps and in thicknesses from 140 mm (for $H$ = 2.4 m) to 240 mm (for $H$ = 6 m) in steps of 20 mm.

To facilitate their joining at the corners, wall panels have in-situ portions up to 1.5 m long (Fig. XI.35).

Wall panels are subjected to bending in only one vertical

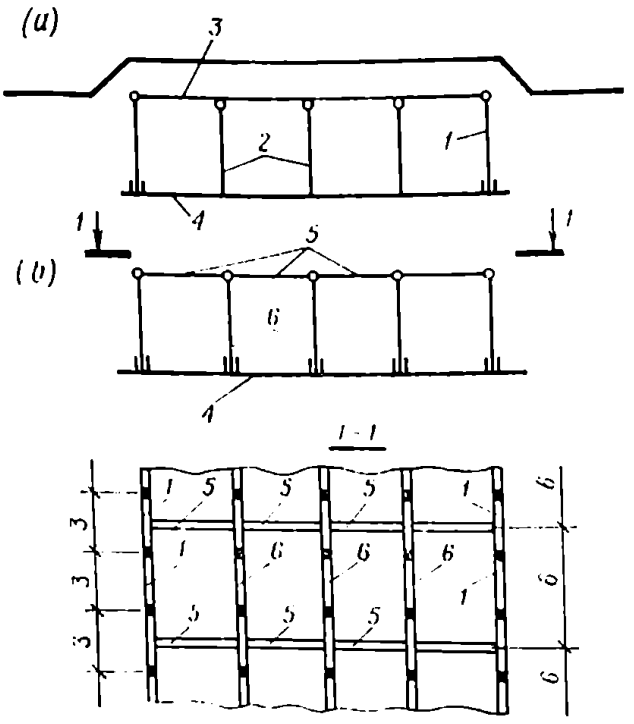



Fig. XI.34. Tanks using beam-type wall panels
(*a*) covered tank; (*b*) open-top tank; *1*—wall panels; *2*—columns; *3*—roof; *4*—floor; *5*—spreaders; *6*—partition panels

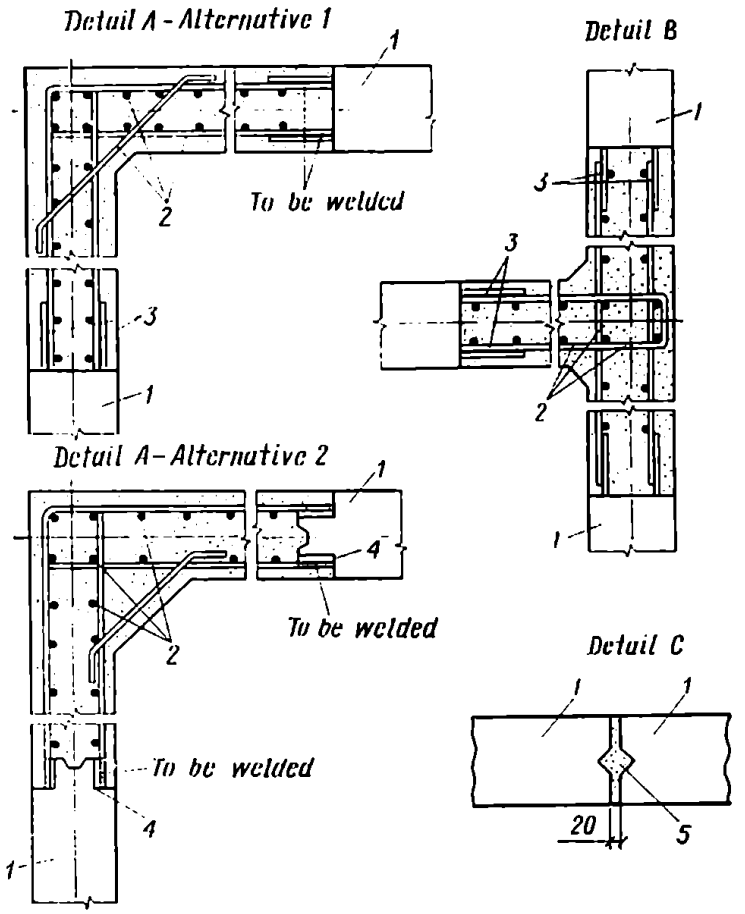



Fig. XI.35. Joints between the wall panels of rectangular tanks (see also Figs. XI.26, XI.27 and XI.28)

*1*—wall panel (reinforced with horizontal bars at the corners); *2*—in-situ portion reinforcement; *3*—panel reinforcement stick-outs; *4*—embedded parts; *5*—cement grout

plane, provided they are sufficiently far away from transverse walls or partitions. Under the circumstances, the vertical bars serve as the load-bearing reinforcement, and the horizontal bars are installed from structural considerations. Some additional horizontal reinforcement is required in the regions where mutually perpendicular walls intersect and small bending moments arise in a horizontal plane as well. The length of such regions is taken equal to the panel

height (for beam-type panels) or half as large again as the panel height (for cantilever-type panels). Because of this, each standard size of panels comes in several reinforcement arrangements.

As already mentioned in Sec. XI.2, new type designs have been developed in the USSR for prefabricated reinforced concrete members fo water supply and sewage disposal systems, which come in a reduced number of standard sizes and require less materials to fabricate and less labour for erection. For example, a concreted vertical joint is replaced by a less labour-consuming key joint with welded stick-outs. Panels more than 3.6 m high now have a varying thickness which is more economical of materials. As an alternative, the corner joints between wall panels can be made flexible (Fig. XI.36). With this type of joint, the deformation of the walls is not constrained, so that wall panels of the same standard size may be used throughout, whereas rigid joints (having in-situ portions) call for panels with additional reinforcement in the corners. The flexible joint owes its water-tightness to the Thiokol sealing compound filling the gap.

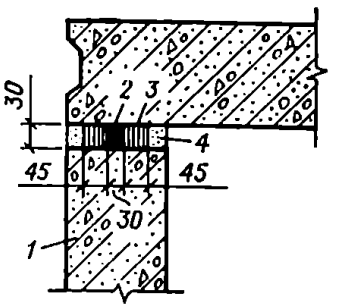



Fig. XI.36. Flexible joint between the wall panels at the corners

*1*—wall panels; *2*—Thiokol sealing compound; *3*—gasket 40 mm in diameter; *4*—cement grout

**Launders and Chutes.** These are used in many tanks and may be placed along the walls (as in settling tanks, clarification tanks, and silt compactors), above the walls (as in aeration tanks), or directly on the ground (as in bunker sand traps). Standardized rectangular launders and chutes have a nominal length of 3 and 6 m, a clear width of 20 to 60 cm, a depth of 30 to 90 cm, and a wall thickness at the bottom of 8 to 11 cm. They are carried in reinforced concrete couplers installed on transverse reinforced concrete beams which embrace the wall (or partition) panels (Fig. XI.37*a*). The beams are secured in place by steel wed-

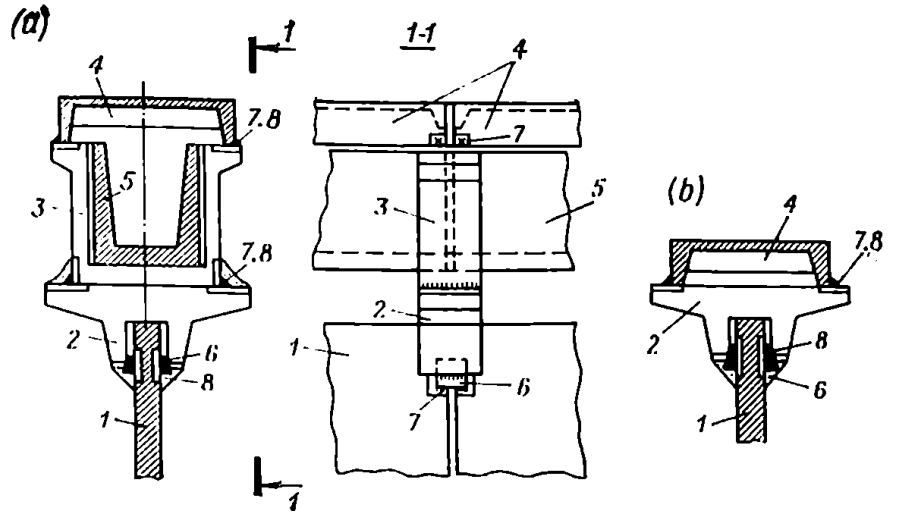



Fig. XI.37. Launders and footwalks in rectangular tanks
(*a*) launder and footwalk; (*b*) footwalk; *1*—wall panel; *2*—transverse beam; *3*—coupling; *4*—slab; *5*—launder; *6*—steel wedges; *7*—embedded parts to be welded during erection; *8*—cement grout

ges. All members are joined together by welding embedded steel parts.

Footwalks for attending personnel are assembled from ribbed slabs ordinarily used in the roofs of industrial buildings. They are carried by transverse reinforced concrete beams (Fig. XI.37*b*) or couplers which join together the individual sections (Fig. XI.37*a*).

## XI.5. Design of Rectangular Tanks

The behaviour under load of, and the choice of a design approach to, the walls of rectangular tanks depend on their construction type and aspect ratio.

The walls of open-top structures whose height is considerably smaller than the length (aeration tanks, oil traps, settlers) are regarded as cantilevers fixed in the floor.

The outer walls are designed:

(1) to resist liquid pressure in the absence of backfill (Fig. 38*a*); and

(2) to resist earth pressure when the tank is empty (Fig. XI.38*b*, *c*). The live load at the ground surface around the tank is taken as 1 tf $m^{-2}$ (10 kN $m^{-2}$).

The formulas necessary to determine the characteristic values of liquid and earth pressure and the respective overload factors have been given in Sec. XI.3.

On finding the design bending moments induced by the specified load (per metre run of the wall), we determine the necessary cross-sectional area of the vertical reinforcing steel as in a rectangular section of effective depth $h_0 = \delta - 2$ cm and width $b = 100$ cm. The reinforcement of area $F_{s,\,in}$ installed on the inside is designed to take up the liquid pressure, and that of area $F_{s,\,out}$ installed on the outside is computed to carry the earth pressure. It should be noted that $F_{s,\,out}$ is smaller than $F_{s,\,in}$.

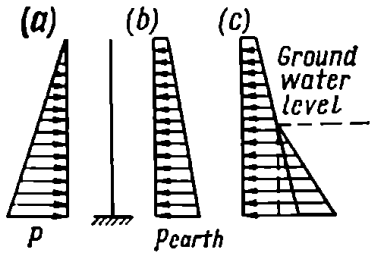



Fig. XI.38. Design system for the exterior walls of extended open-top rectangular tanks
(a) internal water pressure, no backfill; (b) lateral earth pressure, evacuated tank; (c) lateral earth pressure, ground water pressure, evacuated tank

The intermediate walls which experience a liquid pressure from one side or the other (when the adjacent section is evacuated) are reinforced with steel of area $F_{s,\,in}$ on the other side.

To reduce the number of standard sizes used, prefabricated cantilever-type panels have the same reinforcement of area $F_{s,\,in}$ on either side. The adopted vertical reinforcement of precast panels should be checked for ability to withstand erection loads when the panel is subjected to bending due to the self-weight.

Once the necessary amount of reinforcement is found, the walls should be analyzed for tendency to crack-opening, subject to the condition $a_{cr} \leqslant 0.2$ mm. This should be done as prescribed in Sec. VI.7, assuming that all the load is acting for a long time.

As already mentioned, the horizontal reinforcement of the tank walls is placed for structural reasons. Within 1.5 $H_w$ of the end walls (if the longitudinal and transverse walls are rigidly connected), the cross-sectional area of the horizontal reinforcement should be increased so as to meet the condition $F_{s,\,hor} \geqslant \mu_{min}\, bh_0$. This is done to allow for small

bending moments arising in the horizontal plane (see Sec. XI.4).

The intermediate walls which always resist a balanced liquid pressure from both sides (such as inter-corridor walls in aeration tanks, circulation partitions in settlers, and the like) are designed to carry vertical loads due to the liquid-filled launders and chutes, footwalks, and process pipelines resting on them, and also the wind loads imposed on the walls when the tank is empty.

The cross-sectional area of the vertical reinforcement is determined as for mombers in eccentrical compression (see Chap. VII). The horizontal reinforcement is likewise placed from structural considerations. If the wall is assembled from precast members, the adopted steel should be checked for bending due to the self-weight of the panel in transit and erection.

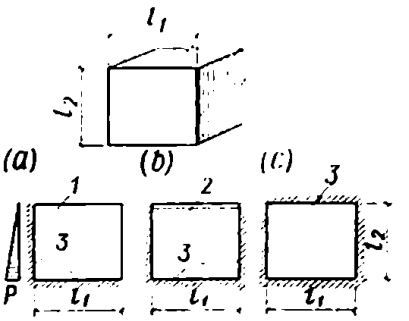



Fig. XI.39. Design systems for wall working in two directions
*1*—free end; *2*—hinged end; *3*—clamped ends

The walls of open-top and covered tanks with an aspect ratio of $l_1/l_2 < 3$ (Fig. XI.39) are regarded as plates clamped at the bottom and along the side faces. How the upper end of the wall is fixed depends on the tank design. In an open-top tank, the wall has a free top (see Fig. XI.39*a*); in a covered tank with a precast roof, the wall top is hinged (Fig. XI.39*b*); and in a covered tank with an in-situ roof, it is fixed rigidly (see Fig. XI.39*c*). The load imposed on the walls by liquid or earth pressure is graphically depicted by a triangle (or a trapezoid) with a maximum ordinate at the bottom. The bending moments are determined using tables for the design of rectangular isotropic slabs, given in Appendix XVIII. Once the bending moments are found, the necessary steel area in each direction is found by the usual procedure, and the walls are checked for tendency to crack-opening.

The walls of extended tanks (Fig. XI.40) have supports at the floor and roof level. An in-situ smooth wall is regar-

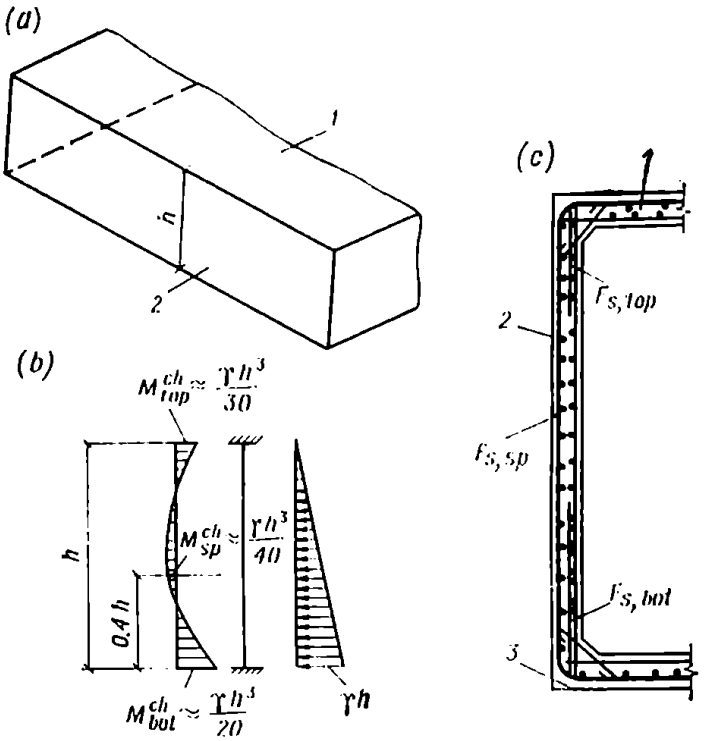



Fig. XI.40. To the design of the walls of extended covered rectangular tanks, rigid joint between the walls, roof, and floor
*1*—roof; *2*—wall; *3*—floor

ded as a beam-type slab elastically fixed at the top and bottom and subjected to the load due to liquid pressure (in the absence of backfill) graphically represented by a triangle (Fig. XI.40*b*). In an evacuated tank, the lateral earth pressure causes a similar distribution of bending moments but opposite in sign. The necessary vertical steel area is found from the design values of the bending moments, and the panel is then checked for tendency to crack-opening. The distribution of reinforcement is shown in Fig. XI.40*c*.

It should be noted that in addition to bending moments, the walls of covered rectangular tanks are subjected to a longitudinal force due to the roof weight, so the walls work in eccentrical compression. However, the effect of the longitudinal force is not taken into consideration in

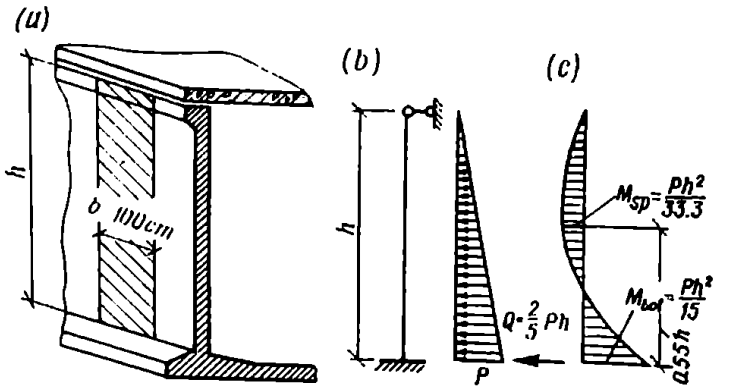



Fig. XI.41. To the design of the walls of extended covered rectangular tanks, hinged joint between the walls and the roof
(a) general view; (b) loading system; (c) bending moment diagram

design calculations and the walls are designed to resist bending only. This results in an overreinforcement, but improves the performance of the structure, because in service it may so happen that the tank has no backfill over its roof, and the longitudinal force will then decrease considerably.

If an in-situ wall has vertical ribs spaced a distance $c \geqslant 0.5$ or $c \geqslant 2h$ apart, the inter-rib portion of the wall should be designed as a slab clamped on four sides, using the tables mentioned above, and the ribs should be designed as a beam elastically fixed at the floor and roof and loaded by the reactions transferred by the slab.

Ribless walls in extended covered tanks with precast roofs (Fig. XI.41) are regarded as clamped in the floor and having a hinged immovable support at roof level. Such walls are designed as vertical beams 1 m wide.

The same design system (see Fig. XI.41) applies to precast beam-type wall panels. For type-design panels, the moment in the span is taken to be somewhat larger than that shown in the figure because of a likely rotation of the joint between the wall and the floor (the moment $M_{sp}$ is determined with $M_{bot}$ reduced by 20%).

When determining the lateral earth pressure, the live load at the ground surface is taken as 1.5 tf $m^{-2}$ (15 kN $m^{-2}$); in the case of tanks with a soil warmth-keeping jacket, the

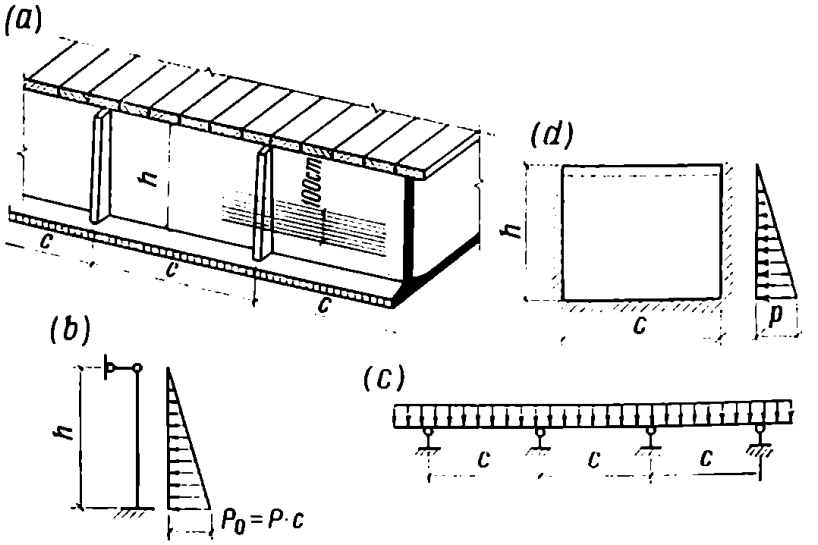



Fig. XI.42. To the design of the ribbed walls of covered rectangular tanks

(*a*) general view; (*b*) loading system for the rib when $c > 2h$; (*c*) loading system for the slab when $h > 2c$; (*d*) loading system for the slab when $c \leqslant 2h$

load is taken as 2.5 tf $m^{-2}$ (25 kN $m^{-2}$). In type-design beam-type panels, this live load is adopted for $H = 3.6$ and 4.8 m. Once the bending moments are found, we determine the required vertical steel area, keeping in mind that the intermediate walls resist liquid pressure from both sides, and the marginal walls are subjected to liquid pressure from one side and earth pressure from the other. Finally, the walls are checked for tendency to crack-opening subject to the condition $a_{cr} \leqslant 0.2$ mm.

If a wall has vertical ribs (Fig. XI.42*a*), then, for $h > 2c$, the moments calculated as in Figs. XI.40 and XI.41 should be multiplied by $c$ and used as guide in finding the necessary rib reinforcement. In this case, the ribbed slab is designed in the horizontal direction as a multispan beam supported on the ribs (Fig. XI.42*c*). The load imposed on the slab varies with height, so the slab is divided into zones 1 m wide, and each zone is designed to carry a load of its own.

If the distance between the ribs is $c \leqslant 2h$, the rib is designed as a beam clamped at the bottom and hinged at the

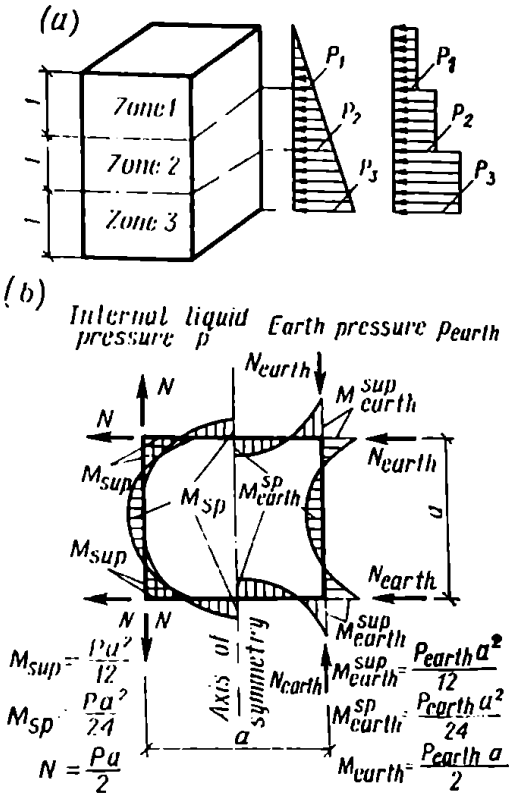



Fig. XI.43. To the design of the walls of rectangular tanks whose height considerably exceeds their dimensions in plan

top and loaded by the reactions of the adjacent slabs supported on four sides (Fig. XI.42*d*).

The walls of small structures whose height considerably exceeds their dimensions in plan are divided into zones 1 m high, and each zone is designed as a closed frame to resist a maximum pressure $p$ (Fig. XI.43*a*). The associated bending moment diagrams, the values of moments and normal forces are shown in Fig. XI.43*b*. In the absence of backfill, the walls are subjected to eccentrical tension due to liquid pressure. In an evacuated tank, the walls work in eccentrical compression induced by the lateral earth pressure. The roof and floor members are designed as prescribed in Sec. XI.3 for circular tanks. For worked examples, the reader should turn to Chap. XII.

## XI.6. Construction Types and Design of Steel Circular Tanks

Steel tanks are classed as sheet structures made of flat or bent sheets connected by welding into a single space thin-wall structure. The welded joints between the sheets should not only be strong but also sufficiently water-tight. Steel tanks mostly experience a moment-free state of stress where they are subjected only to axial (mainly tensile) forces which give rise to stresses uniformly distributed across the thickness of the sheets. Bending moments only arise where the sheets meet stiffening rings, flat floors, etc. They are rapidly damped and affect the state of stress only within limited portions (edge disturbance).

Sheet structures subjected to tensile stresses induced by internal pressure (pipelines, circular tanks, and so on) remain serviceable even after the stresses in the sheets have reached the yield point. For this case, the design tensile strength of the steel is based on its ultimate rupture strength (see Chap. IX).

Steel circular tanks are mostly used in water towers. They may have either a flat or a space floor (Figs. XI.44 and XI.45). The weight of the roof is usually insignificant and has no effect on the wall behaviour.

Tanks with a flat floor are the easiest to manufacture, but they have certain disadvantages in service: the floor is difficult of access for inspection and repair, and atmospheric precipitation has no free run-off and speeds up steel corrosion.

For ease of welding and better corrosion resistance, the minimum sheet thickness is taken to be 4 mm. In tanks having a relatively small capacity (from 100 to 1 000 $m^3$), the walls and the flat floor have a constant thickness $\delta = 4$ mm. In large-capacity tanks (2 000 to 5 000 $m^3$), the wall thickness varies from 4 to 10 mm, the flat floor has a constant thickness of 4 to 5 mm in the middle and a thickness of 6 to 8 mm at the edges. A wall is assembled from several individual horizontal rings (tiers) installed one on top of another. The sheets in the tiers are joined by butt-welds. In the adjacent tiers, these welded joints are staggered, the stagger being half the sheet length, but not less

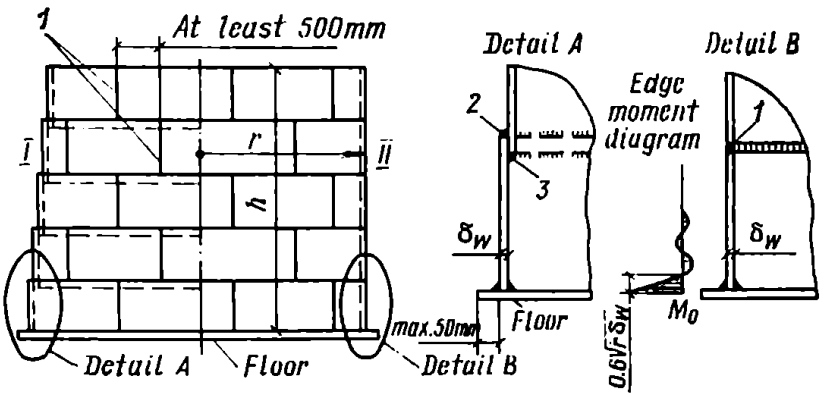



Fig. XI.44. Steel tank with a flat slab

*I*—lap-welded tiers; *II*—butt-welded tiers; *1*—butt weld; *2*—continuous weld; *3*—intermittent weld

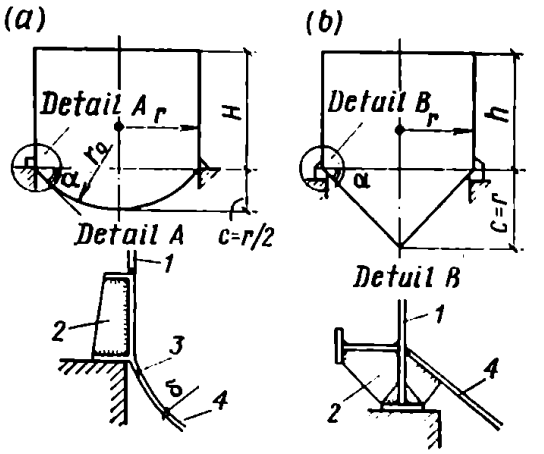



Fig. XI.45. Steel tanks with a spatial floor supported around the periphery

(*a*) spherical floor; (*b*) conical floor; *1*—wall; *2*—supporting ring; *3*—butt weld; *4*—floor

than 500 mm. The tiers are joined by butt- or lap-welds (see Fig. XI.44).

The sheets of a flat floor are joined by butt- or lap-welds along each strip, and the strips are lap-welded to one another.

The walls and the floor are joined by two continuous circumferential T-butt welds (see Fig. XI.44). There is no merit in using an additional angle or straps at the joint between the wall and floor or in extending the floor beyond the wall

for more than 50 mm, as this only leads to a considerable stress concentration at the joint.

In tanks with space floors, the joint between the walls and the floor has a support ring (see Fig. XI.45).

In present-day practice, ever wider application is being found by steel circular tanks whose component members are prefabricated and transported to the site in rolls. In the course of erection, the members are unrolled and welded together*. The sheet thickness in such tanks is limited so that the steel can be rolled. The walls of large-capacity tanks may be prestressed, for which purpose high-strength wire is wrapped around the wall and the entire structure is shotcreted.

The internal hydrostatic pressure causes the wall of a circular tank to work in axial tension. The design circular tensile force $T$ acting at a depth $h$ from the top of the wall in a circular tank of radius $r$ is given by Eq. (XI.3). If the tank is filled with water, we have

$$T = 1.1hr$$

For each tier, the depth $h$ is taken as the distance from the top of the wall to the bottom of the tier (Fig. XI.46*a*).

The walls of thickness $\delta_w$ are checked for strength at the vertical joints between the sheets in the tiers

$$T/\delta_w \leqslant R_{ten}^{weld} \qquad \text{(XI.24)}$$

At the joints between the wall and the floor, the welds restrain the free deformation of the wall, thereby giving rise to an edge bending moment $M_0$. The design value of $M_0$ may approximately be determined as

$$M_0 \approx 0.11Hr\delta_w \qquad \text{(XI.25)}$$

where $H$ is the height of the circular wall.

The edge moment rapidly subsides with height (Fig. XI.46) and mainly affects the welds at the joint between the wall and floor. The stresses induced by the edge moment in the welds should satisfy the following condition

$$\sigma_{weld} = M_0/W_{weld} \leqslant R_f^{weld} \qquad \text{(XI.26)}$$

* This is the main type of tanks for petroleum products.

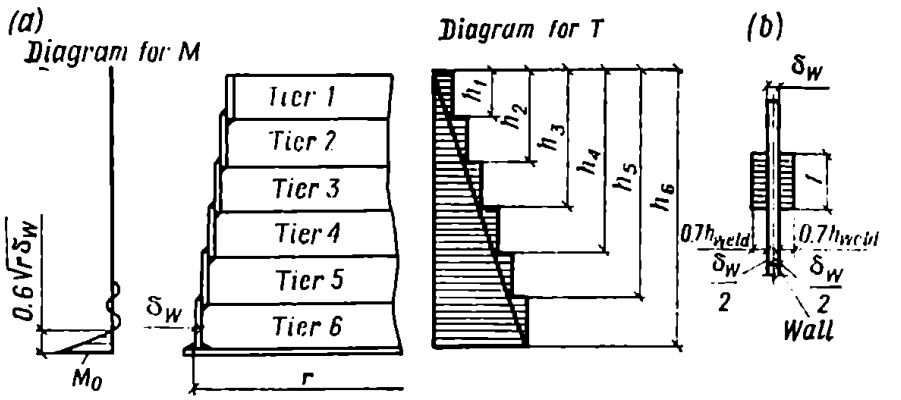



Fig. XI.46. To the design of a steel circular tank
(a) wall; (b) welded joint between wall and floor

where $W_{weld}$ is the moment of resistance of two fillet welds, in cm$^3$ (see Fig. XI.46*b*):

$$W_{weld} = J_{weld}/(\delta_w/2 + 0.7h_{weld}) \qquad \text{(XI.27)}$$

Here, $J_{weld}$ is the moment of inertia of the welds in cm$^4$:

$$J_{weld} = 2\,[1 \times (0.7h_{weld})^3/12 + 1 \times 0.7h_{weld} \times (0.7h_{weld}/2 + \delta_w/2)^2] \qquad \text{(XI.28)}$$

These stresses are not added to the tensile stresses which the weld experiences as a member of the lower tier of the wall, because the vectors of these stresses are mutually perpendicular.

Flat floors resting on a continuous foundation are exposed to insignificant forces, so they are not usually analyzed.

The hydrostatic pressure acting on a spherical floor supported around the periphery (see Fig. IX.45) gives rise to tensile forces. Such a floor is checked for strength at the weld in the lower zone where the tensile stresses reach a maximum:

$$n\gamma\,(H + c)\,r_0/2\delta \leqslant R_{ten}^{weld} \qquad \text{(XI.29)}$$

where $n$ is the overload factor, $\gamma$ is the unit weight of the liquid contained in the tank, and the meaning of $H$, $c$, $r_0$, and $\delta$ is explained in Fig. XI.45.

In a conical floor with an angle of slope $\alpha$ (see Fig. XI.45*b*), the circular tensile stresses reach a maximum at the sup-

porting ring. The weld is checked for strength subject to the condition

$$nH\gamma r \operatorname{ctg} \alpha/\delta \sin \alpha \leqslant R_{ten}^{weld} \qquad \text{(XI.30)}$$

The supporting ring of a tank supported around the periphery experiences the following compressive forces

$$N = nG^{ch} \operatorname{tg} \alpha/2\pi \qquad \text{(XI.31)}$$

where $G^{ch}$ is the weight of the water and floor.

The ring is said to have an adequate strength if

$$N/F_r \leqslant R \qquad \text{(XI.32)}$$

where $F_r$ is the cross-sectional area of the ring.

The supporting ring is also checked for stability subject to the condition

$$N \leqslant N_{cr} \qquad \text{(XI.33)}$$

where

$$N_{cr} = 3EJ_y/F_r r_r^2 \qquad \text{(XI.34)}$$

Here, $E$ is the modulus of elongation for steel, $J_y$ is the moment of inertia of the ring section about the vertical axis passing through the centroid of the section, and $r_r$ is the ring radius.

## XI.7. Reinforced Concrete Pipes and Wells for Water Supply and Sewage Disposal Systems

Reinforced concrete pipes may bo of the nonpressure and pressure types. The former are used to carry fluids in gravity flow such as in sanitary sewers and storm drains, the latter are used in conduits working under a hydrostatic pressure of up to 20 atmospheres.

Nonpressure pipes are nonprestressed. They have a length of up to 5 m, a diameter of $D = 400$ to 2 500 mm, and a wall thickness of 50 to 150 mm. The pipe reinforcement (with $D \leqslant 900$ mm) consists of one or two cylindrical welded cages made up of spiral reinforcement (Fig. XI.47). With a stepped bell, the gap between the bell and the spigot is filled with a mixture of asbestos and cement for water-carrying pipes, and a tarred rope is additionally used for sewer pipes (Fig. XI.48*b* and *c*). With a tapered bell, a solid

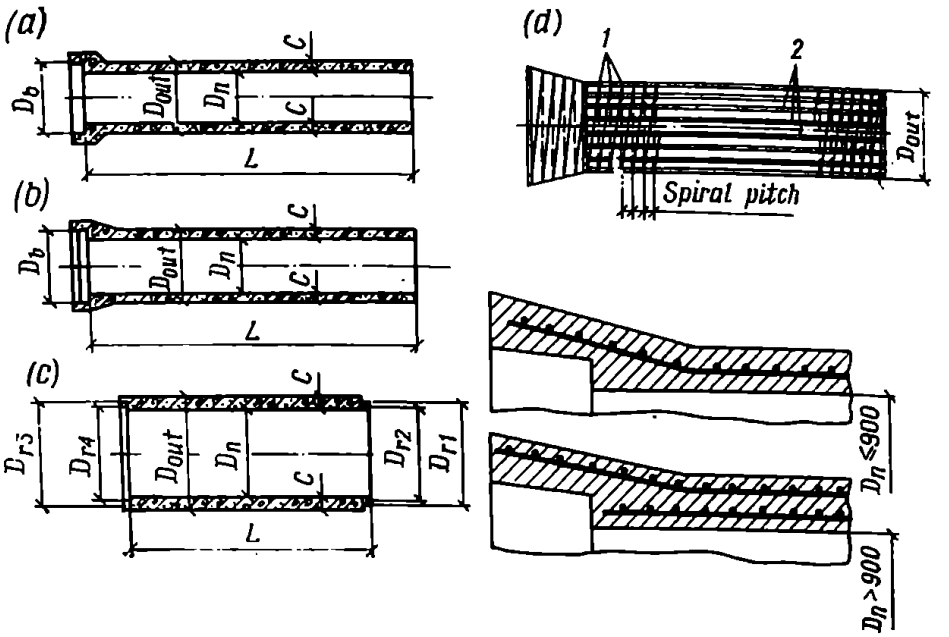



Fig. XI.47. Reinforced concrete nonpressure pipes

(*a*) stepped-bell pipe; (*b*) tapered-bell pipe; (*c*) flush-connected pipe; (*d*) distribution of pipe reinforcement; *1*—spiral reinforcement; *2*—longitudinal bars

rubber gasket is used to seal the joint against leakage (Fig. XI.48*a*). The joints in flush-connected pipes are caulked with a mixture of asbestos and cement or shotcreted (Fig. 48*d*).

The load-bearing capacity, settlement, and, as a consequence, the tightness of joints largely depend on how pipes are laid on the foundation.

Type-design nonpressure pipes with $D \leqslant 500$ mm designed for backfill 4 and 6 m deep may be laid directly on a flat soil foundation and the trench may be backfilled with the soil. If $D \geqslant 600$ mm, the shape of the trench cross section should be such that the angle of contact is at least 90° (Fig. XI. 49*a*). The trench is backfilled with sand as deep as the pipe axis, and each layer of sand should properly be compacted. In soils having a design strength of less than 1.5 kgf cm$^{-2}$ (0.15 MPa), use is made of concrete footings (Fig. XI.49*b*). If the soil is likely to settle unevenly, reinforced concrete footings are used. In coarse-grained rocks, hard clays and loams, pipes are laid on a layer of sand.

Pressure pipes should be highly crack-resistant, which is why they are prestressed. The lateral prestress is produced by tensioning the spiral reinforcement, and the longitudinal prestress by tensioning the longitudinal bars.

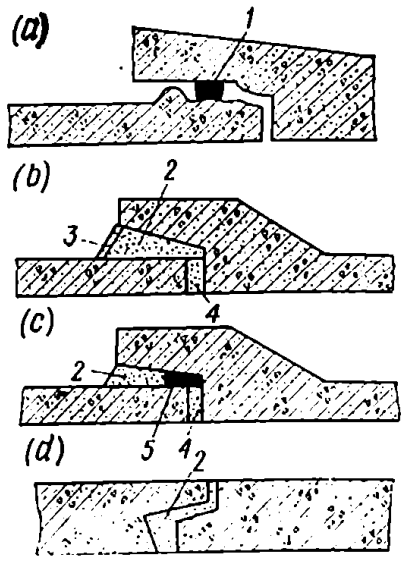



Fig. XI.48. Joints in nonpressure pipes

*1*—solid rubber gasket; *2*—asbestos-cement mixture; *3*—asphalt putty; *4*—cement grout; *5*—tarred rope

Type-design reinforced concrete pressure pipes are fabricated at concrete yards by a vibrohydraulic process. They range between 500 and 1 600 mm in diameter, their walls are from 55 to 105 mm thick, and the length is 5 m. The pipes are reinforced with class B-II (spiral reinforcement) and class Bp-II (longitudinal steel) high-strength wire. The pipe bells are tapered, and the joints are sealed with rubber gaskets (see Fig. XI.48*a*). Pressure pipes are laid in trenches similarly to nonpressure pipes.

Some designs incorporate a light-gauge steel cylinder in the wall of the pipe as a waterproof diaphragm. Such pipes intended for a pressure of up to 10 atm are nonprestressed, and those intended for a pressure of 10 to 20 atm have prestressed spiral reinforcement.

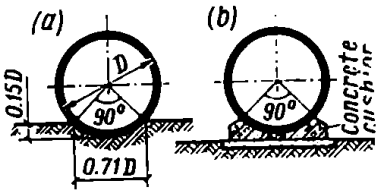



Fig. XI.49. Installation of pipes

Prestressed pipes are checked for strength and incipient cracking, nonprestressed are checked for strength and crack width.

In service, pipes are subjected to internal pressure to which may be added a water hammer, self-weight and weight of liquid, soil pressure, and live load on the ground surface induced by vehicles. These loads bring about bending moments and axial forces both in the lateral (circular) and in the longitudinal sections.

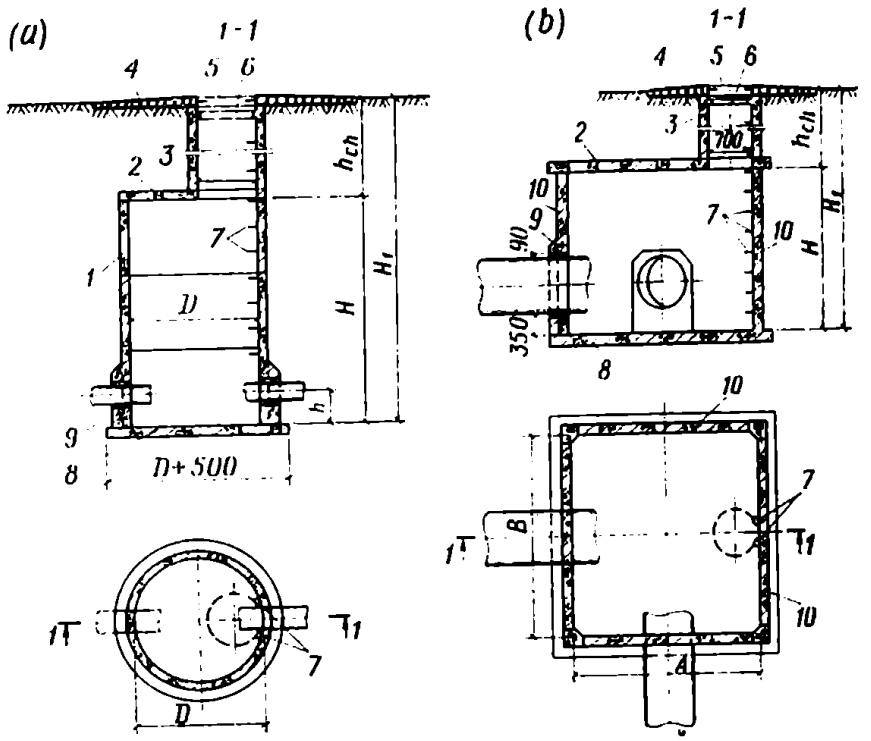



Fig. XI.50. Reinforced concrete wells

(*a*) circular well; (*b*) rectangular well; *1*—wall rings; *2*—roof slab; *3*—chimney ring; *4*—pavement; *5*—cast iron manhole; *6*—supporting ring for manhole; *7*—steps; *8*—floor slab; *9*—in-situ concrete around pipelines; *10*—flat wall slabs

Design procedures for pipes are not covered in this text.

**Reinforced Concrete Wells for Water Supply and Sewage Disposal Systems.** Most commonly, these are circular in plan (Fig. XI.50*a*). Standardized wells have an inside diameter of 1 000, 1 500, and 2 000 mm, and a chimney diameter of 700 mm. The well depth depends on the depth at which the pipeline is laid and may vary over a wide range as a multiple of 300 mm.

The well walls are assembled from reinforced concrete rings 600 or 900 mm in diameter and 80 to 100 mm thick. Bottom rings have four openings on either side for inlet and outlet pipelines. After the pipes have been installed, the remaining openings and gaps between the pipes and walls are concreted.

The wall rings are installed on a flat floor slab 100 to 120 mm thick and roofed over by a flat slab 150 mm thick having a hole 700 mm in diameter, on which the chimney is erected. The floor slab diameter should exceed the inside diameter of the wall rings by 500 mm, and is taken as 1 500, 2 000, and 2 500 mm. The diameter of the roof slab is equal to the outside diameter of the wall rings and is taken as 1 160, 1 680, or 2 200 mm.

The chimney is covered by a reinforced concrete slab 840 mm in diameter and 70 mm thick. It has a hole 580 mm in diameter, over which a standard-size cast iron manhole and cover are installed. If a well is located on a highway or an industrial site where heavy trucks are likely to pass, the round chimney roof slab is replaced by a rectangular highway slab 1 150 × 2 500 mm large and 220 mm thick. In the middle, the slab has a hole 580 mm in diameter serving as a manhole. Outside the chimney, the highway slab rests on a compacted sand foundation 250 mm thick.

The bottom part of wells may also be rectangular in plan (Fig. XI.50*b*). In this case, use is made of standardized wall panels for rectangular tanks (see Sec. XI.4).

The load-bearing capacity of type-design reinforced concrete structures is such that the floor slabs may be embedded as deep as 7 m, and the roof slabs as deep as 0.5 to 4 m depending on the live load at the ground surface.

## XI.8. Construction Types and Design of Water Towers

The function of water towers is to maintain the required water head in the associated lines.

The main part of a water tower is a tank. The capacity of the tank is chosen to suit water consumption requirements, and the height of the tank above ground level depends on the design water head required.

In capacity, water towers range between 15 and 3 000 $m^3$, and in height, between 6 and 50 m.

Water towers may be of the shelter type (Fig. XI.51*a*) or the shelterless type (Fig. XI.51*b*). In the former type, the tank is enclosed in a shelter to keep it from freezing at subzero temperatures and at low rates of water circula-

tion, or to keep the water from heating at high ambient temperatures. In the latter type, thermal insulation is applied (if necessary) directly to the tank walls. Nowadays, the most popular tanks in the USSR are made of the shelterless or semi-shelter type. This simplifies construction, reduces the weight of support structures, and cats down the cost.

Instead of one tank (see Fig. XI.51*a* and *b*), a water tower may have several tanks (Fig. XI.51*c*), if water of different purity and temperature is required.

Existing evidence shows that, given the same height and construction type, the cost of water towers varies only insignificantly with tank capacity. For example, if the effective

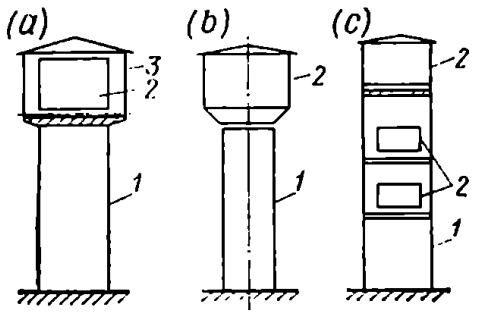



Fig. XI.51. Construction types of water towers
*1*—support; *2*—tank; *3*—shelter

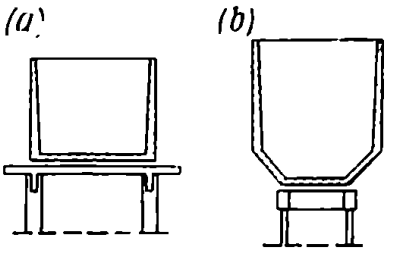



Fig. XI.52. Water tower tanks

tank capacity is increased by 30% to 40%, the cost of the tower will rise only by 3% to 6%. Given the same capacity and construction type, the cost of towers varies in proportion to the height of the support.

On the basis of a technical and economic analysis. the following type-design structures have been developed in the USSR: towers with a tank capacity of 25 and 50 $m^3$ for a height of 9 to 27 m, and towers with a tank capacity of 150, 250, 500, and 1 000 $m^3$ for a height of 12 to 42 m (taken as a multiple of 3 m).

The tanks of water towers are made either of reinforced concrete or of steel. Among reinforced concrete tanks, the simplest structure consists essentially of a cylindrical shell

and a flat non-bearing bottom lying on a solid floor installed in the supporting structure (Fig. XI.52*a*). Tanks made up of a cylindrical shell changing at the bottom into a truncated cone and a load-bearing bottom are more economical of material and, therefore, cost by 30% to 40% less (Fig. 52*b*).

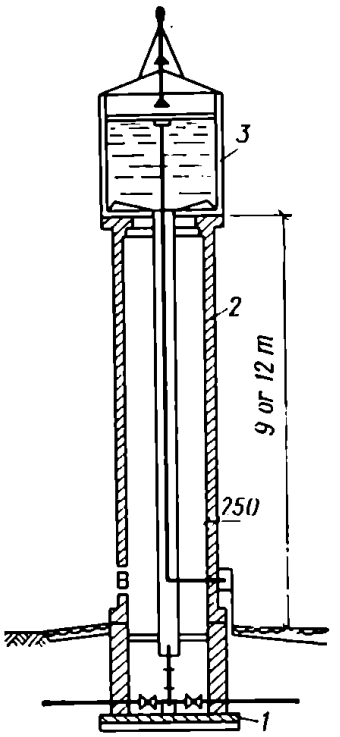



Fig. XI.53. Water tower with a brick support
*1*—footing; *2*—brick support; *3*—metal tank

To ensure the required crack resistance, the walls of large reinforced concrete tanks ought to be prestressed. However, steel tensioning at a great height is inconvenient and not always warranted. Because of this, tanks are often made of steel instead of reinforced concrete (see Sec. XI.6).

The supports of water towers are mostly made of reinforced concrete, but for small-capacity tanks they may be made of metal or brick depending on the local conditions. Masonry supports are warranted for a tower height of 9 to 12 m and are built as a cylindrical shaft with walls 250 to 380 mm thick. Figure XI.53 schematically shows a shelterless tower with a masonry support and a metal tank.

According to exposure conditions, reinforced concrete supports may be made as a solid monolithic cylindrical shell (Fig. XI.54*a*), as a space frame (Fig. XI.54*b*), or as an open-work lattice structure (Fig. XI.55). Other things being equal, precast r. c. open-work lattice supports are less expensive than the other types. The cost of such supports is from two thirds to one half the cost of a solid in-situ support, and one half to one third that of masonry supports. The cost of precast reinforced concrete and metal supports is about the same.

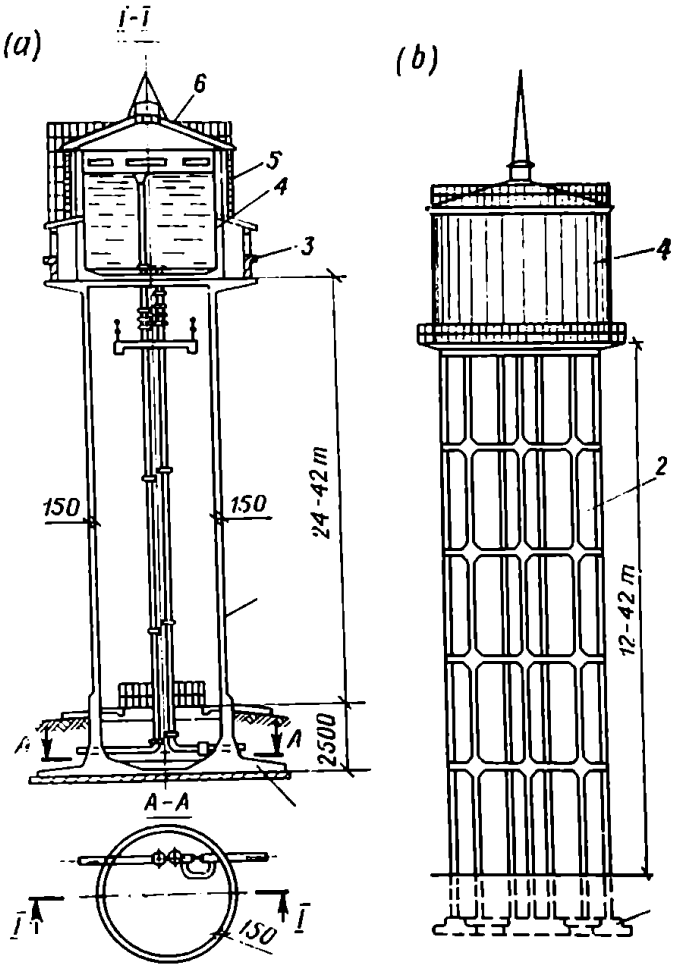



Fig. XI.54. (*a*) Water tower with a solid in-situ reinforced concrete support, and (*b*) water tower with a frame-type precast reinforced concrete support

*1*—reinforced concrete footing; *2*—support; *3*—semi-shelter (heated gallery); *4*—reinforced concrete tank; *5*—warmth-keeping jacket around tank wall; *6*—roof

Figure XI.54*a* shows a water tower using a reinforced concrete support which is a monolithic shell cast in a reusable travelling form. The minimum shell thickness permitting the form to be moved without causing tears in the concrete considerably exceeds the value required by strength analysis so the cost of the entire structure rises considerably. Such supports are attractive for towers at least 24 m high using

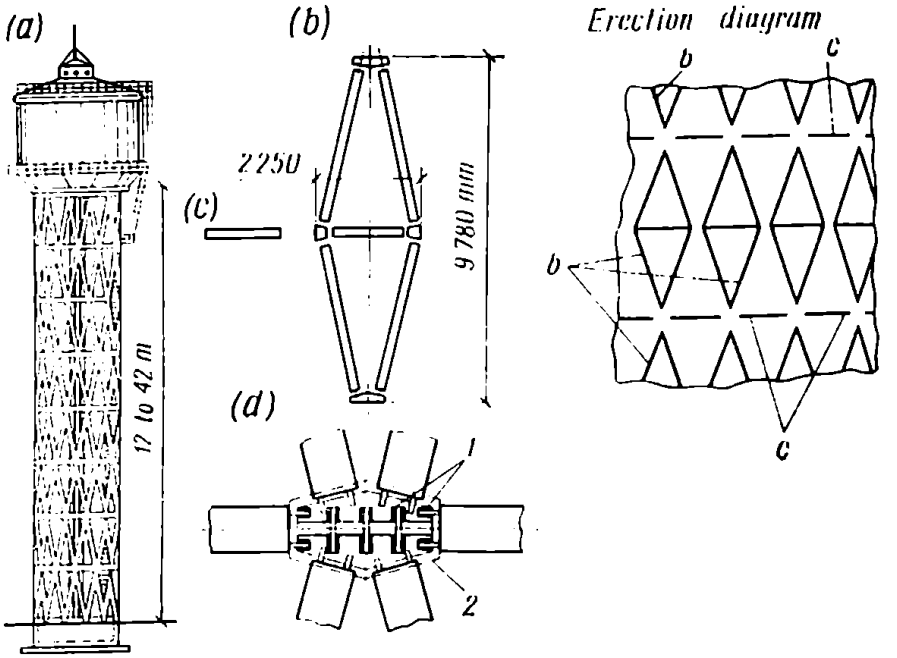



Fig. XI.55. Water tower with an open-work lattice support
*1*—to be welded during erection; *2*—outer surface of in-situ concrete

large-capacity tanks (at least 800 m³). The shaft of the tower rests on a in-situ reinforced concrete footing having a circular rib running all the way round the shaft.

Water towers using frame r. c. supports (Fig. XI.54*b*) are more economical of material than those with solid in-situ r.c. supports. They, however, are more labour-consuming because complex rigid joints have to be made between prefabricated members.

Water towers using open-work lattice supports (Fig. XI.55*a*) are assembled from rhombic rod units (Fig. XI.55*b*) and chord units (Fig. XI.55*c*). Each rhombic unit is preassembled from inclined struts and chords. All prefab members have reinforcement stick-outs which are welded at the joints to steel plates during erection. After welding, the joints are concreted (Fig. XI.55*d*).

The tanks, supports, footings, and shelters of water towers should be designed as prescribed in relevant chapters of this text.

For the supports and footings, the main loads (Fig. XI.56) are the weight of the water-filled tank $G_1$, the dead weight

of the support $G_2$, the dead weight of the footing and backfill over its top $G_3$, the lateral wind pressure applied to the shelter (tank) $P_1$, and to the support $P_2$.

As a whole, the support is subjected to eccentrical compression due to $G_1$ and $G_2$ and to the bending moment induced by $P_1$ and $P_2$, which reaches its maximum at the footing. If the support is solid, strength analysis is carried out for its circular cross section. Frame and open-work lattice supports are analyzed as space rod systems.

The size of the footing base is designed to carry the normal force and moment, depending on the bearing capacity of the soil.

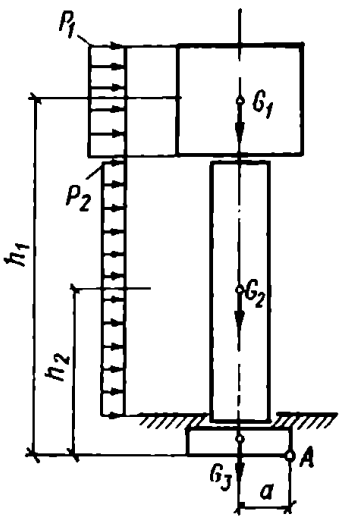



Fig. XI.56. To the design of a water tower

The tower as a whole is checked for stability, mentally assuming that it might overturn about the external side of the footing on the leeside (point $A$ in Fig. XI.56). The overturning moment due to the wind and the moment of stability due to the weight of the component parts of the structure are found by

$$M_{ov} = \sum P_i h_i \quad \text{and} \quad M_{st} = \sum G_i a_i$$

The wind load is multiplied by the overload factor $n = 1.3$ and the self-weight of the structure (with the tank empty) is multiplied by the reduction factor $n = 0.9$.

The coefficient of stability against overturning

$$k = M_{st}/M_{ov}$$

should be at least 1.5.

# CHAPTER TWELVE

# DESIGN OF REINFORCED CONCRETE STRUCTURES. WORKED EXAMPLES

## XII.1. General

This chapter is concerned with the design and engineering of typical reinforced concrete structures used in water supply and sewage disposal systems. Most examples are given for rectangular reinforced concrete tanks assembled from prefabricated members. In particular we shall consider a 6000-$m^3$ tank 36 m square (Fig. XII.1). This tank may have one of two precast, r.c. roofs:

(1) a slab and girder roof consisting of ribbed slabs and girders with a column grid 6 m square (Fig. XII.1*b*), or

(2) a roof made up of slabs supported directly on columns with a column grid 4 m square (Fig. XII.1*c*).

The slab and girder roof may have either non-prestressed slabs (see Sec. XII.2) or prestressed slabs (see Sec. XII.3), and a precast multispan girder (see Sec. XII.5).

The roof with a column grid size of $4 \times 4$ m incorporates a square slab with ribs around the periphery (see Sec. XII.4). It also includes columns, footings (see Sec. XII.6), and walls assembled from prefab reinforced concrete panels (see Sec. XII.7).

In addition, we shall discuss the design and engineering of a prestressed wall for a buried circular precast reinforced concrete tank (see Sec. XII.8).

## XII.2. Nonprestressed Ribbed Roof Slab

We set out to design a nonprestressed ribbed slab for the roof of Fig. XII.1*b*, according to data in Fig. XII.2*a*. The weight of a square metre of the slab is 275 kg. The concrete brand number is M-300, the service factor for the concrete is $m_{c1} = 1$, and the load-bearing reinforcement of the ribs is class A-III bars.

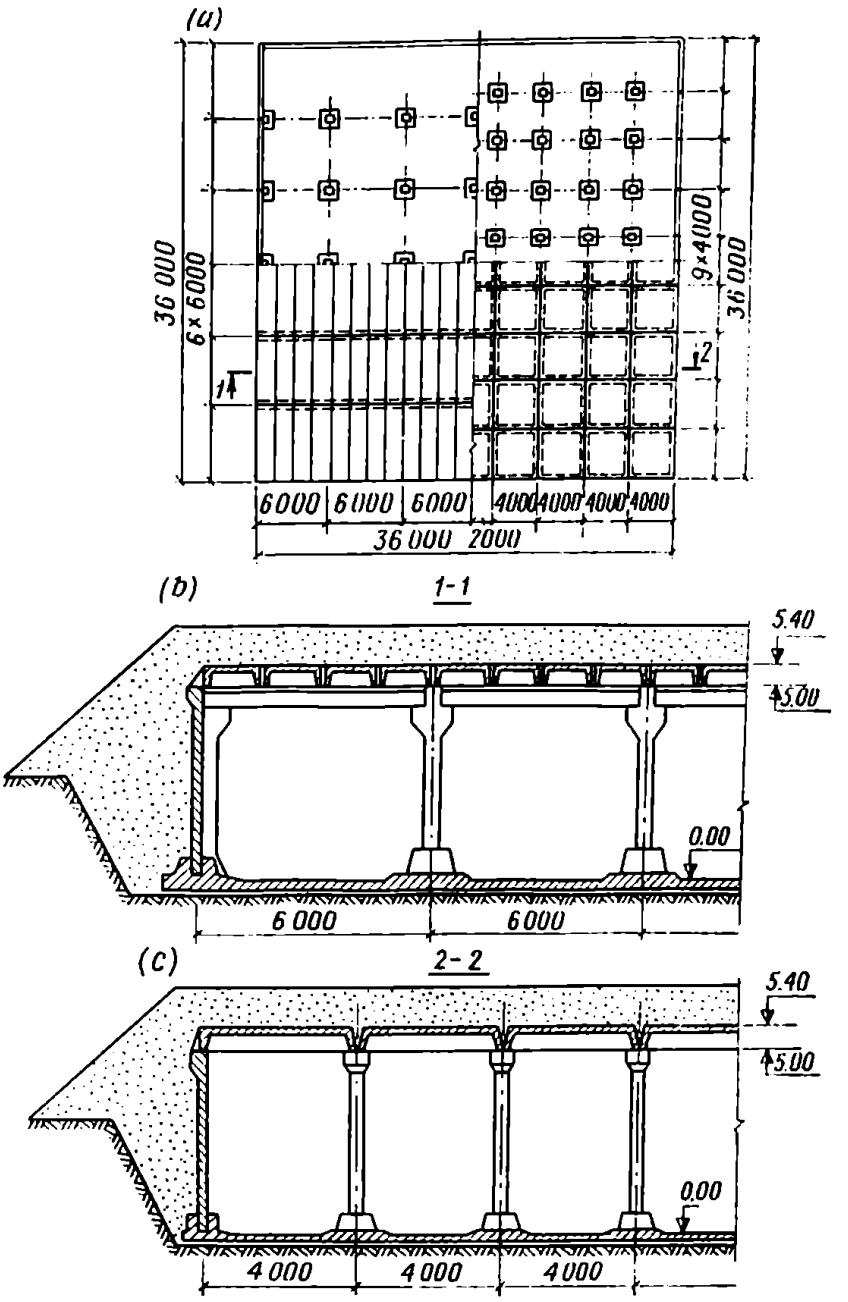
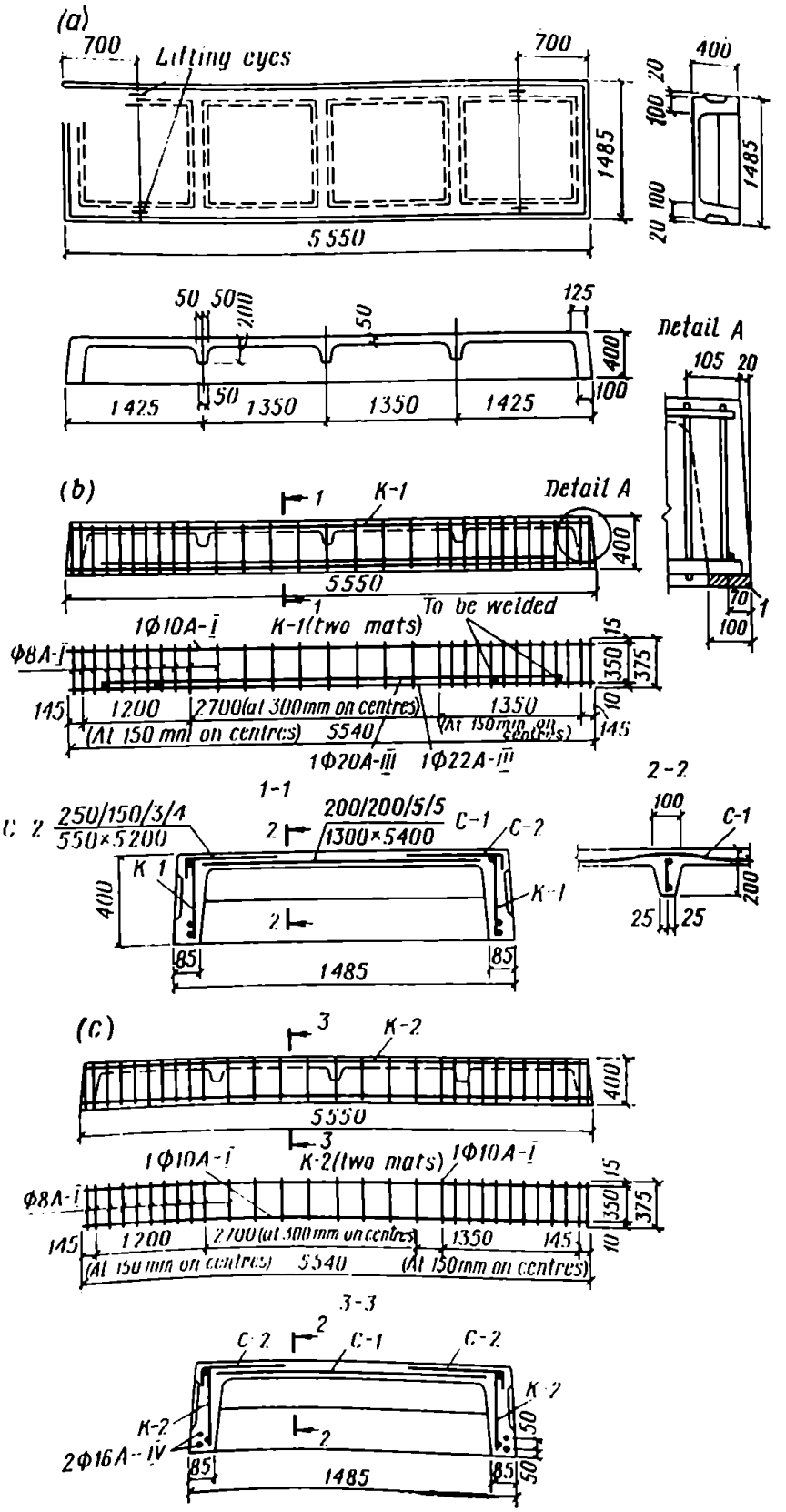



Fig. XII.1. Precast reinforced concrete rectangular tank
(*a*) plan; (*b*) and (*c*) sectional views


(a)
700
Lifting eyes
700
400
20
100
20
100
1485
1485
5 550
50 50
200
50
125
400
100
1425
50
1350
1350
1425
Detail A
105
20
(b)
1
K-1
Detail A
400
5550
1
To be welded
1Φ10A-I
K-1(two mats)
Φ8A-I
15
350
375
145
1200
2700(at 300mm on centres)
1350
10
145
(At 150 mm on centres)
5540
(At 150mm on centres)
1Φ20A-III
1Φ22A-III
1-1
C-2
250/150/3/4
550×5200
2
200/200/5/5
1300×5400
C-1
C-2
K-1
K-1
400
2
85
1485
85
2-2
100
C-1
200
25
25
(c)
3
K-2
400
5550
3
1Φ10A-I
K-2(two mats)
1Φ10A-I
Φ8A-I
15
350
375
145
1200
2700(at 300mm on centres)
1350
145
10
(At 150 mm on centres)
5540
(At 150mm on centres)
3-3
C-2
2
C-1
C-2
K-2
K-2
50
2Φ16A-IV
2
85
1485
85
50

**Effective Span and Loads.** As a first trial, we shall assume the following cross-sectional dimensions of the girder: $h = l/10 = 600/10 = 60$ cm and $b = 0.5h = 0.5 \times 60 = 30$ cm. Since the slab rests on the girder flanges,

**Table XII.1. Characteristic and Design Loads (in Terms of the First Group of Limit States) per Square Metre of the Roof**

| Loads | Characteristic load, kgf m$^{-2}$ (N m$^{-2}$) | Overload factor | Design load, kgf m$^{-2}$ (N m$^{-2}$) |
|---|---|---|---|
| *Dead*: | | | |
| precast r.c. roof slabs | 275 (2 750) | 1.1 | 303 (3 030) |
| cement covering 0.025× ×2 200 | 55 (550) | 1.3 | 71 (710) |
| water proofing | 10 (100) | 1.3 | 13 (130) |
| backfill over roof 1× ×1 700 | 1 700 (17 000) | 1.2 | 2 040 (20 400) |
| **Total** | 2 040 (20 400) | — | 2 430 (24 300) |
| *Live*: | | | |
| snow | 100 (1 000) | 1.4 | 140 (1 400) |
| or as on maintenance platform | 150 (1 500) | 1.4 | 210 (2 100) |
| Maximum total load | 2 190 (21 900) | — | 2 640 (26 400) |

according to Fig. XII.6, we take the slab length as $l_{sl} = 555$ cm and the length of the support area as 10 cm. The effective span of the slab will consequently be $l_0 = 555 - 10 = 545$ cm.

The charcteristic and design loads are given in Table XII.1.

In terms of the first group of limit states, the total design load per metre run of the slab of nominal width 1.5 m is

$$q = 2\,640 \times 1.5 = 3\,960 \text{ kgf m}^{-1} \text{ (39 600 N m}^{-1}\text{)}$$

Fig. XII.2. Ribbed roof slab

(*a*) formwork dimensions; (*b*) distribution of reinforcement in non-prestressed slab; (*c*) distribution of reinforcement in prestressed slab; *1*—embedded steel plate at support

The characteristic load per metre run of the slab in kgf $m^{-1}$ (N $m^{-1}$) is as follows:

| | |
|---|---|
| —dead load | $q^{ch} = 2\,040 \times 1.5 = 3\,060 (30\,600)$ |
| —long-time live snow load | $p^{ch}_{l\text{-}t} = (100 - 70) \times 1.5 = 45 (450)$ |
| —short-time live load (as on a maintenance platform) | $p^{ch}_{sh\text{-}t} = 150 \times 1.5 = 225 (2\,250)$ |
| —maximum total characteristic load | $q^{ch} = 3\,285 (32\,850)$ |

*Materials.* For M-300 concrete, Appendices III, IV, and V give: $R^{ch}_{pr} = R_{pr\,II} = 170$ kgf $cm^{-2}$ (17 MPa), $R^{ch}_{ten} = R_{ten\,II} = 15$ kgf $cm^{-2}$ (1.5 MPa), $R_{pr} = 135$ kgf $cm^{-2}$ (13.5 MPa), $R_{ten} = 10$ kgf $cm^{-2}$ (1 MPa), and $E_c = 290\,000$ kgf $cm^{-2}$ (29 000 MPa).

The reinforcing steel (welded-bar mats) for the ribs is as follows. Longitudinal load-bearing reinforcement: class A-III bars at least 10 mm in diameter, $R_s = 3\,600$ kgf $cm^{-2}$ (360 MPa), and $E_s = 2\,000\,000$ kgf $cm^{-2}$ (200 000 MPa); transverse and erection reinforcement: class A-I bars, $R_s = 2\,100$ kgf $cm^{-2}$ (210 MPa), $R_{s,\,tr} = 1\,700$ kgf $cm^{-2}$ (170 MPa), and $E_{s,\,tr} = 2\,100\,000$ kgf $cm^{-2}$ (210 000 MPa) (see Appendix VI). The reinforcing steel (welded-wire fabric) for the slab comprises class B-I wire with $R_s = 3\,150$ kgf $cm^{-2}$ (315 MPa) (see Appendix VII).

### A. Design in Terms of the First Group of Limit States

**Determining the Rib Cross-Sectional Area.** Experience shows that the optimal depth of nonprestressed ribs, $h$, ranges between $l_0/14$ and $l_0/18$ (where $l_0$ is the effective span). Because the load imposed on the slab is heavy, we adopt $h = l_0/14 = 545/14 \approx 40$ cm and assign the width of the rib at the bottom as 8.5 cm.

Assuming that the tensile reinforcement is placed in two rows, we get the effective depth of the rib as $h_0 = h - a = 40 - 7 = 33$ cm. The total width of the ribs in the slab (with allowance for the rib bevels) is $b \approx 22$ cm.

Now, we check the ribs for compliance with the conditions in Eqs. (VI.34) and (VI.35):

$$k_1 R_{t\,en} b h_0 \leqslant Q \leqslant 0.35 R_{pr} b h_0 \text{ (where } Q = 0.5 q l_0)$$

$$0.6 \times 10 \times 22 \times 33 \leqslant 0.5 \times 3\,960 \times 5.45 \leqslant 0.35 \times 135 \times 22 \times 33$$

$$4\,350 \text{ kgf (45.3 kN)} \leqslant 10\,800 \text{ kgf (108 kN)} \leqslant 34\,300 \text{ kgf (343 kN)}$$

As is seen, the cross-sectional dimensions of the ribs are acceptable, and the transverse reinforcement in them is required by design.

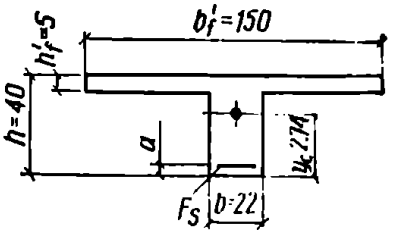



Fig. XII.3. Design cross section of the slab

**Normal-Section Strength Analysis.** The maximum bending moment (at midspan) is given by

$$M = q l_0^2/8 = 3\,960 \times 5.45^2/8 = 14\,700 \text{ kgf m (147 kN m)}$$

For design purposes, we adopt a T-section (Fig. XII.3) with all of the flange width $b_f' = 150$ cm taken into consideration (see Sec. VI.3). The section is analyzed as a rectangle of width $b_f'$ and depth $h_0$. Equation (VI.13) yields

$$A_0 = M/b_f' h_0^2 R_{pr} = 1\,470\,000/(150 \times 33^2 \times 135) = 0.067$$

For $A_0 = 0.067$, Table VI.1 gives $\xi = 0.07$ and $\eta = 0.965$. Thus, the depth of the compression zone $x = \xi h_0 = 0.07 \times 33 = 2.3$ cm is smaller than the flange depth $h_f' = 5$ cm.

Equation (VI.14) gives the following cross-sectional area of the longitudinal reinforcement

$$F_s = M/\eta h_0 R_s = 1\,470\,000/(0.965 \times 33 \times 3\,600) = 12.8 \text{ cm}^2$$

We adopt two class A-III bars 20 mm in diameter and two class A-III bars 22 mm in diameter of total area $F_s = 13.88$ cm$^2$ (see Appendix VIII), and distribute them in the ribs in two rows as shown in Fig. XII.2*b*.

**Inclined-Section Strength Analysis.** The maximum shearing force (at the support) is given by

$$Q = 0.5 q l_0 = 0.5 \times 3\,960 \times 5.45 = 10\,800 \text{ kgf (108 kN)}$$

Each rib is assumed to be reinforced with one welded-bar mat (Fig. XII.2*b*). We adopt two transverse bars of diameter $d_{tr} = 8$ mm ($f_{tr} = 0.5$ cm$^2$) and of total area $F_{tr} = f_{tr}n = 0.5 \times 2 = 1$ cm$^2$.

According to Eq. (VI.43), the design force per metre run of the rib, carried by the transverse bars, is

$$q_{tr} = Q^2/4k_2bh_0^2R_{ten} = 10\,800^2/(4 \times 2 \times 22 \times 33^2 \times 10)$$
$$= 61 \text{ kgf cm}^{-1} \text{ (610 N cm}^{-1}\text{)}$$

It should not be less than

$$q_{tr} = R_{ten}b/2 = 10 \times 22/2 = 110 \text{ kgf cm}^{-1} \text{ (1 100 N cm}^{-1}\text{)}$$

Equation (VI.44) gives the centre-to-centre distance between the transverse bars as

$$u = R_{s,\,tr}F_{tr}/q_{tr} = 1\,700 \times 1/110 = 15.5 \text{ cm}$$

According to Eq. (VI.45)

$$u_{max} = 0.75k_2R_{ten}bh_0^2/Q = 0.75 \times 2 \times 10 \times 22 \times 33^2/10\,800$$
$$= 33 \text{ cm}$$

For structural reasons (see Sec. VI.1), $u \leqslant h/2 = 40/2 = 20$ cm, or $u \leqslant 15$ cm.

We adopt the least of these values, $u = 15$ cm, for the rib portion at the support, $l_0/4$ long.

In the middle of the span, we take $u = 3/4\,h = 3 \times 40/4 = 30$ cm (see Fig. XII.2*b*).

**Slab Design.** The clear space between the longitudinal ribs is $l_1 = 148.5 - 2 \times 12 \approx 125$ cm and that between the transverse ribs is $l_2 = 135 - 2 \times 5 = 125$ cm. The aspect ratio is $l_1/l_2 = 1$, so the slab is regarded as working two-way. For design purposes, we shall allow for the fixed ends of the slab, and assume that the reinforcement in the span and at the supports is the same.

Taking into consideration the self-weight of the slab and recalling the data of Table XII.1, we may write the design load per square metre of the slab 5 cm thick as

$$q = 0.05 \times 2\,500 \times 1.1 + (71 + 13 + 2\,040) + 210$$
$$\approx 2\,475 \text{ kgf m}^{-2} \text{ (24.75 kN m}^{-2}\text{)}$$

Assuming that the slab is reinforced with welded-wire fabric we determine the design bending moment at the support and in the span as prescribed in Sec. X.2:

$$M = ql_1^2/42 = 2\,475 \times 1.25^2/42 = 81 \text{ kgf m } (810 \text{ N m})$$

The design cross-sectional dimensions of the slab are $h_0 = h - a = 5 - 2 = 3$ cm and $b = 100$ cm. Equation (VI.13) gives

$$A_0 = M/bh_0^2R_{pr} = 8\,100/(100 \times 3^2 \times 135) = 0.067$$

For this value of $A_0$, Table VI.1 gives $\eta = 0.965$. Equation (VI.14) gives the cross-sectional area of the reinforcing steel

$$F_s = M/\eta h_0 R_s = 8\,100/(0.965 \times 3 \times 3\,150) = 0.89 \text{ cm}^2$$

Because the ribs contribute to the strength of the slab as a whole, this cross-sectional area may be reduced by 20% (see Chap. X).

In the span, we adopt a welded-wire fabric 200/200/5/5, 1 300 mm long, 5 400 mm wide, and having load-bearing wires of area $F_s = 0.98$ cm$^2$ in either direction. Above the longitudinal ribs, we adopt a welded-wire fabric 250/150/3/4, 550 mm wide, 5 200 mm long, and of area $F_s = 0.84$ cm$^2$ in the transverse direction.

### B. Design in Terms of the Second Group of Limit States

The short-time characteristic load accounts only for

$$p_{sh-t}^{ch}/q^{ch} = (225/3\,285) \times 100 = 6.8\%$$

of the total characteristic load. So, we shall assume that all of the load is applied for a long time.

**Slab Sag.** The total design lod is equal to the characteristic load taken with the overload factor $n = 1$ applied, and is regarded as a long-time load $q_{II} = 3\,285$ kgf m$^{-1}$.

The sag is determined at midspan where the bending moment is

$$M_{II} = q_{II}l_0^2/8 = 32.85 \times 545^2/8$$
$$= 1\,220\,000 \text{ kgf cm } (122 \text{ kN m})$$

For design purposes, we shall assume a T-section (Fig. XII.3). Since the reinforcing steel present does not practically affect the results, we shall not take it into consideration in design calculations.

The cross-sectional area of the slab is

$$F_{tr} \approx F = 22 \times 40 + 128 \times 5 = 880 + 640 = 1\,520 \text{ cm}^2$$

The static moment about the lower side of the section is given by

$$S_{tr} \approx S = 880 \times 20 + 640 \times 37.5$$
$$= 17\,600 + 24\,000 = 41\,600 \text{ cm}^3$$

The distance from the bottom side of the section to the centroid is

$$y_c = S/F = 41\,600/1\,520 = 27.4 \text{ cm}$$

The moment of inertia of the section is given by

$$J_{tr} \approx J = 22 \times 40^3/12 + 22 \times 40 \times 7.4^2 + 128 \times 5^3/12$$
$$+ 128 \times 5 \times 10.1^2 = 226\,520 \text{ cm}^4$$

The moment of resistance about the tensile face of the section is

$$W_0 = J/y_c = 226\,530/27.4 = 8\,250 \text{ cm}^3$$

The moment of resistance of the section with allowance for the inelastic strain in the concrete of the tension zone is found from Eq. (VI.67) where $\gamma = 1.5$ (see Table VI.2):

$$W_{cr} = \gamma W_0 = 1.5 \times 8\,250 = 12\,380 \text{ cm}^3$$

Referring to Fig. VI.21, $e_{0.\,pr} = y_c - a = 27.4 - 7 = 20.4$ cm

Using Eqs. (VI.81) through (VI.86) and the dimensions of the design cross section (see Fig. XII.3), we find the necessary auxiliary quantities. In doing so, we must remember that the compression zone has no reinforcement, $F'_s = 0$, and that the slab is nonprestressed and carries no

longitudinal force:

$$M_{eq} = M^h_{ext} = M, \quad N_0 = 0, \quad M^h_{pr} = 0$$

$$n = E_s/E_c = 2 \times 10^6/(0.29 \times 10^6) = 6.9$$

$$\mu = F_s/bh_0 = 13.88/(22 \times 33) = 0.019$$

$$\gamma' = (b'_f - b)\, h'_f/bh_0 = (150 - 22) \times 5/(22 \times 33) = 0.88$$

$$T = \gamma' (1 - h'_f/2h_0) = 0.88\,[1 - 5/(2 \times 33)] = 0.81$$

$$L = M_{II}/bh_0^2 R_{prII} = 1\,220\,000/(22 \times 33^2 \times 170) = 0.3$$

$$\xi = 1/\{1.8 + [1 + 5\,(L + T)]/10\,\mu n\}$$
$$= 1/\{1.8 + [1 + 5\,(0.3 + 0.81)]/(10 \times 0.019 \times 6.9)\}$$
$$= 0.147$$

$$z_1 = h_0\,[1 - (\gamma' h'_f/h_0 + \xi^2)/2\,(\gamma' + \xi)]$$
$$= 33\,\{1 - (0.88 \times 5/33 + 0.147^2)/[2\,(0.88 + 0.147)]$$
$$= 33 \times 0.92 = 30.3 \text{ cm}$$

$$m = R_{tenII} W_{cr}/M^h_{ext} = 15 \times 12\,380/1\,220\,000 = 0.152$$

$$\psi_s = 1.25 - sm = 1.25 - 0.8 \times 0.152 = 1.13$$

which is larger than 1, so we adopt $\psi_s = 1$.

Equations (VI.89) and (VI.89*a*) give the flexural rigidity of the slab

$$B = h_0 z_1 E_s F_s/\psi_s\,(1 - z_1/e_{s,\,av}) + \psi_c r_{s,\,c}]$$
$$= 33 \times 30.3 \times 2 \times 10^6 \times 13.88/(1 + 0.9 \times 0.86)$$
$$= 15.6 \times 10^9 \text{ kgf cm}^2\,(1.56 \times 10^9 \text{ MPa cm}^4)$$

where $e_{s,\,av} = \infty$ (no prestress, $N_0 = 0$), $\nu = 0.15$ (for long-time loading), $\psi_c = 0.9$, and

$$r_{s,\,c} = E_s F_s/(\gamma' + \xi)\, bh_0 E_c \nu$$
$$= 2 \times 10^6 \times 13.88/[(0.88 + 0.147) \times 22 \times 33 \times 0.29 \times$$
$$\times 10^6 \times 0.15] = 0.86$$

The maximum slab sag at midspan, induced by the uniformly distributed load $q_{II} = 3\,285$ kgf m$^{-1}$ (32.85 kN m$^{-1}$)

is calculated as for a simply supported single-span beam:

$$f = (5/384)\, q_{II} l_0^4/B = 5 \times 32.85 \times 545^4/(384 \times 15.6 \times 10^9)$$
$$= 2.41 \text{ cm}$$

which is less than the maximum safe value $f = 2.5$ cm for $5 \text{ m} \leqslant l \leqslant 10$ m (see Sec. VI.6).

**Crack-Opening Analysis of the Slab Ribs.** This structure has to meet the requirements of the third category of cracking-resistance. This means that the maximum width of cracks under long-time loading with the overload factor $n = 1$ is $a_{cr,\,l\text{-}t} = 0.3$ mm.

Equation (VI.90), where $N_0 = 0$ (no prestress), gives the stress in the reinforcing steel at midspan:

$$\sigma_s = M_{II}/F_s z_1 = 1\,220\,000/(13.88 \times 30.3)$$
$$= 2\,900 \text{ kgf cm}^{-2} \text{ (290 MPa)}$$

The width of normal cracks is found from Eq. (V.13), where $k = 1$, $c_{l\text{-}t} = 1.5$ (long-time loading), $\eta = 1$ (deformed bars), and $\mu = 0.019$:

$$a_{cr}\,(\text{mm}) = k c_{l\text{-}t} \eta\, (\sigma_s/E_s)\, 20\,(3.5 - 100\mu)\, \sqrt[3]{d\,(\text{mm})}$$
$$= 1 \times 1.5 \times 1 \times [2\,900/(2 \times 10^6)]$$
$$\times 20\,(3.5 - 100 \times 0.019)\, \sqrt[3]{22} = 0.195 \text{ mm}$$

which is smaller than the maximum safe value, $a_{cr,\,l\text{-}t} = 0.3$ mm.

The width of inclined cracks at the design rib section lying within the distance $h_0$ of the support is found from Eqs. (VI.91) and (VI.94) where the load is assumed to be acting for a long time, $N_0 = 0$ (no prestress), $\mu_0 = 0$ (no bent-up bars), and $d_{max} = d_{st} = 8$ mm (stirrup diameter):

$$Q_{II} = q_{II}\,(0.5 l_0 - h_0) = 32.85\,(0.5 \times 545 - 33)$$
$$= 7\,870 \text{ kgf (78.7 kN)}$$
$$t = Q_{II}/bh_0 = 7\,870/(22 \times 33) = 10.8$$
$$\mu_{transv} = \mu_{st} = F_{st}/bu = 1/(22 \times 15) = 0.003\,03$$
$$k = (20 - 1\,200\,\mu_{transv})\, 10^3$$
$$= (20 - 1\,200 \times 0.003\,03) \times 10^3 = 16.3 \times 10^3$$

$$a_{cr}\ (\text{mm}) = c_{t\text{-}t} k \left[h_0 + 30 d_{\max}\ (\text{mm})\right] \eta t^2/\mu_{transv} E_s^2$$
$$= 1.5 \times 16.3 \times 10^3\ (330 + 30 \times 8) \times 1$$
$$\times\ 10.8^2/(0.003\,03 \times 2.1^2 \times 10^{12}) = 0.12\ \text{mm}$$

which is smaller than the maximum safe value, $a_{cr,\ t\text{-}t} = 0.3$ mm.

**Supplementary Calculations.** In addition to the design calculations carried out above, it is necessary to calculate the anchorage of the longitudinal reinforcement of the slab ribs at the supports, determine the dimensions of the welds connecting the longitudinal steel to the supporting embedded steel parts, check the slab for erection loads, find the necessary cross-sectional area of lifting eyes, check the slab for strength in transit and erection, etc.

No matter how important they may be, these calculations do not usually affect the design of the slab. They are obligatory in the design of real structures, but are not discussed in detail in this text.

## XII.3. Prestressed Ribbed Roof Slab

We are to design a ribbed roof slab with the ribs congested with prestressed longitudinal reinforcement The necessary dimensions are given in Fig. XII.2*a*.

The concrete brand is M-400, $R_{pr} = 175$ kgf cm$^{-2}$ (17.5 MPa), $R_{pr\,II} = 225$ kgf cm$^{-2}$ (22.5 MPa), $R_{ten} = 12$ kgf cm$^{-2}$ (1.2 MPa), $R_{ten\,II} = 18$ kgf cm$^{-2}$ (1.8 MPa), $E_c = 0.33 \times 10^6$ kgf cm$^{-2}$ ($0.33 \times 10^5$ MPa) (Appendices V and VI), $m_{c1} = 1$ (the slab is to be exposed to an ambient humidity of more than 75%), and the transfer strength of the concrete is $R_0 = 300$ kgf cm$^{-2}$ (30 MPa).

The load-bearing reinforcement of the slab consists of class A-IV deformed bars with $R_s = 5\,000$ kgf cm$^{-2}$ (500 MPa), $R_{s\,II} = 6\,000$ kgf cm$^{-2}$ (600 MPa), and $E_s = 2 \times 10^6$ kgf cm$^{-2}$ ($2 \times 10^5$ MPa) (Appendix VI). The service factor for steel having no definite yield is $m_{s4} > 1$. The slab is pretensioned by electrical heating. The slab is heat-cured, and the difference in temperature between the steel and the forms (according to Manufacturer's data) is $\Delta t = 20$°C. In the zone subjected to compression under

load, the slab has no prestressed reinforcement, $F'_{pr} = 0$. The structure should meet the third category of cracking-resistance requirements, that is, cracks may open for a long time as wide as $a_{cr,\ l\text{-}t} = 0.3$ mm, with the load multiplied by the overload factor $n = 1$.

**A. Design in Terms of the First Group of Limit States**

We determine the cross-sectional dimensions of the ribs, check the ribs for inclined-section strength, and design the slab proper according to Sec. XII.2.

Then we check the slab for normal-section strength. On the basis of trial calculations (omitted here), we adopt four class A-IV longitudinal bars 16 mm in diameter of area $F_{pr} = 8.04$ cm$^2$ (see Appendix VIII). The design section for normal-section strength analysis is that at midspan.

As a first trial, we determine the depth of the compression zone for $m_{s4} = 1$, using Eqs. (VI.2) and (VI.9):

$$x_{(m_{s4}=1)} = F_{pr}R_s m_{s4}/b'_f R_{pr}$$
$$= 8.04 \times 5\,000 \times 1/(150 \times 175) = 1.54 \text{ cm}$$

For $h_0 = 35$ cm, the relative depth of the compression zone is

$$\xi = x/h_0 = 1.54/35 = 0.044$$

Equations (VI.5) through (VI.7) give $\xi_R$

$$\sigma_A = R_s + 5\,000 = 5\,000 + 5\,000$$
$$= 10\,000 \text{ kgf cm}^{-2} (1\,000 \text{ MPa})$$

$$\xi_0 = 0.85 - 0.000\,8R_{pr} = 0.85 - 0.0008 \times 175 = 0.71$$

$$\xi_R = \xi_0/[1 + \sigma_A (1 - \xi_0/1.1)/5\,000]$$
$$= 0.71/[1 + 10\,000 (1 - 0.71/1.1)/5\,000] = 0.415$$

Using Eq. (VI.4), we determine the service factor for high-strength steel, where $\bar{m}_{s4}$ for class A-IV reinforcement is taken as 1.2 (see Sec. VI.3):

$$m_{s4} = \bar{m}_{s4} - (\bar{m}_{s4} - 1)\,\xi/\xi_R$$
$$= 1.2 - (1.2 - 1)\,0.044/0.415 = 1.18$$

Equation (VI.9) gives the depth of the compression zone $x$ for $m_{s4} = 1.18$

$$x = x_{(m_{s4}=1)}\, m_{s4} = 1.54 \times 1.18 = 1.82 \text{ cm}$$

which is smaller than

$$\xi_R h_0 = 0.415 \times 35 = 14.5 \text{ cm}$$

The slab is checked for strength subject to the condition in Eq. (VI.11). The load-bearing capacity of the slab in terms of the bending moment

$$F_{pr} R_s m_{s4} (h_0 - 0.5x) = 8.04 \times 5\,000 \times 1.18\,(35 - 0.5 \times 1.82)$$
$$= 1\,615\,000 \text{ kgf cm } (161.5 \text{ kN m})$$

exceeds the design bending moment $M = 1\,470\,000$ kgf cm (147 kN m).

### B. Design in Terms of the Second Group of Limit States

**Determining the Prestressing Force.** The tensile prestress $\sigma_0$ in the prestressed reinforcement is adopted as prescribed in Sec. III.2 as a maximum safe prestress for reinforcing bars of length $l \approx 6$ m, pretensioned by electrical heating. The inaccuracy of tensioning taken in design calculations is

$$p = 300 + 3\,600/l = 300 + 3\,600/6 = 900 \text{ kgf cm}^{-2} \text{ (90 MPa)}$$

Equation (III.8) gives the prestress

$$\sigma_0 = R_{sII} - p = 6\,000 - 900 = 5\,100 \text{ kgf cm}^{-2} \text{ (510 MPa)}$$

*Early Losses.* The losses due to the relaxation of stress in bar reinforcement tensioned by electrical heating are given by

$$\sigma_1 = 0.03\,\sigma_0 = 0.03 \times 5\,100 = 153 \text{ kgf cm}^{-2} \text{ (15.3 MPa)}$$

The losses due to the temperature difference in the course of curing are

$$\sigma_2 = 12.5\Delta t = 12.5 \times 20 = 250 \text{ kgf cm}^{-2} \text{ (25 MPa)}$$

The losses due to the deformation of anchors $\sigma_3^{(pre)}$ and forms $\sigma_5$ are not taken into consideration for tensioning by electrical heating, because they have already been accounted for in determining steel elongation.

With allowance for all the above losses, the prestress and the prestressing force are given by

$$\sigma_{01} = \sigma_0 - \sigma_1 - \sigma_2 = 5\,100 - 153 - 250$$
$$= 4\,697 \text{ kgf cm}^{-2} \text{ (469.7 MPa)}$$
$$N_{01} = \sigma_{01} F_{pr} = 4\,697 \times 8.04 = 37\,800 \text{ kgf (378 kN)}$$

With the values of $F$, $J$, and $y_c$ taken from Sec. XII.2, the stress in the concrete at the centroid of the tensile steel is given by

$$\sigma_{c,pr} = (N_{01}/F + N_{01} e_{01}^2/J)$$
$$= (37\,800/1\,520 + 37\,800 \times 22.4^2/226\,530)$$
$$= 108.6 \text{ kgf cm}^{-2} \text{ (10.86 MPa)}$$

where $e_{01} = y_c - a_{pr} = 27.4 - 5 = 22.4$ cm.

Because $\sigma_{c,pr}/R_0 = 108.6/300 = 0.36 < 0.6$, the losses due to high-rate (instantaneous) creep are calculated using the first line of Eq. (III.21) with the coefficient 0.85 which accounts for the heat curing of the concrete:

$$\sigma_6 = 0.85 \times 500 \sigma_{c,pr}/R_0 = 0.85 \times 500 \times 0.36$$
$$= 153 \text{ kgf cm}^{-2} \text{ (15.3 MPa)}$$

Thus, the early losses are equal to

$$\sigma_{loss\ 1} = \sigma_1 + \sigma_2 + \sigma_6 = 153 + 250 + 153$$
$$= 556 \text{ kgf cm}^{-2} \text{ (55.6 MPa)}$$

*Late Losses.* According to Table III.2, the losses due to shrinkage are

$$\sigma_8^{(pre)} = 350 \text{ kgf cm}^{-2} \text{ (35 MPa)}$$

Recalling the previous calculations, the losses due to creep are found from Eq. (III.22)

$$\sigma_9^{(pre)} = 0.85 \times 2\,000 \times 0.36 = 611 \text{ kgf cm}^{-2} \text{ (61.1 MPa)}$$

So, the late losses are given by

$$\sigma_{loss\ 2} = \sigma_8^{(pre)} + \sigma_9^{(pre)} = 350 + 611$$
$$= 961 \text{ kgf cm}^{-2} \text{ (96.1 MPa)}$$

The prestress in the reinforcing steel with allowance for all losses is

$$\sigma_{02} = \sigma_0 - (\sigma_{loss\ 1} + \sigma_{loss\ 2})$$
$$= 5\ 100 - (556 + 961) = 3\ 583 \text{ kgf cm}^{-2} \text{ (358.3 MPa)}$$

The prestressing force is

$$N_{02} = \sigma_{02} F_{pr} = 3\ 583 \times 8.04 \approx 29\ 000 \text{ kgf (290 kN)}$$

**Normal-Cracking Analysis of the Slab Ribs.** Equation (VI.65) (see Fig. VI.21 and the data for $W_0$ and $F$ in Sec.XII.2) gives

$$r_k = 0.8 W_0 / F_{tr} \approx 0.8 \times 8\ 250/1\ 520 = 4.4 \text{ cm}$$

According to Eq. (VI.66), the moment induced by the internal forces in the tension zone immediately before cracking is

$$M_{cr} = R_{tenII} W_{cr} + M_{pr}^k = R_{tenII} W_{cr} + m_{ac} N_{02} (e_{01} + r_k)$$
$$= 18 \times 12\ 380 + 0.9 \times 29\ 000\ (22.4 + 4.4)$$
$$= 223\ 000 + 700\ 000 = 923\ 000 \text{ kgf cm (92 kN m)}$$

The moment induced by the external forces $M_{ext}^k = M_{II} = 1\ 220\ 000$ kgf cm (122 kN m) (see Sec. XII.2). Since $M_{ext}^k > M_{cr}$ [see the condition of Eq. (VI.64)], normal cracks occur in the slab.

**Slab Sag.** The design bending moment and the geometrical characteristics of the slab section have been found in Sec. XII.2:

$$M_{II} = 1\ 220\ 000 \text{ kgf cm (122 kN m)}$$
$$F = 1\ 520 \text{ cm}^2$$
$$S = 41\ 600 \text{ cm}^3$$
$$y_c = 27.4 \text{ cm}$$
$$J = 226\ 530 \text{ cm}^4$$
$$W_0 = 8\ 250 \text{ cm}^3$$
$$W_{cr} = 12\ 380 \text{ cm}^3$$
$$e_{0,pr} = 20.4 \text{ cm}$$

Equations (VI.81) through (VI.88) give the necessary auxiliary quantities

$$n = E_s/E_c = 2 \times 10^6/(0.33 \times 10^6) = 6.1$$

$$\mu = F_s/bh_0 = 8.04/(22 \times 35) = 0.01$$

$$\gamma' = (b_f' - b)\, h_f'/bh_0 = (150 - 22) \times 5/(22 \times 35) = 0.83$$

$$T = \gamma' (1 - h_f'/2h_0) = 0.83\,[1 - 5/(2 \times 35)] = 0.77$$

$$L = M_{II}/bh_0^2 R_{prII} = 1\,220\,000/(22 \times 35^2 \times 225) = 0.2$$

$$\xi = 1/\{1.8 + [1 + 5\,(L + T)]/10\mu n\}$$

$$= 1/\{1.8 + [1 + 5\,(0.2 + 0.77)]/(10 \times 0.01 \times 6.1)\}$$

$$= 0.088$$

$$z_1 = h_0\,[1 - (\gamma' h_f'/h_0 + \xi^2)/2\,(\gamma' + \xi)]$$

$$= 35\,\{1 - (0.83 \times 5/35 + 0.088^2)/[2\,(0.83 + 0.088)]\}$$

$$= 35 \times 0.93 = 32.5 \text{ cm}$$

$$e_{s,\,av} = M_{eq}/N_0 = 1\,220\,000/29\,000 = 42 \text{ cm}$$

$e_{s,\,av}/h_0 = 42/35 = 1.2$, but, according to a relevant standard, this ratio should be not less than $1.2/s = 1.2/0.8 = 1.5$ (where $s$ is taken as 0.8 for long-time loading);

$$m = R_{tenII}\, W_{cr}/(M_{ext}^k - M_{pr}^k)$$

$$= 18 \times 12\,380/(1\,220\,000 - 720\,000) = 0.45$$

where

$$M_{pr}^k = N_{02}\,(e_{0,\,pr} + r_k) = 29\,000\,(20.4 + 4.4)$$

$$= 720\,000 \text{ kgf cm } (72 \text{ kN m})$$

$$\psi_s = 1.25 - sm - (1 - m^2)/[(3.5 - 1.8\,m)\, e_{s,\,av}/h_0$$

$$= 1.25 - 0.8 \times 0.45 - (1 - 0.45^2)/[(3.5 - 1.8 \times 0.45)1.5]$$

$$= 0.69$$

Equations (VI.89*a*) and (VI.89) give the flexural rigidity of the slab

$$r_{s,c} = E_s F_s/(\gamma' + \xi)\, bh_0 E_c \nu$$

$$= 2 \times 10^6 \times 8.04/[(0.83 + 0.088)\, 22 \times 35 \times 0.33 \times 10^6 \times 0.15] = 0.46$$

$$B = h_0 z_1 E_s F_s/[\psi_s (1 - z_1/e_{s,\,av}) + \psi_c r_{s,\,c}]$$
$$= 35 \times 32.5 \times 2 \times 10^6 \times 8.04/[0.69 (1 - 32.5/42) + 0.9 \times 0.46] = 32.2 \times 10^9 \text{ kgf cm}^{-2} (3.22 \times 10^9 \text{ MPa})$$

At $q_{II} = 32.85$ kgf cm$^{-1}$ (328.5 N cm$^{-1}$), the maximum sag at midspan is

$$f = (5/384)\, q_{II} l_0^4/B = 5 \times 32.85 \times 545^4/(384 \times 32.2 \times 10^9) = 1.17 \text{ cm}$$

This is smaller than the maximum safe value $f = 2.5$ cm (see Sec. VI.6) for 5 m $\leqslant l \leqslant$ 10 m, even without taking into consideration the hog due to prestressing, so it need not be calculated.

**Normal Crack-Opening Analysis of the Slab Ribs.** The stress in the longitudinal reinforcement is determined from Eq. (VI.90) where $M = M_{II}$, $N_0 = N_{02}$, and $e_{s,\,pr} = 0$ (the distance from $N_0$ to the tensile reinforcement):

$$\sigma_s = [M - N_0(z_1 - e_{s,\,pr})]/F_s z_1$$
$$= (1\,220\,000 - 29\,000 \times 32.5)/(8.04 \times 32.5)$$
$$= 1\,070 \text{ kgf cm}^{-2} \quad (107 \text{ MPa})$$

The width of normal cracks is found from Eq. (V.13), where $k = 1$, $c_{l\text{-}t} = 1.5$, $\eta = 1$, $\mu = 0.01$, and $d = 16$ mm

$$a_{cr}\,(\text{mm}) = k c_{l\text{-}t} \eta\, (\sigma_s/E_s)\, 20\, (3.5 - 100\mu) \sqrt[3]{d\,(\text{mm})}$$
$$= 1 \times 1.5 \times 1\, [1\,070/(2 \times 10^6)] \times [20\,(3.5 - 100 \times 0.01)] \sqrt[3]{16} = 0.1 \text{ mm}$$

which is smaller than the maximum safe value, $a_{cr,\,l\text{-}t} = 0.3$ mm.

**Inclined Crack-Opening Analysis.** According to an appropriate standard, inclined crack-opening analysis is carried out for sections lying within at least the distance $h_0$ of the support. At this distance, the shearing force (for $n = 1$) is $Q_{II} = 7\,870$ kgf (78 700 N) (see Sec. XII.2). The region to be checked is chosen at the centroid of the section. The normal stress induced by the prestressing force in the concrete is

$$\sigma_x = N_{02}/F_{tr} \approx 29\,000/1\,520 = 19 \text{ kgf cm}^{-2} (1.9 \text{ MPa})$$

The tangential stress is

$$\tau_{xy} = Q_{II} S_{tr}/J_{tr} b \approx 7\,870 \times 41\,600/(226\,530 \times 22)$$
$$= 65.7 \text{ kgf cm}^{-2} \text{ (6.57 MPa)}$$

The principal tensile and compressive stresses are given by Eq. (VI.68), where $\sigma_y = 0$:

$$\left.\begin{matrix}\sigma_{princ,\,com}\\ \sigma_{princ,\,ten}\end{matrix}\right\} = 0.5\sigma_x \pm \sqrt{0.25\sigma_x^2 + \tau_{xy}^2}$$
$$= 0.5 \times 19 \pm \sqrt{0.25 \times 19^2 + 65.7^2}$$
$$= \begin{cases} 75.2 \text{ kgf cm}^{-2} \text{ (7.52 MPa) (compression)} \\ -56.2 \text{ kgf cm}^{-2} \text{ (} -5.62 \text{ MPa) (tension)} \end{cases}$$

Since $\sigma_{princ,\,ten} = 56.2$ kgf cm$^{-2}$ (5.62 MPa) $> R_{tenII} =$ $= 18$ kgf cm$^{-2}$ (1.8 MPa), we may infer that inclined cracks will occur in the slab.

**Inclined Crack-Opening Analysis.** The width of inclined sections in the rib is given by Eqs. (VI.91) through (VI.94) where $\mu_0 = 0$, $\mu_{transv} = 0.003$, $k = 16.4 \times 10^3$, and $d_{max}$ $= 8$ mm as in Sec. XII.2:

$$t = Q/bh_0 - 0.25N_0/F$$
$$= 7\,870/(22 \times 35) - 0.25 \times 29\,000/1\,520$$
$$= 5.4 \text{ kgf cm}^{-2} \text{ (0.54 MPa)}$$

$$a_{cr} = c_{l\text{-}t} k (h_0 + 30d_{max}) \eta t^2/\mu_{transv} E_s^2$$
$$= 1.5 \times 16.4 \times 10^3 (350 + 30 \times 8) \times 1 \times 5.4^2/[0.003 (2.1 \times 10^6)^2] = 0.032 \text{ mm}$$

which is smaller than the maximum safe value $a_{cr,\,l\text{-}t}$ $= 0.3$ mm.

**Supplementary Calculations.** We shall omit supplementary calculations concerned with the strength and cracking-resistance analysis of the ribs near supports and that of the slab proper in the transfer of the prestress, transit, and erection.

## XII.4. Square Roof Slab

We are to design a square slab for a tank roof, having nominal dimensions of $4 \times 4$ m, with ribs round the perimeter, and supported directly on columns at the corners (see Figs. XII.1 and XII.4).

As a first trial, we choose the following rib dimensions $h = l/10 = 400/10 = 40$ cm and $b = 0.4h = 0.4 \times 40 = 16$ cm.

We assume that the rib is 15 cm wide at the bottom and 20 cm wide at the slab bottom. The slab is taken to be 10 cm thick. The weight of a square metre of the slab is 250 kg.

The concrete brand is M-300, $R_{pr} = 135$ kgf cm$^{-2}$ (13.5 MPa), $R_{ten} = 10$ kgf cm$^{-2}$ (1 MPa) (see Appendix IV), and $m_{c1} = 1$. The welded fabric and longitudinal load-bearing reinforcement of the welded-bar mats in the ribs

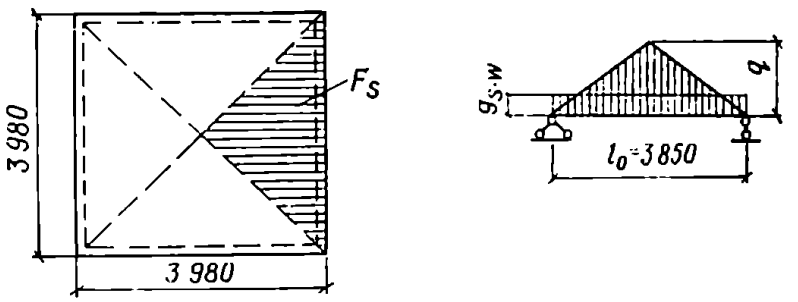



Fig. XII.4. To the design of a square roof slab

consists of class A-III deformed bars: $R_s = 3\,400$ kgf cm$^{-2}$ (340 MPa) for $d = 6$ to 8 mm and $R_s = 3\,600$ kgf cm$^{-2}$ (360 MPa) for $d = 10$ to 40 mm. The transverse and erection reinforcement is class A-I steel: $R_s = 2\,100$ kgf cm$^{-2}$ (210 MPa) and $R_{s.tr} = 1\,700$ kgf cm$^{-2}$ (170 MPa) (see Appendix VI).

**Strength Analysis of the Slab Proper.** The design load per square metre of the slab is determined as for the ribbed slab of Sec. XII.2 (see Table. XII.1). In this case, we have $q = 2\,620$ kgf m$^{-2}$ (26 200 N m$^{-2}$) (for the first group of limit states).

The effective span of the slab is taken as the clear space between the longitudinal ribs, $l_0 = 398 - 20 \times 2 = 358$ cm.

We determine the bending moment in the span, $M_{sp}$ and the bending moment at the support, $M_{sup}$, as prescribed in Sec. X.2, assuming that the moments at the suppor

and in the span are equal and that half of the lower bars are cut off within 1/4 of the supports:

$$M = M_{sp} = M_{sup} = ql_0^2/42$$

$$= 2\,620 \times 3.58^2/42 = 800 \text{ kgf m } (8\,000 \text{ N m})$$

The effective depth and the design width of the slab are

$$h_0 = h - a = 10 - 2 = 8 \text{ cm}$$

$$b = 100 \text{ cm}$$

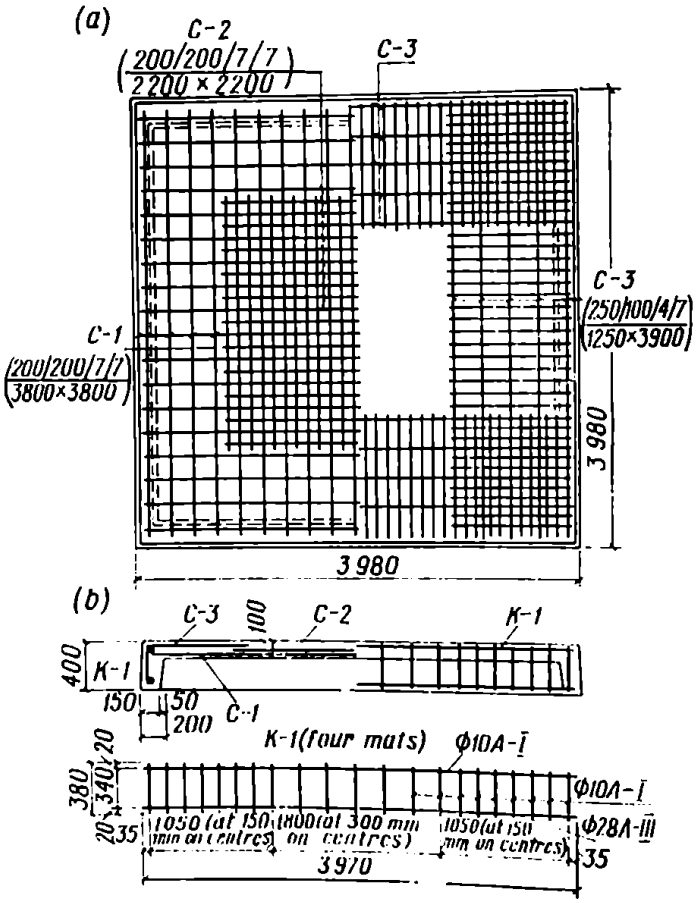



Fig. XII.5. Square roof slab
(a) slab reinforcement; (b) rib reinforcement

Equation (VI.13) and Table VI.1 give

$$A_0 = M/R_{pr}bh_0^2$$
$$= 80\,000/(135 \times 100 \times 8^2) = 0.093$$
$$\eta = 0.952$$

Equation (VI.14) yields

$$F_a = M/R_a\eta h_0 = 80\,000/(3\,400 \times 0.952 \times 8) = 3.1 \text{ cm}^2$$

We adopt two pieces of welded-wire fabric

$$\frac{200/200/7/7}{3\,800 \times 3\,800} \text{ and } \frac{200/200/7/7}{2\,200 \times 2\,200}$$

of area $F_a = 1.93 \times 2 = 3.86$ cm² in the span, and one piece of welded-wire fabric

$$\frac{250/100/4/7}{1\,250 \times 3\,900}$$

of area $F_a = 3.86$ cm² at the supports.

The distribution of reinforcement in the slab is shown in Fig. XII.5*a*.

**A. Design in Terms of the First Group of Limit States**

The design load per metre run in the middle of the slab rib (see Fig. XII.4) is as follows:

—the maximum distributed triangular load due to the slab proper

$$q = 2\,620 \times 2 = 5\,240 \text{ kgf m}^{-1} (524 \text{ kN m}^{-1})$$

—the uniformly distributed load due to the self-weight of the rib

$$g_{s\text{-}w} = 0.5\,(0.15 + 0.2)\,0.3 \times 2\,500 \times 1.1$$
$$= 168 \text{ kgf m}^{-1} (1.68 \text{ kN m}^{-1})$$

The effective span of the slab rib is

$$l_0 = 400 - 15 = 385 \text{ cm}$$

The bending moment and shearing force induced by the design loads are given by

$$M = ql_0^2/12 + g_{s\text{-}w}l_0^2/8$$
$$= 5\,240 \times 3.85^2/12 + 168 \times 3.85^2/8$$
$$= 6\,480 + 310 = 6\,790 \text{ kgf m } (67.9 \text{ kN m})$$
$$Q = ql_0/4 + g_{s\text{-}w}l_0/2$$
$$= 5\,240 \times 3.85/4 + 168 \times 3.85/2$$
$$= 5\,050 + 325 = 5\,375 \text{ kgf } (53.75 \text{ kN})$$

**Normal-Section Strength Analysis.** As prescribed in Sec. VI.3, analysis is carried out for the flange of a T-section

$$b_f' = b + 6h_f' = 20 + 6 \times 10 = 80 \text{ cm}$$

which is smaller than $l_0/3 = 385/3 = 128$ cm.

The effective depth of the rib is

$$h_0 = h - a = 40 - 4 = 36 \text{ cm}$$

The T-section is analyzed as a rectangular section of width $b = 80$ cm by Eqs. (VI.13) and (VI.14) and using the data of Table VI.1, because the neutral axis obviously lies within the flange. Accordingly, we get

$$A_0 = M/R_{pr}bh_0^2 = 679\,000/(135 \times 80 \times 36^2) = 0.049$$
$$\eta = 0.975$$
$$F_s = M/R_s\eta h_0 = 679\,000/(3\,600 \times 0.975 \times 36) = 5.37 \text{ cm}^2$$

We adopt one class A-III bar 28 mm in diameter of area $F_s = 6.16$ cm$^2$ (Appendix VIII).

**Inclined-Section Strength Analysis of the Rib.** Let us check if the conditions defined by Eqs. (VI.34) and (VI.35) are satisfied at $b_r = 17.5$ cm (the average width of the rib):

$$k_1R_{tcn}b_rh_0 \leqslant Q \leqslant 0.35\,R_{pr}b_rh_0$$
$$0.6 \times 10 \times 17.5 \times 36 < 5\,375 < 0.35 \times 135 \times 17.5 \times 36$$
$$3\,780 \text{ kgf } (37.8 \text{ kN}) < 5\,375 \text{ kgf } (53.75 \text{ kN}) < 29\,700 \text{ kgf } (297 \text{ kN})$$

As is seen, the rib is sufficiently wide and should have transverse reinforcement. For each rib, we adopt one welded-

bar mat ($n = 1$) (see Fig. XII.5*b*) with class A-I transverse bars of diameter $d_{tr} = 10$ mm ($F_{tr} = 0.785$ cm$^2$) [the condition for weldability with longitudinal reinforcement 28 mm in diameter is met (see Table II.2)]. Equation (VI.43) gives

$$q_{tr} = Q^2/4k_2bh_0^2R_{ten} = 5\,375^2/(4 \times 2 \times 17.5 \times 36^2 \times 10)$$
$$= 16 \text{ kgf cm}^{-1} \quad (160 \text{ N cm}^{-1})$$

but not less than

$$q_{tr} = R_{ten}b_r/2 = 10 \times 17.5/2 = 87.5 \text{ kgf cm}^{-1} \ (875 \text{ N cm}^{-1})$$

The spacing of the transverse bars is given by Eqs. (VI.44) and (VI.45) as

$$u = R_{s,\,tr}F_{s,\,tr}/q_{tr} = 1\,700 \times 0.785/87.5 = 15.3 \text{ cm}$$
$$u_{max} = 0.75k_2R_{ten}bh_0^2/Q$$
$$= 0.75 \times 2 \times 10 \times 17.5 \times 36^2/5\,375 = 63 \text{ cm}$$

For structural reasons, the spacing should be taken to be not larger than $h/2 = 40/2 = 20$ cm, nor larger than 15 cm, whichever is the smaller.

We adopt $u = 15$ cm (the least value) at the supports and $u = 3/4\ h = 30$ cm at midspan.

The distribution of the rib reinforcement is shown in Fig. XII.5*b*.

The slab is not analyzed in terms of the second group of limit states, because from past experience it is known that this design of slab meets the applicable conditions always.

## XII.5. Roof Girder

We are to design a six-span girder having 6-m spans (the centre-to-centre distance between the columns) for the roof of the tank shown in Fig. XII.1*a* and *b*. The concrete brand is M-300, $m_{c1} = 1$, $R_{pr} = 135$ kgf cm$^{-2}$ (13.5 MPa), and $R_{ten} = 10$ kgf cm$^{-2}$ (1 MPa) (Appendix IV). The longitudinal load-bearing reinforcement is class A-III deformed bars with $R_s = 3\,600$ kgf cm$^{-2}$ for $d \geqslant 10$ mm. The transverse reinforcement is class A-II steel with $R_s = 2\,700$ kgf cm$^{-2}$ (270 MPa) and $R_{s,\,tr} = 2\,150$ kgf cm$^{-2}$ (215 MPa) (Appen-

dix VI). The load per square metre of the roof is taken from Table XII.1.

The girder is designed only in terms of the first group of limit states, because it is known to satisfy the conditions in terms of the second group always.

Since the share of the live load is insignificant, we regard all of the load as dead and sustained.

With 6-m spans, the design load per metre run of the girder is:

— due to the roof,

$$2\,640 \times 6 = 15\,840 \text{ kgf m}^{-1} (158.4 \text{ kN m}^{-1})$$

—due to the self-weight of the girder,

$$0.4 \times 0.7 \times 2\,500 \times 1.1 = 770 \text{ kgf m}^{-1} (7.7 \text{ kN m}^{-1})$$

The total load is $q = 16\,610$ kgf m$^{-1} \approx 16.6$ tf m$^{-1}$ = $= 166$ kN m$^{-1}$.

The effective span is taken as the centre-to-centre distance between the columns, that is, $l = 6$ m.

**Design Bending Moments and Shearing Forces.** With a large number of spans, we adopt a five-span design system (Fig. XII.6*a*). Assuming that the structure is in an elastic state ($\alpha_B = 0.105$ and $\alpha_C = 0.071$ according to relevant reference sources), the bending moments in the girder above the intermediate supports $B$ and $C$ are given by

$$M_B = \alpha_B q l^2 = -0.105 \times 16.6 \times 6^2$$

$$= -62.7 \text{ tf m } (627 \text{ kN m})$$

$$M_C = \alpha_C q l^2 = -0.071 \times 16.6 \times 6^2 = -41.8 \text{ tf m } (418 \text{ kN m})$$

In statically indeterminate reinforced concrete structures, it is legitimate to re-distribute the internal moments to a certain extent.

To standardize the design of joints between the component members of the girder (Fig. XII.6 and XII.7), the moments at the supports may be equalized as

$$M_B^{eq} = M_C = -41.8 \text{ tf m } (418 \text{ kN m})$$

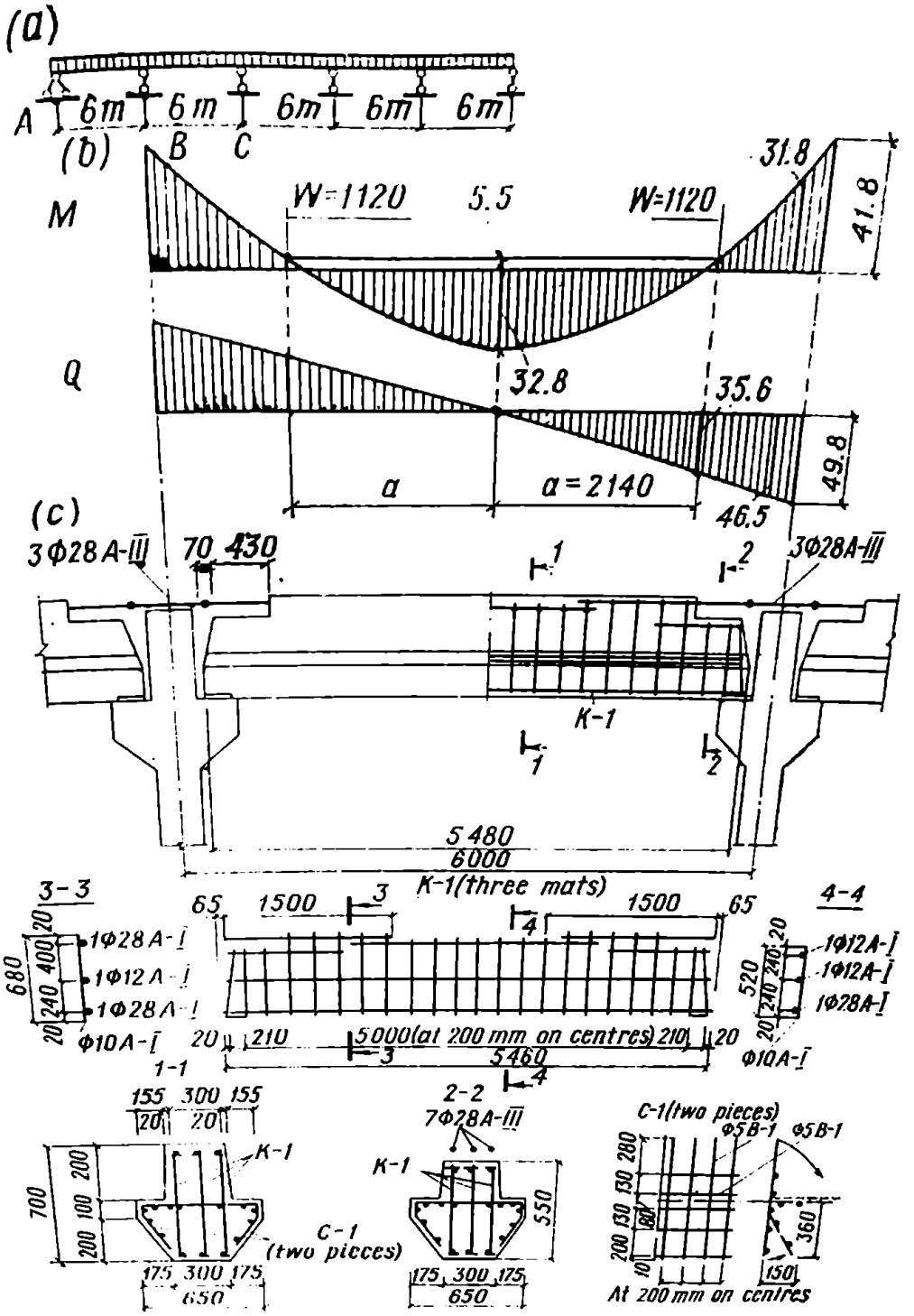



Fig. XII.6. Roof girder

(a) loading system; (b) bending moment and shearing force diagram in intermediate spans; (c) precast girder member for intermediate spans

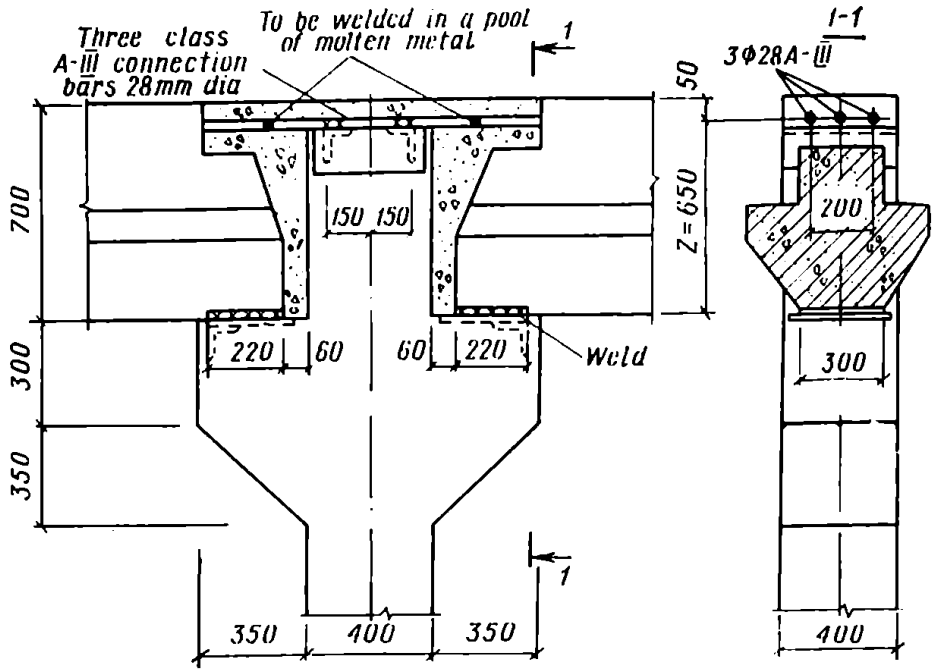



Fig. XII.7. Joint between the girder and the column

With equalized bending moments, the shearing forces are given by

$$Q_A = ql/2 + M_B^{eq}/l$$
$$= 16.6 \times 6/2 - 41.8/6 = 42.8 \text{ tf } (428 \text{ kN})$$
$$Q_B^{left} = -ql/2 + M_B^{eq}/l$$
$$= -16.6 \times 6/2 - 41.8/6 = -56.8 \text{ tf } (568 \text{ kN})$$
$$Q_B^{right} = -Q_C^{left} = ql/2$$
$$= 16.6 \times 6/2 = 49.8 \text{ tf } (498 \text{ kN})$$

Now, we determine the maximum bending moments $M_1$, $M_2$, and $M_3$ in the spans. The maximum bending moment $M_1$ is applied in the marginal span within the distance $x_0$ of the marginal support

$$x_0 = Q_A/q = 42.8/16.6 = 2.58 \text{ m}$$
$$M_1 = Q_A x_0 - qx_0^2/2 = 42.8 \times 2.58 - 16.6 \times 2.58^2/2$$
$$= 55.2 \text{ tf m } (552 \text{ kN m})$$

The moments in the intermediate spans are

$$M_2 = M_3 = ql^2/8 - |M_B^{eq}|$$
$$= 16.6 \times 6^2/8 - 41.8 = 32.8 \text{ tf m } (328 \text{ kN m})$$

The design bending moment and shearing force diagrams for the intermediate spans are shown in Fig. XII.6*b*. The figure gives the ordinates corresponding to the faces of the supports. Assuming that the columns are 40 × 40 cm in cross section, we get

$$M_{B,f} = - |M_B^{eq}| + Q_B^{right} \times 0.2$$
$$= -41.8 + 49.8 \times 0.2 = 31.8 \text{ tf m } (-318 \text{ kN m})$$
$$Q_{B,f}^{right} = Q_B^{right} (0.5l - 0.2)/0.5l$$
$$= 49.8 (3 - 0.2)/3 = 46.5 \text{ tf } (465 \text{ kN})$$

The maximum shearing force in the marginal span is

$$Q_{B,f}^{left} = Q_B^{left} [(l - x_0) - 0.2]/(l - x_0)$$
$$= -56.8 [(6 - 2.58 - 0.2)]/(6 - 2.58)$$
$$= -53.5 \text{ tf } (-535 \text{ kN})$$

**Determining the Cross-Sectional Dimensions of the Girder.** The cross-sectional dimensions of the girder are adopted so as to carry the moments and shearing forces in the marginal spans where they reach their maximum. According to Sec.VI.3, we assume the relative depth of the compression zone as $\xi = 0.4$. In Table VI.1, this value of $\xi$ corresponds to $A_0 = 0.32$. Assuming $b = 30$ cm, Eq. (VI.17) yields

$$h_0 = \sqrt{M_1/A_0 b R_{pr}} = \sqrt{5\,520\,000/(0.32 \times 30 \times 135)} = 65 \text{ cm}$$

The total depth of the girder is

$$h = h_0 + a = 65 + 5 = 70 \text{ cm}$$

We take the girder depth to be $h = 70$ cm, because it fits a standard size. Thus, the effective depth of the girder is 65 cm.

Next, we check the cross section of the girder for $Q_{B,l}^{left}$ subject to the conditions defined by Eqs. (VI.34) and (VI.35)

$$k_1 R_{ten} b h_0 \leqslant Q_{B,l}^{left} \leqslant 0.35 R_{pr} b h_0$$

$$0.6 \times 10 \times 30 \times 65 < 53\,500 < 0.35 \times 135 \times 30 \times 65$$

$$11\,700 \text{ kgf (117 kN)} < 53\,500 \text{ kgf (535 kN)}$$

$$< 92\,000 \text{ kgf (920 kN)}$$

As is seen, the conditions of Eqs. (VI.34) and (VI.35) are satisfied, the cross-sectional dimensions are acceptable, and transverse reinforcement is required by design. The same dimensions are adopted for the intermediate spans of the girder.

**Normal-Section Strength Analysis.** According to Eqs. (VI.13) and (VI.14) and the data of Table VI.1, we have for the marginal span:

$$A_0 = M_1/bh_0^2 R_{pr} = 5\,520\,000/(30 \times 65^2 \times 135) = 0.323$$

$$\eta = 0.797$$

$$F_s = M_1/\eta h_0 R_s = 5\,520\,000/(0.797 \times 65 \times 3\,600)$$

$$= 29.6 \text{ cm}^2$$

We adopt three class A-III bars 28 mm in diameter and three class A-III bars 22 mm in diameter of total area $F_s = 29.9$ cm$^2$ (see Appendix VIII).

In the intermediate spans:

$$A_0 = 3\,280\,000/(30 \times 65^2 \times 135) = 0.192$$

$$\eta = 0.892$$

$$F_s = 3\,280\,000/(0.892 \times 65 \times 3\,600) = 15.7 \text{ cm}^2$$

We take three class A-III bars 28 mm in diameter of area $F_s = 18.47$ cm$^2$.

The moment at the supports, $M_f = 31.8$ tf m (318 kN m), is close to $M_2 = 32.8$ tf m (328 kN m), so here we likewise adopt three class A-III bars 28 mm in diameter.

**Inclined-Section Strength Analysis.** We shall limit ourselves to the intermediate span of the girder (see Fig. XII.6*c*). For a welded-bar mat with longitudinal bars 28 mm in diameter, we adopt transverse bars of $d_{tr} = 10$ mm and $f_{tr} =$

$= 0.785$ cm$^2$ (Appendix VIII). This diameter meets the conditions for welding stated in Table II.2. The number of transverse bars in the section (the number of welded-bar mats) is $n = 3$. Therefore, $F_{tr} = f_{tr}n = 0.785 \times 3 = 2.355$ cm$^2$.

The design load per unit length of the girder, carried by the transverse bars, is given by Eq. (VI.43) as

$$q_{tr} = Q^2/(4k_2bh_0^2R_{ten})$$
$$= 46\,500^2/(4 \times 2 \times 30 \times 65^2 \times 10) = 213 \text{ kgf cm}^{-1}$$
$$(2\,130 \text{ N cm}^{-1})$$

It ought not to be smaller than

$$q_{tr} = R_{ten}b/2 = 10 \times 30/2$$
$$= 150 \text{ kgf cm}^{-1} (1\,500 \text{ N cm}^{-1})$$

Equation (VI.44) gives the spacing between the transverse bars

$$u = R_{s,\,tr}F_{tr}/q_{tr}$$
$$= 2\,150 \times 2.355/213 = 24 \text{ cm}$$

The maximum spacing is given by Eq. (VI.45) as

$$u_{\max} = 0.75k_2R_{ten}bh_0^2/Q$$
$$= 0.75 \times 2 \times 10 \times 30 \times 65^2/46\,500 = 41 \text{ cm}$$

From structural considerations (see Sec. VI.1), for $h > 45$ cm, the spacing between the transverse bars should be not larger than $h/3 = 70/3 = 23$ cm.

For the portions near the supports, a quarter of the span long, we adopt the least value $u = 20$ cm. In the middle, this distance may be taken as $u = 2 \times 20 = 40$ cm which is smaller than $3/4\ h = 52$ cm.

The intermediate span of the girder is shown in Fig. XII.6c

Next, we locate the cutoff point for the longitudinal load bearing reinforcement above the supports. The distance from midspan to the theoretical cutoff point (see Fig. XII.6*l* is given by

$$a = \sqrt{2(M_2 + M_{3\varnothing 12})/q} = \sqrt{2(32.8 + 5.5)/16.6} = 2.14 \text{ m}$$

where $M_{3\phi 12}$ is the moment carried by the upper bars 12 mm in diameter, belonging to three welded-bar mats and placed for structural reasons [the calculations are omitted (see Fig. XII.6c)].

The shearing force at this section is $Q_{a=2.14} = 35.6$ tf (356 kN).

Equations (VI.51) and (VI.52) give the embedment length for the cut-off bars:
at

$$q_{tr,w} = R_s F_{tr}/u$$
$$= 2\,700 \times 2.355/20 = 318 \text{ kgf cm}^{-1} \quad (3\,180 \text{ N cm}^{-1})$$

we have

$$w = Q_{(a=2.14)}/2q_{tr,w} + 5d$$
$$= 35\,600/(2 \times 318) + 5 \times 2.8 = 70 \text{ cm}$$
$$w = 20d = 20 \times 2.8 = 56 \text{ cm}^2$$

We adopt the largest value $w = 70$.

**The Joint between a Girder and a Column.** The precast components of a girder rest on column cantilevers (Fig. XII.7). At the joint, the internal couple of the support moment of the girder is resisted by the connection bars at the top, and by the welds of the embedded steel parts in the girder and column cantilevers at the bottom. The arm of the internal couple is

$$z = 70 - 5 = 65 \text{ cm}$$

The couple components are

$$N = M_{B,f}/z = 31.8/0.65 = 49 \text{ tf (490 kN)}$$

The minimum cross-sectional area of the embedded parts in the girder and cantilevers, made of class C38/23 steel [$R = 2\,100$ kgf cm$^{-2}$ (210 MPa)], is

$$F = N/R = 49\,000/2\,100 = 23.4 \text{ cm}^2$$

The shearing force between the supports of the girder and cantilever is given by

$$N_{sh} = N - Qf = 49 - 46.5 \times 0.15 = 42 \text{ tf (420 kN)}$$

where $Q$ is the normal force applied to the support and $f$ is the coefficient of friction for steel against steel.

The welds connecting the supports should be designed to carry this force.

## XII.6. Column and Footing

We are to design a column and a column footing for the tank of XII.1. The standard column is 40 × 40 cm in cross section and has an effective length of $l_0 = 5 - 0.6 - 0.2 = 4.2$ m. The concrete brand is M-200, $m_{c1} = 1$, $R_{pr} = 90$ kgf cm$^{-2}$ (9 MPa), and $R_{ten} = 7.5$ kgf cm$^{-2}$ (0.75 MPa) (Appendix IV). The reinforcement is class A-II deformed bars with $R_{s,com} = 2\,700$ kgf cm$^{-2}$ (270 MPa) (Appendix VI). The foundation material is coarse dense sands. According to Appendix XVI, the design bearing strength of the soil is $R_0 = 6$ kgf cm$^{-2}$ (0.6 MPa).

**Design of the Column.** Since the share of live load is insignificant, all of the load is assumed as dead and sustained. The load is composed of the girder reaction [the maximum force $Q_B^{left}$ and $Q_B^{right}$ at the first intermediate support as reckoned from the girder end (see Sec. XII.5)]:

$$56.8 + 49.8 = 106.6 \text{ tf (1 066 kN)}$$

and the self-weight of the column:

$$0.4 \times 0.4 \times 5.1 \times 2.5 \times 1.1 = 2.2 \text{ tf (22 kN)}$$

So, the total load is

$$N_{l\text{-}t} \approx 109 \text{ tf (1 090 kN)}$$

The column resists a longitudinal axial force. It is a member of a statically indeterminate structure, so the design eccentricity of the longitudinal force is equal to the accidental eccentricity (see Sec. IV.1). Since $l_0/h = 420/40 = 10.5$, the design may be carried out as prescribed in Sec. IV.2.

Let us check if the column should be reinforced. According to Eqs. (IV.2) and (IV.3), at $F_s = 0$ and $m = 1$ (the size of the column cross section exceeds 20 cm), the following

condition should be satisfied

$$N \leqslant \varphi R_{pr} F$$

or

$$109\ 000 \text{ kgf } (1\ 090 \text{ kN}) < 0.882 \times 90 \times 40 \times 40 = 127\ 000 \text{ kgf } (1\ 270 \text{ kN})$$

where $\varphi = \varphi_c = 0.882$ is looked up in Table IV.1.

As is seen, no reinforcing steel is required by design.

To suit structural considerations, we adopt a minimum percentage of reinforcement $\mu = 0.4\%$, that is, the cross-sectional area of the reinforcing steel should be at least $F_s = 0.004 \times 40 \times 40 = 6.4$ cm$^2$. As a result, we take four class A-II bars 16 mm in diameter of area $F_s = 8.04$ cm$^2$ (Appendix VIII). The transverse bars in the reinforcing cage are chosen to be 6 mm in diameter, which meets the conditions for welding specified in Table II.2. The spacing between the transverse bars (see Sec. IV.1) should be $u \leqslant 20d = 20 \times 1.6 = 32$ cm, so we adopt $u = 30$ cm.

The column is shown in Fig. XII.8.

**Design of the Column Cantilever.** The maximum girder reaction is applied to the intermediate column from the marginal span (see Sec. XII.5), $Q_B^{left} = 56.8$ tf (568 kN). With the roof slabs arranged as shown in Fig. XII.1*b*, one-eight of the load is carried directly by the column. The cantilever has to resist the force applied by the girder

$$Q_{cant} = 56.8 \times 7/8 = 49.7 \text{ tf } (497 \text{ kN})$$

The supporting platform of the girder has the following dimensions: $b_g = 30$ cm and $b_{pl} = 22$ cm (Figs. XII.7 and XII.9). The pressure beneath the platform is

$$\sigma = Q_{cant}/(b_g b_{pl}) = 49\ 700/(30 \times 22) = 75 \text{ kgf cm}^{-2} \text{ (7.5 MPa)}$$

which is smaller than

$$R_{pr} = 90 \text{ kgf cm}^{-2} \text{ (9 MPa)}$$

The supporting platform of the girder is welded to its longitudinal reinforcement, and that of the cantilever to its upper horizontal reinforcement.

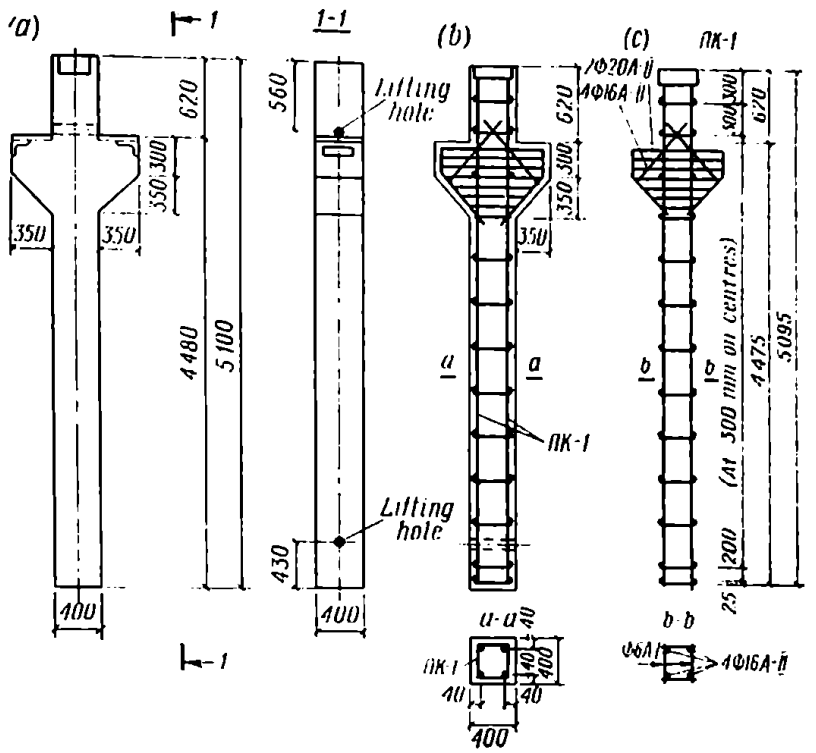



Fig. XII.8. Column

(a) formwork dimensions; (b) distribution of reinforcement; (c) welded-bar reinforcing cage

A relevant standard requires that short cantilevers ($l_{cant} \leqslant 0.9h_{0,cant}$) at the joints of frame structures should be designed subject to

$$Q_{cant} \leqslant 1.25 \times 1.2R_{ten}bh_{0,cant}^2/a$$

and

$$Q_{cant} \leqslant 2.5R_{ten}bh_{0,cant}$$

Hence, the necessary effective depth of the cantilever is

$$h_{0,cant} = \sqrt{Q_{cant}a/(1.5R_{ten}b)}$$
$$= \sqrt{49\,700 \times 17/(1.5 \times 7.5 \times 40)} = 43 \text{ cm}$$

or

$$h_{0,cant} = Q_{cant}/2.5R_{ten}b$$
$$= 49\,700/(2.5 \times 7.5 \times 40) = 60 \text{ cm}$$

We adopt $h_{cant} = 65$ cm (and $h_{0,cant} = 60$ cm).

The upper horizontal reinforcement of area $F_s$ is subjected to compression due to the force $N_g$ induced by the moment applied by the girder, $N_g = M_{B,f}/z = 31.8/(70-4) = -48$ tf (—480 kN) (see Sec. XII.5 and Fig. XII.9), and to tension caused by the force $N_{cant}$ induced by the cantilever moment, equal (at $\eta \approx 0.9$) to

$$N_{cant} = Qa/\eta h_{0,cant}$$

$$= 46.5 \times 0.17/(0.9 \times 0.6) = 14.7 \text{ tf } (147 \text{ kN})$$

Because $N_g$ is larger than $N_{cant}$, the horizontal reinforcement in the cantilever should be placed for structural reasons. Its area should be at least $F_s = 0.002 \times h_{0,cant} b = 0.002 \times 60 \times 40 = 4.8$ cm$^2$, so we adopt two class A-II bars 20 mm in diameter of area $F_s = 6.28$ cm$^2$.

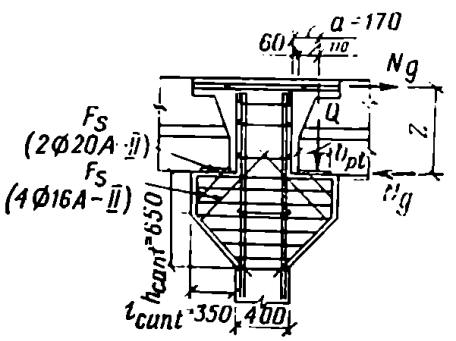



Fig. XII.9. Column head reinforcement

An appropriate standard requires that, irrespective of the results yielded by calculations, cantilevers with $h > 2.5a$ must be reinforced with bent-up bars and horizontal stirrups throughout. The cross-sectional area of these bars should be not less than $F_0 = 0.002bh_{0,cant} = 0.002 \times 40 \times 60 = 4.8$ cm$^2$. We adopt four class A-II bars 16 mm in diameter of area $F_0 = 8.04$ cm$^2$, and class A-II stirrups 6 mm in diameter spaced 100 mm apart.

The distribution of reinforcement in the column top is shown in Fig. XII.9.

**Determining the Dimensions of the Footing.** The footing dimensions are determined so as to carry the design pressure $R_s$ exerted on the soil by the characteristic load multiplied by the overload factor $n = 1$.

Assuming that the footing may be about 1.5 m square and the embedment depth is $h = 0$, the equation given in Chap. X, where $k_1 = 0.125$ (for sands) and $b_1 = 1$ m, gives

the design pressure exerted on the foundation material:

$$R_s = R_0 [1 + k_1 (b - b_1)/b_1] (h + h_1)/2h_1$$
$$= 6 [1 + 0.125 (1.5 - 1)/1] 1/2$$
$$= 3.2 \text{ kgf cm}^{-2} (0.32 \text{ MPa})$$

The loads imposed on the footing base include the characteristic load $N_{II} = 92$ tf (920 kN) (including the weight of the footing) and the design load $N = 109$ tf (1 090 kN) (leaving out the weight of the footing).

With a square footing, the footing base area and the footing side are

$$F_f' = N_{II}/R_s = 92/32 = 2.87 \text{m}^2$$

$$a = \sqrt{F_f} = \sqrt{2.87} = 1.7 \text{ m}$$

We adopt a footing base 170 cm square of area $F_f = 2.89 \text{ cm}^2$.

**Design of the Footing.** Let us check the footing for strength. The soil pressure beneath the footing base caused by the design load (leaving out the weight of the footing) is

$$p_s = N/F_f = 109\,000/(170 \times 170)$$
$$= 3.8 \text{ kgf cm}^{-2} (0.38 \text{ MPa})$$

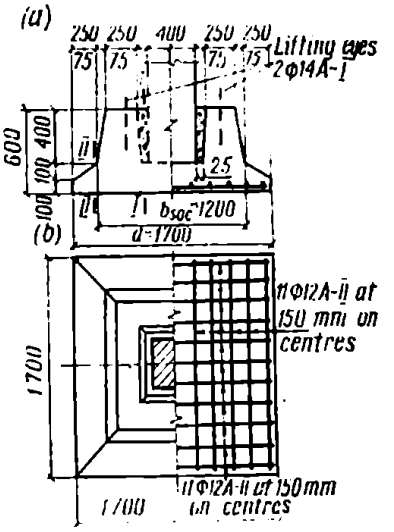



Fig. XII.10. Column footing
(a) side view and sectional view; (b) plan and distribution of reinforcement at the bottom

The effective depth of the footing, necessary to resist crushing is

$$h_0 = -0.5 b_{col} + 0.5 \sqrt{N/(R_{ten} + p_s)}$$
$$= -0.5 \times 40 + 0.5 \sqrt{109\,000/(7.5 + 3.8)} = 29 \text{ cm}$$

The total depth of the footing is

$$h_f = h_0 + a = 29 + 5 = 34 \text{ cm}$$

To this must be added the embedment depth of the column in the socket and the minimum base thickness in the socket,

so the overall footing depth should be

$$h_f = h_{col} + 20 \text{ cm} = 40 + 20 = 60 \text{ cm}$$

Thus, the overall depth of the footing is $h_f = 60$ cm ($h_0 = 53$ cm); the other dimensions are given in Fig. XII.10.

The bending moments at sections *I-I* (at the column face) and *II-II* (where the footing base meets the socket) are as follows

$$\begin{aligned} M_{\text{I}} &= 0.125 p_s (a - b_{col})^2 a \\ &= 0.125 \times 3.8 (170 - 40)^2 \times 170 \\ &= 1\,360\,000 \text{ kgf cm (136 kN m)} \\ M_{\text{II}} &= 0.125 p_s (a - b_{soc})^2 a \\ &= 0.125 \times 3.8 (170 - 120)^2 \times 170 \\ &= 202\,000 \text{ kgf cm (20.2 kN m)} \end{aligned}$$

The amount of reinforcement necessary at the footing base is

$$\begin{aligned} F_{s1} &= M_{\text{I}}/(0.9 R_s h_0) \\ &= 1\,360\,000/(0.9 \times 2\,700 \times 53) = 10.6 \text{ cm}^2 \\ F_{s2} &= M_{\text{II}}/0.9 R_s h_0' \\ &= 202\,000/(0.9 \times 2\,700 \times 13) = 6.4 \text{ cm}^2 \end{aligned}$$

We adopt eleven class A-II bars 12 mm in diameter of area $F_s = 12.4$ cm$^2$ (Appendix VIII).

The percent of reinforcement at sections I-I and II-II is

$$\begin{aligned} \mu_1 &= 100 F_{s1}/(b_{soc} h_0) \\ &= 100 \times 12.4/(120 \times 53) = 0.19\% > \mu_{min} = 0.1\% \\ \mu_2 &= 100 F_{s2}/a h_0' \\ &= 100 \times 12.4/(170 \times 13) = 0.56\% > \mu_{min} = 0.1\% \end{aligned}$$

## XII.7. Wall of a Precast Reinforced Concrete Rectangular Tank

We are to design the wall of the precast reinforced concrete tank shown in Fig. XII.1*a* and *b*. The wall is 20 cm thick, and the wall panels have a nominal width (the centre-to-

centre distance between joints) of 3 m and a structural width of 2.8 m. The vertical joints between the wall panels are 20 cm wide. They are filled with M-300 concrete and given a layer of shotcrete about 50 cm wide applied from the inside. The wall panels are installed into a slot in the tank floor at the bottom, and the welded fittings in the panels are welded to those in the precast roof at the top.

Panel material: M-200 concrete, $R_{pr} = 90$ kgf cm$^{-2}$ (9 MPa), $m_{c1} = 1$ (see Appendix IV).

Panel reinforcement: welded-wire fabric with class A-III vertical load-bearing reinforcement, $R_s = 3\,600$ kgf cm$^{-2}$ (360 MPa) for $d \geqslant 10$ mm (Appendix VI).

**Loads Imposed on the Wall.** When the tank is evacuated, the wall resists the earth pressure from the outside. During hydrostatic tests, it carries the internal liquid pressure from the inside in the absence of backfill.

The wall panel separately should be designed to carry the bending moment induced by the earth and water pressure. For design purposes it is regarded as a single beam fixed at the floor and hinged at the roof (Fig. XII.11*a*). The roof does not always impose any vertical load on the wall, so its effect is not taken into account in design calculations.

The design load due to the water pressure per metre run of the wall at the section where the wall is fixed in the floor (Fig. XII.11*b*) is

$$p_w = n\gamma h = 1.1 \times 1 \times 4.3 = 4.7 \text{ tf m}^{-1} (47 \text{ kN m}^{-1})$$

The design load due to the earth pressure applied respectively at the top and bottom (Fig. XII.11*c*) is as follows

$$p_{s1} = n\gamma_s h_1 \operatorname{tg}^2 (45° - \varphi/2)$$

$$= 1.2 \times 1.8 \times 1.4 \times \operatorname{tg}^2 (45° - 30°/2)$$

$$= 1.1 \text{ tf m}^{-1} (11 \text{ kN m}^{-1})$$

$$p_{s2} = p'_{s2} + p_{s1} = n\gamma_s h \operatorname{tg}^2 (45° - \varphi/2) + p_{s1}$$

$$= 1.2 \times 1.8 (4.8 - 0.5) \operatorname{tg}^2 (45° - 30°/2) + 1.1$$

$$= 4.3 \text{ tf m}^{-1} (43 \text{ kN m}^{-1})$$

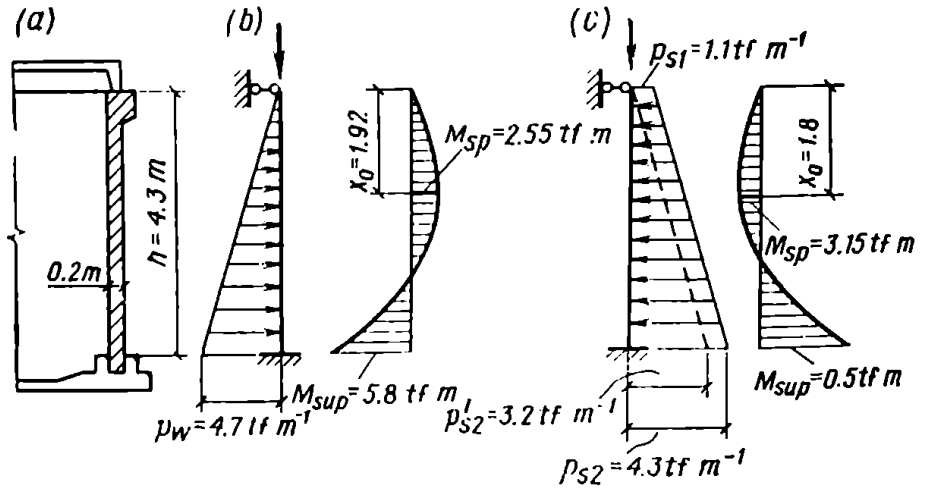



Fig. XII.11. To the design of the wall of a rectangular tank
(*a*) sectional view; (*b*) loading system and bending moment diagram due to water pressure; (*c*) loading system and bending moment diagram due to lateral earth pressure

**Bending Moments in the Wall, Amount of Reinforcement.** The internal water pressure causes the following bending moments at the bottom (Fig. XII.11*b*).

$$M_{sup,\,w} = -p_w h^2/15$$
$$= -4.7 \times 4.3^2/15 = -5.8 \text{ tf m } (-58 \text{ kN m})$$

The maximum moment in the span (at the distance $x_0 = 0.447h = 0.447 \times 4.3 = 1.92$ m from the top) is

$$M_{sp,\,w} = p_w h^2/33.3$$
$$= 4.7 \times 4.3^2/33.3 = 2.55 \text{ tf m } (25.5 \text{ kN m})$$

The bending moment induced by the earth pressure at the bottom (Fig. XII.11*c*) is

$$M_{sup,\,s} = -p'_{s2} h^2/15 - p_{s1} h^2/8$$
$$= -3.2 \times 4.3^2/15 - 1.1 \times 4.3^2/8$$
$$= -6.5 \text{ tf m } (-65 \text{ kN m})$$

The maximum bending moment in the span is

$$M_{sp,\,s} = (p'_{s2}/10 + 3p_{s1}/8)\,hx_0 - p'_{s2}x_0^3/6h - p_{s1}x_0^2/2$$
$$= (3.2/10 + 3 \times 1.1/8) \times 4.3 \times 1.8$$
$$-3.2 \times 1.8^3/(6 \times 4.3) - 1.1 \times 1.8^2/2$$
$$= 3.15 \text{ tf m } (31.5 \text{ kN m})$$

Here, the distance from the top to the point of moment application $x_0 = 1.8$ m is found from the following equation

$$p'_{s2}h/10 + 3p_{s1}h/8 - p'_{s2}x_0^2/2l - p_{s1}x_0 = 0$$

Since the bending moments induced by the internal water pressure and the lateral earth pressure are almost the same, we adopt the same amount of reinforcement near both the

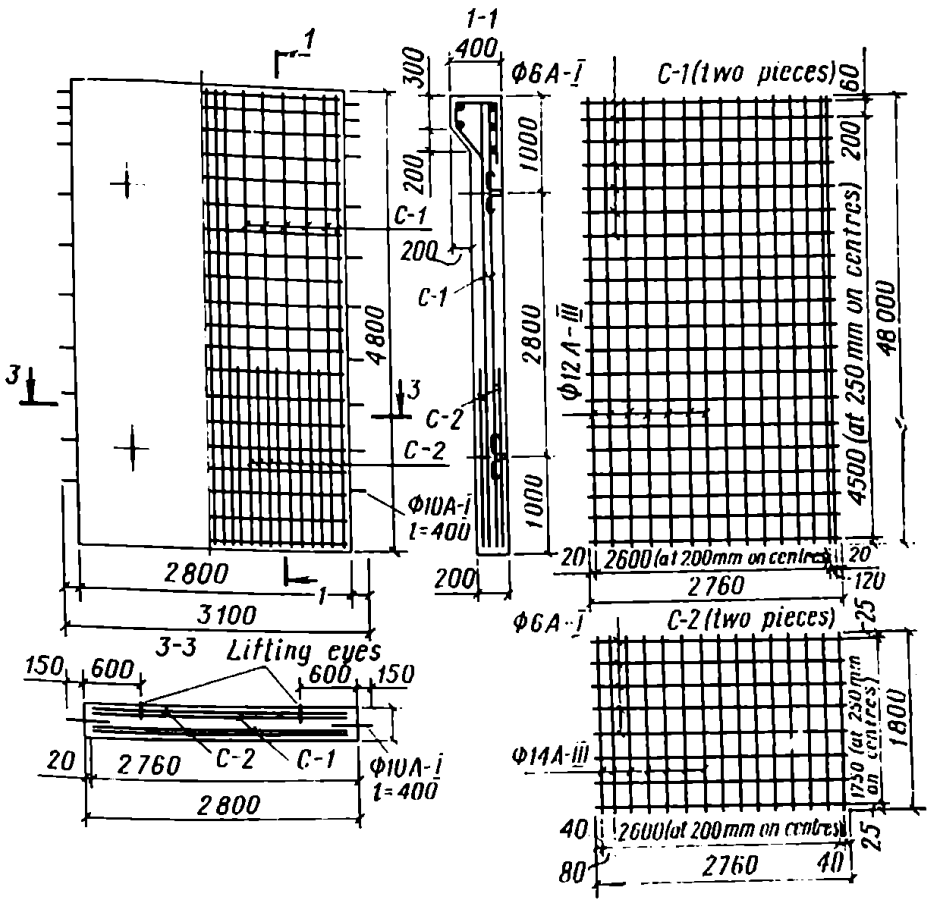



Fig. XII.12. Wall panel of a rectangular tank

inner and the outer surfaces of the wall. In design calculations, we assume the larger values of the bending moments. The wall is checked for normal-section strength as a singly reinforced slab having the following dimensions: $b = 100$ cm and $h_0 = h - a = 20 - 3 = 17$ cm.

For the section at floor level, Eqs. (VI.13) and (VI.14) and Table VI.1 give

$$A_0 = M/(bh_0^2 R_{pr}) = 650\,000/(100 \times 17^2 \times 90) = 0.25$$

$$\eta = 0.852$$

$$F_s = M/(\eta h_0 R_s)$$

$$= 650\,000/(0.852 \times 17 \times 3\,600) = 12.5 \text{ cm}^2$$

For the section in the span, we get

$$A_0 = 315\,000/(100 \times 17^2 \times 90) = 0.121$$

$$\eta = 0.935$$

$$F_s = 315\,000/(0.935 \times 17 \times 3\,600) = 5.5 \text{ cm}^2$$

We adopt double welded-wire fabric on either side of the section. One of the pieces consists of five class A-III wires 12 mm in diameter of area $F_s = 5.65$ cm$^2$ and covers the entire height of the wall, the other consists of five class A-III wires 14 mm in diameter of area $F_s = 7.69$ cm$^2$ and is installed at the bottom.

Horizontal bars are placed for structural reasons.

Crack-opening analysis is omitted.

The wall panel is shown in Fig. XII.12.

## XII.8. Wall of a Precast Circular Tank

We are to design the prestressed concrete wall of a covered semi-buried circular tank. The tank is surrounded by backfill, its diameter is 18 m ($r = 9$ m), the height is $H = 4.8$ m, and the wall thickness is $\delta = 14$ cm. The wall is hinged to the floor. The vertical pressure per metre run of the wall bottom is taken as $N_1 = 3$ tf m$^{-1}$ (30 kN m$^{-1}$) in the absence of backfill, and as $N_2 = 10$ tf m$^{-1}$ (100 kN m$^{-1}$) in the presence of backfill.

Panel material: M-200 concrete, $m_{c1} = 1$. The joints between the panels are filled with M-300 concrete.

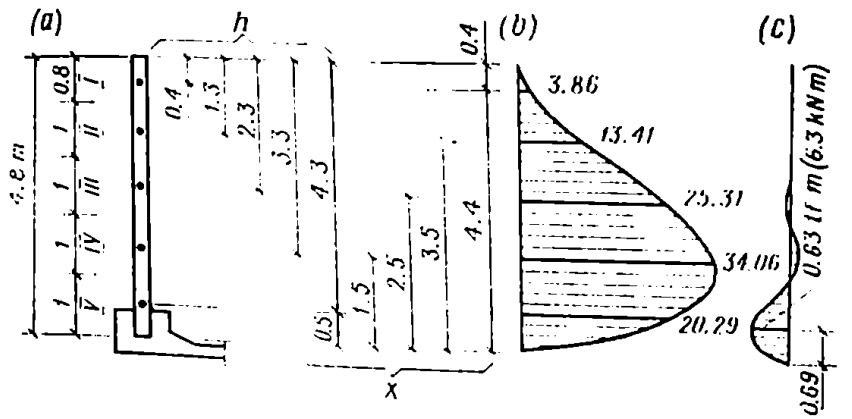



Fig. XII.13. To the design of the wall of a circular tank
(a) division into zones; (b) circular force diagram (the forces are stated in t); (c) vertical bending moment diagram

Hoop reinforcement: prestressed class A-IV deformed bars, $R_{sII} = 6\,000$ kgf cm$^{-2}$ (600 MPa), $R_s = 5\,000$ kgf cm$^{-2}$ (500 MPa), and $E_s = 2 \times 10^6$ kgf cm$^{-2}$ ($2 \times 10^5$ MPa) (Appendix V). Each hoop consists of three sections (see Fig. XII.14*b*). The reinforcing steel is posttensioned by electric heating.

The prestress is transferred to the wall when the concrete of the wall panels attains a strength $R_0 = R = 200$ kgf cm$^{-2}$ (20 MPa) and that in the joints between the panels (compacted by vibration) attains a strength $R_0 = 0.7R = 0.7 \times 300 = 210$ kgf cm$^{-2}$ (21 MPa).

**Circular Forces and Vertical Bending Moments in the Wall.** Equation (XI.9) gives the design circular tensile forces in the wall to be

$$T = T_0 - (2r/s)\, Q_{fr}\eta_1$$

where $T_0$ and $Q_{fr}$ are found from Eqs. (XI.3) and (XI.8) and $\eta_1$ is looked up in Table XI.2.

Let us divide the wall into zones 1 m high, except the upper zone which is 0.8 m high (Fig. XII.13*a*). For each zone, the design force is taken equal to that in the middle of the zone.

Equation (XI.5) gives the value of $s$ to be

$$s = 0.76\sqrt{r\delta} = 0.76\sqrt{9 \times 0.14} = 0.86 \text{ m}$$

**Table XII.2. Circular Forces in the Tank Wall**

| Zone Nos. | $h$ (from wall top), m | $x$ (from wall bottom), m | $T_0 = np_\omega r$, tf | $\varphi = x/s$ | $\eta_1$ | $(2r/s)\, Q_{fr}\eta_1$ | | $T = T_0 - (2r/s)Q_{fr}\eta_1$ | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | $Q_{fr1} = 1.5$ | $Q_{fr2} = 2.28$ | $T_1$ | $T_2$ |
| | | | | | | tf or 10 kN | | | |
| I | 0.4 | 4.4 | 3.96 | 5.1 | 0.002 2 | 0.069 | 0.1 | 3.89 | 3.86 |
| II | 1.3 | 3.5 | 12.87 | 4.06 | −0.011 4 | −0.036 | −0.54 | 12.91 | 13.41 |
| III | 2.3 | 2.5 | 22.77 | 2.9 | −0.053 5 | −1.58 | −2.54 | 24.35 | 25.31 |
| IV | 3.3 | 1.5 | 32.67 | 1.75 | −0.029 2 | −0.91 | −1.39 | 33.56 | 34.06 |
| V | 4.3 | 0.5 | 42.67 | 0.58 | 0.488 | 15.28 | 22.28 | 26.58 | 20.29 |
| | 4.8 | 0 | 47.52 | 0 | 1 | 31.32 | 47.52 | 16.2 | 0 |

The friction force $Q_{fr}$ at the bottom of the wall is determined in the absence and in the presence of backfill with the coefficient of friction $f = 0.5$:

$$Q_{fr1} = fN_1 = 0.5 \times 3 = 1.5 \text{ tf (15 kN)}$$

$$Q_{fr2} = fN_2 = 0.5 \times 10 = 5 \text{ tf (50 kN)}$$

Now, we check the condition

$$Q_{fr} \leqslant p_w s/2 = 5.28 \times 0.86/2 = 2.28 \text{ tf (22.8 kN)}$$

where

$$p_w = n\gamma H = 1.1 \times 1 \times 4.8 = 5.28 \text{ tf m}^{-1} \text{ (52.8 kN m}^{-1}\text{)}$$

Since $Q_{fr2} = 5$ tf (50 kN) exceeds $Q_{fr2} = 2.28$ tf (22.8 kN), in our further calculations we adopt, according to Eq. (XI.10), $Q_{fr2} = 2.28$ tf (22.8 kN). The circular forces are given in Table XII.2.

The circular force diagram is shown in Fig. XII.13*b*.

Equation XI.11 gives the vertical (meridional) bending moment in the wall as

$$M = Q_{fr} s\eta_2$$

where $\eta_2$ is looked up in Table XI.2.

The maximum meridional moment (Fig. XII.13*c*) appears when $Q_{fr}^{max} = Q_{fr2} = 2.28$ tf (22.8 kN) and acts in the section with the ordinate $x_0$ for which $\eta_2^{max} = 0.322$ and $\varphi = 0.8$. Consequently,

$$x_0 = \varphi s = 0.8 \times 0.86 = 0.69 \text{ m}$$

whence

$$M_{max} = Q_{fr}^{max} s\eta_2^{max}$$

### A. Design in Terms of the First Group of Limit States

Equation (V.26) gives the cross-sectional area of the hoop reinforcement to be

$$F_{pr} = T/R_{s,pr}$$

where $R_{s,pr} = R_s m_{s4} = 5\,000 \times 1.2 = 6\,000$ kgf cm$^{-2}$ (600 MPa) ($m_{s4} = 1.2$ is taken from Sec. VI.3).

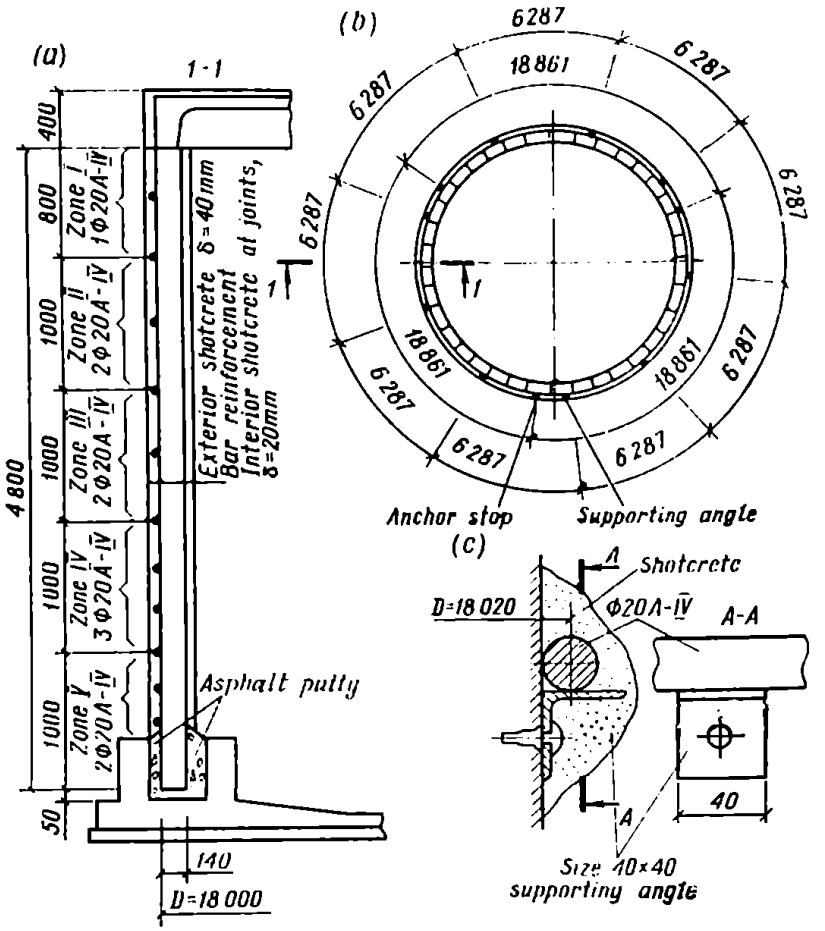



Fig. XII.14. Circular tank wall

(a) vertical sectional view; (b) plan showing joints between wall panels and between prestressed bars and supporting angles for reinforcement; (c) detail

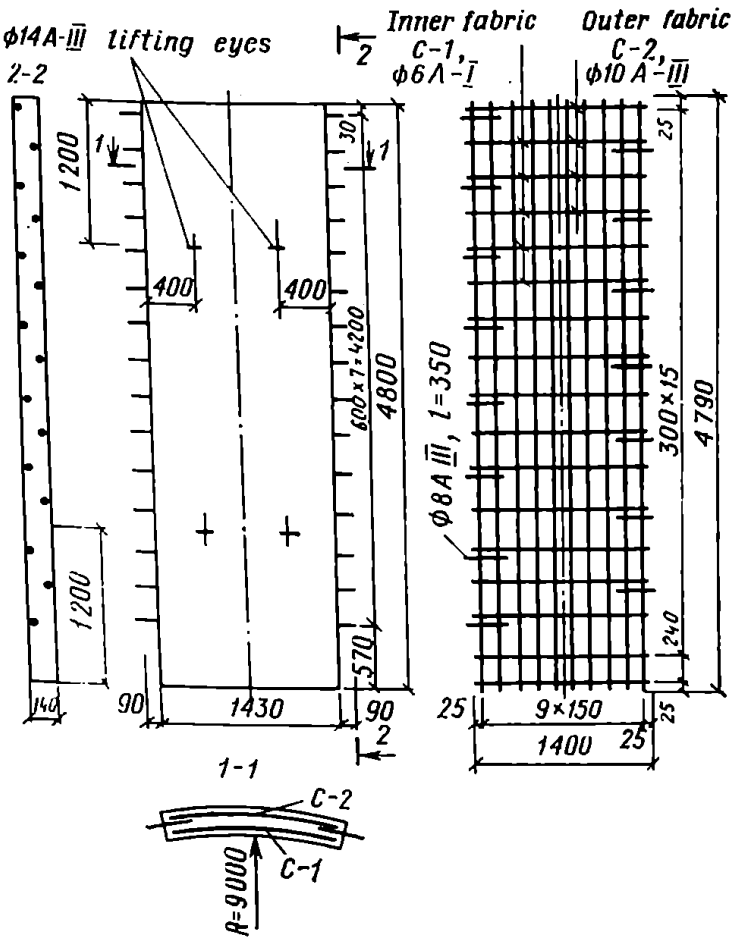



Fig. XII.15. Wall panel of a circular tank

Since the requirements for the cracking resistance of the structure are very stringent, the cross-sectional area of the reinforcement should be increased by about 40% (see Table XII.3).

The distribution of reinforcement in each zone of the wall is shown in Fig. XII.14*a*.

According to the bending moment diagram (Fig. XII.13*c*), the wall requires an insignificant amount of vertical reinforcement. Therefore, we adopt the steel area from structural considerations or according to the results of a check on erec-

**Table XII.3. Cross-Sectional Area of the Hoop Reinforcement in the Tank Wall**

| Zone Nos. | $T$, tf | $F_{pr} = 1.4T/R_{s,\,pr}$, cm² | Bars adopted and their cross-sectional area for each zone, cm² |
|---|---|---|---|
| I | 3.86 | 0.9 | 1 bar 20 mm in dia. 3.14 |
| II | 13.41 | 3.1 | 2 bars 20 mm in dia. 6.28 |
| III | 25.31 | 5.9 | 2 bars 20 mm in dia. 6.28 |
| IV | 34.06 | 8 | 3 bars 20 mm in dia. 9.41 |
| V | 20.29 | 4.7 | 2 bars 20 mm in dia. 6.28 |

*Note.* $R_{s,\,pr} = 6\,000$ kgf cm$^{-2}$ (600 MPa).

tion loads. The construction of the precast wall panel is shown in Fig. XII.15.

### B. Design in Terms of the Second Group of Limit States

The wall is to meet the first category of cracking resistance requirements (see Sec. III.1). In our case, no cracks are allowed under the design loads multiplied by the same overload factor $n > 1$ as in strength analysis (see Sec. III.1). Design in terms of the force $T$ is done for zone IV as the most stressed one (see Table XII.2).

**Prestressing Force.** For a prestressed bar whose length $l = 2\pi r/3 = 2 \times 3.14 \times 9/3 = 18.8$ m is equal to the distance between the bar joints, the maximum prestress with allowance for the permissible deviations given by Eq. (III.10) is found from Eq. (III.8)

$$p = 300 + 3\,600/18.8 \approx 490 \text{ kgf cm}^{-2} \text{ (49 MPa)}$$

and

$$\sigma_0 = R_{sII} - p = 6\,000 - 490$$
$$= 5\,510 \text{ kgf cm}^{-2} \text{ (551 MPa)}$$

*Early losses.* Equation (III.18) gives the losses due to the deformed anchorages

$$\sigma_3^{post} = E_s (\lambda_1 + \lambda_2)/l = 2\,000\,000\,(1 + 1)/18\,800 = 212 \text{ kgf cm}^{-2} (21.2 \text{ MPa})$$

Since the steel is tensioned by electric heating, there is no friction of the reinforcing bars against the tank wall. So,

$$\sigma_{loss1} = \sigma_3^{post} = 212 \text{ kgf cm}^{-2} (21.2 \text{ MPa})$$

*Late losses.* The losses due to the relaxation of stresses in the reinforcing bars (in tensioning by electric heating) are

$$\sigma_1 = 0.03\sigma_0 = 0.03 \times 5\,510 = 165 \text{ kgf cm}^{-2} (16.5 \text{ MPa})$$

According to Table III.2, the losses due to the shrinkage of M-200 concrete are

$$\sigma_8^{(post)} = 300 \text{ kgf cm}^{-2} (30 \text{ MPa})$$

Equation (III.22) gives the losses due to creep in concrete to be

$$\sigma_9^{(post)} = 2\,000\,\sigma_{pr}/R_0 = 2\,000 \times 33.7/200 = 337 \text{ kgf cm}^{-2} (33.7 \text{ MPa})$$

where

$$\sigma_{pr} = N_{01}/F_{tr} = (\sigma_0 - \sigma_{loss1})\,F_{pr}/(F_c + nF_{pr}) = (5\,510 - 212) \times 9.41/(14 \times 100 + 8.33 \times 9.41) = 33.7 \text{ kgf cm}^{-2} (3.37 \text{ MPa})$$

which is less than $0.6R_0 = 0.6 \times 200 = 120$ kgf cm$^{-2}$ (12 MPa); and

$$n = E_s/E_c = 2 \times 10^6/(0.24 \times 10^6) = 8.33$$

Thus, the late losses are

$$\sigma_{loss2} = \sigma_1 + \sigma_8^{(post)} + \sigma_9^{(post)} = 165 + 300 + 337 = 802 \text{ kgf cm}^{-2} (80.2 \text{ MPa})$$

and the *total losses* are

$$\sigma_{loss} = \sigma_{loss1} + \sigma_{loss2}$$
$$= 212 + 802 = 1\,014 \text{ kgf cm}^{-2} (101.4 \text{ MPa})$$

which is larger than the minimum value 1 000 kgf $cm^{-2}$ (100 MPa) that must be taken into consideration in design calculations.

With allowance for the total loss of prestress and the tensioning accuracy factor $m_{ac}$, the prestressing force is

$$N_0 = m_{ac} (\sigma_0 - \sigma_{loss}) F_{pr}$$
$$= 0.9 (5\,510 - 1\,014) 9.41 = 38\,100 \text{ kgf (381 kN)}$$

Here, according to Eq. (III.25), for tensioning by electric heating

$$\Delta m = 0.5p (1 - 1/\sqrt{n_b})/\sigma_0$$
$$= 0.5 \times 500 (1 - 1/\sqrt{3})/5\,500 = 0.07$$

For design purposes, $\Delta m$ is taken to be not less than 0.1. So, the tensioning accuracy factor in Eq. (III.24) is $m_{ac} = 1 - \Delta m = 1 - 0.1 = 0.9$.

**Incipient-Cracking Analysis.** The wall is checked subject to the condition in Eq. (V.27)

$$T \leqslant N_0$$

For Zone IV where the stress is maximal (see Table XII.3), $T = 34\,060$ kgf (340.6 kN) which is less than $N_0 = 38\,100$ kgf (381 kN). Consequently, the cracking-resistance requirements are met and no cracks will develop in the wall.

In the other zones, the condition $T \leqslant N_0$ is satisfied, too (the calculations are omitted).

# CHAPTER THIRTEEN

# STRUCTURES FOR HEAT SUPPLY SYSTEMS

## XIII.1. General

Heat is supplied to residential areas and industrial consumers by trunk and distributing heating systems. These are thermally insulated steel pipelines carrying superheated water or steam. To avoid stresses that might be caused by thermal deformations on heating up and cooling down, the pipelines have movable and immovable supports.

As a rule, heating systems are laid under ground. Pipelines are placed in conduits or headers to protect the thermal insulation and facilitate their movement during temperature variations. Sometimes, pipelines are laid without any conduits directly in the ground. In this case, they have protective jackets. The cost of a heat pipeline may considerably be reduced if it is run in a common header together with other underground services (such as electric and telephone cables, and water supply and sewage disposal pipelines).

Within the premises of industrial projects where the ground water level is high and also in permafrost regions, use is made of overhead heat pipelines installed on low supports (piers), scaffold bridges, masts, or brackets.

Along its length, a heat pipeline has servicing chambers to accommodate gate valves and instrumentation, movable and immovable supports, and also compensating niches for expansion U-bends enabling the pipes to elongate between immovable supports.

Heat pipelines should be laid with a longitudinal slope of not less than 0.002, to facilitate the release of water (at the lowest points) and to vent air (at the highest points). Because water may accidentally leak into a conduit or a header, pits are made at every 100 to 150 m through which the water can be discharged into gutters or bodies of water. If a buried pipeline is laid in an area with the presence of

ground water, a parallel tube drainage is usually run to lower the ground water level. In this case, the longitudinal slope of the pipeline is 0.003, and the transverse slope of the conduit towards the drain-pipe is 0.01.

If a pipeline is laid in a dry soil or has a tube drainage, the outer surfaces of the conduit walls and roof are water-proofed by two layers of hot bitumen.

When a pipeline crosses saturated-soil areas and a drain-pipe is unfeasible to install, the conduit is water-proofed by pasted insulation.

With approaches to buildings, heat pipeline conduits should have a reverse slope to prevent water from leaking into the basements and wetting the soil beneath the foundations.

The internal height of low conduits is less than 1 200 mm and that of medium-height conduits is from 1 400 to 2 100 mm. Access to low conduits can be had only from the top after the overburden and the roof slabs have been removed. Access to medium-height conduits is from chambers whose roofs have manholes brought out to the ground surface. The internal height of high conduits (also called passageway conduits, tunnels, or headers if they additionally carry other services) is from 2 100 to 3 000 mm. They provide access for the erection, inspection, and repair of all the services contained.

The roofs of buried conduits and headers should be at least 0.7 m below the surface, or 0.5 m if they run below paved roads. Heat pipelines may sometimes be half-buried. With this type, the top of the conduits is from 200 to 400 mm above the surface. The thermal insulation of such conduits consists of slag-filled walls and a warmth-keeping jacket over the roof.

When choosing a route for a heat pipeline, its length and the number of intersections with water obstacles and rail and motor roads should be kept to a minimum, as they significantly increase the cost of construction.

Along their length, conduits have movement joints to facilitate independent settlement of their individual portions and reduce the temperature strain of the structures. Movement joints are installed where conduits adjoin servicing chambers and compensating niches, and where the

soil conditions undergo sudden changes. They should be spaced not more than 50 m apart in buried conduits and not more than 30 m apart in half-buried conduits.

The structural members of buried and half-buried heat pipeline conduits use brick and concrete, both plain and reinforced. The pier supports of overhead pipelines are made of brick and plain concrete, and their masts and scaffold bridges employ reinforced concrete and metal.

## XIII.2. Construction Types of Conduits and Headers

**Low Conduits.** A simple low conduit is shown in Fig. XIII.1. Its bottom is a layer of M-75 concrete 100 mm thick placed on planed ground. After the pipes have been erected and thermally insulated, concrete wall blocks are installed on the bottom, and the conduit is roofed by precast reinforced concrete slabs. The conduit walls may also be made of well burned brick. According to the diameter of the pipeline,

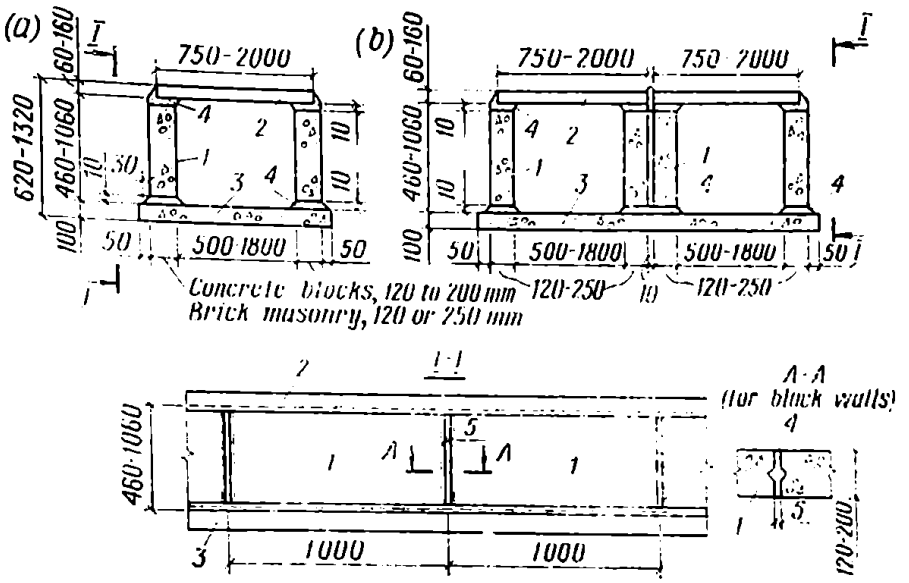



Fig. XIII.1. Construction types of low conduits with concrete block or brick walls

(*a*) single-duct conduit; (*b*) double-duct conduit; *1*—concrete wall blocks or brick masonry; *2*—precast reinforced concrete roof slab; *3*—concrete layer; *4*—brand 50 cement grout

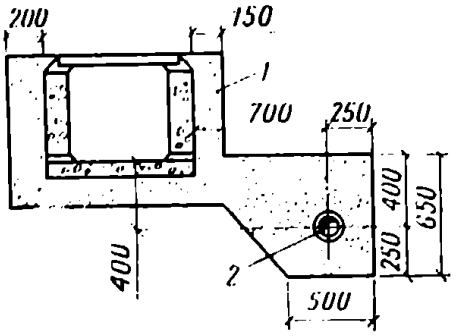



Fig. XIII.2. Conduit with a parallel drain-pipe
*1*—gravel and sand filter; *2*—drain-pipe

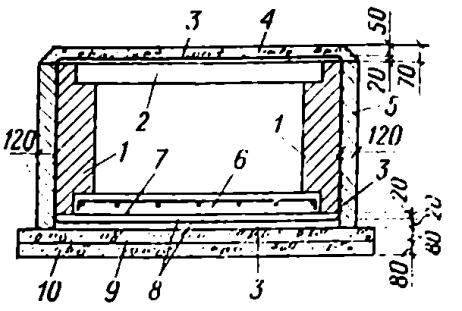



Fig. XIII.3. Conduit with pasted insulation
*1*—conduit walls; *2*—precast reinforced concrete roof slabs; *3*—pasted insulation; *4*—concrete layer; *5*—protective brick wall; *6*—reinforced concrete floor; *7*—floor load-bearing reinforcement; *8*—cement coat; *9*—concrete layer; *10*—broken stone cushion

the height of such conduits may range between 460 and 1 060 mm and the width between 500 and 1 800 mm.

The thickness of the concrete wall blocks depends on the conduit height and may be anywhere between 120 and 200 mm. The thickness of brick walls is 120 and 250 mm. The walls can resist lateral earth pressure only when they are reliably fixed at the top, so the conduit trench may not be backfilled until after the roof slabs have been installed. When the conduit roof is removed for inspection or repair, the walls should securely be held in place by spacers installed at the top.

Flat reinforced concrete roof slabs are fabricated 60 to 160 mm thick, depending on their span (conduit width).

Figure XIII.2 shows a conduit with a parallel drain-pipe intended to lower the ground water level. The asbestos-cement or ceramic drain-pipe with water inlet holes 150 to 200 mm in diameter is placed in a layer of filtering sand and gravel 400 mm below the conduit bottom and within 700 mm of the conduit wall.

As already mentioned in Sec. XIII.1, if it is laid in saturated soil and no drain-pipes can be used, a conduit must appropriately be water-proofed (Fig. XIII.3). Under the

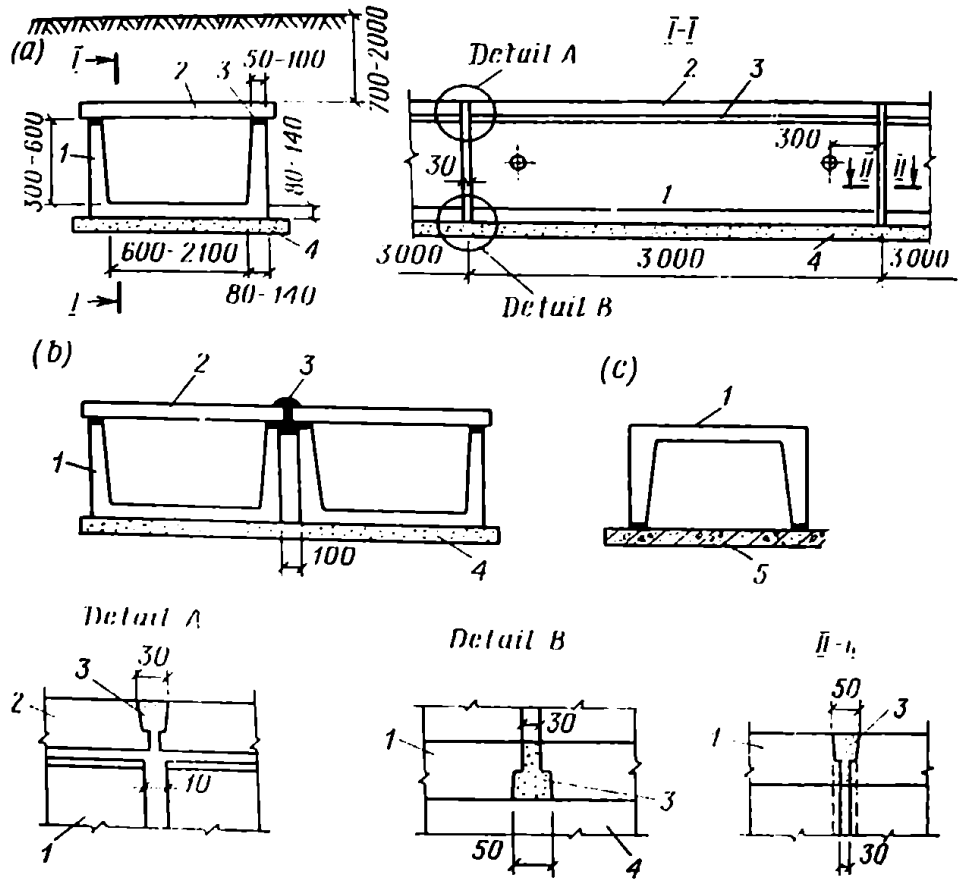



Fig. XIII.4. Low conduit assembled from precast reinforced concrete U-shaped members and flat slabs

*1*—U-shaped member; *2*—roof slab; *3*—cement grout; *4*—sand cushion; *5*—concrete footing

circumstances, the conduit bottom is made of reinforced concrete, because it must resist the ground water upthrust and work in bending between the walls. To avoid floating-up, the weight of the conduit should exceed the resultant of the ground water upthrust [see Eq. (XI.21)].

In Soviet practice, low conduits are customarily assembled from standardized precast reinforced concrete members which are more economical of material and are less labour-consuming than their in-situ counterparts.

Figure XIII.4 shows a buried single-duct low conduit assembled from prefabricated reinforced concrete U-shaped

members laid on a layer of sand and roofed by reinforced concrete deck slabs. Double-duct conduits are built of two single-duct conduits installed with a gap of 100 mm filled with sand (Fig. XIII.4*b*). Sometimes, U-shaped members are placed upside down on a layer of concrete as shown in Fig. XIII.4*c*.

Standard U-shaped members are made 300, 450, and 600 mm high and 600 to 2 100 mm wide. Their nominal length is 3 m and the structural length is 2.97 m.

So that the length of straight portions between servicing chambers and compensating niches may be varied, make-up members with a nominal length of 0.6 m (the structural length being 0.57 m) have been designed for all types of U-shaped elements and slabs. The walls of U-shaped elements are intended to carry lateral earth pressure as cantilevers and have, therefore, a variable thickness of 80 to 140 mm at the bottom and 50 to 100 mm at the top. If necessary, the roof slabs may be removed with the backfill left around the conduit.

The U-shaped members are made of M-300 concrete reinforced with welded-wire fabric using class A-III and B-I steel (Fig. XIII.5*a*).

The width of the deck slabs in the direction of the effective span (across the U-shaped member) is from 0.85 to 2.4 m, the nominal length is 3 m (0.6 m for make-up slabs), and the thickness is from 70 to 160 mm. The slabs are made of M-200 concrete and use class A-III load-bearing reinforcement and class B-I distribution reinforcement (Fig. XIII.5*b*).

Low conduits may also be assembled from two U-shaped members, one serving as the bottom and the other as the roof. The top U-shaped member is installed after the pipeline has been erected and insulated (Fig. XIII.6). The height of such conduits is equal to the sum of the heights of the two U-shaped members plus the gap between them (about 20 mm). The gap is filled with brand 50 cement grout. To align the U-shaped members, two steel channels (one size 12, and the other, size 16) welded together to form an I-section are installed in the gap. The walls of each U-shaped member carry lateral earth pressure as cantilevers fixed at the member bottom.

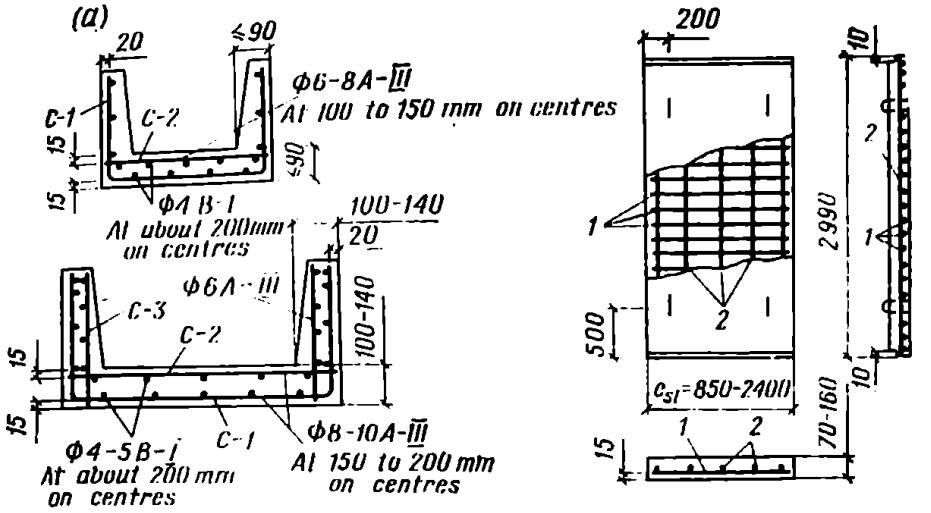



Fig. XIII.5. Distribution of reinforcement in prefab U-shaped members and roof slabs

*1*—load-bearing reinforcement; *2*—distribution reinforcement

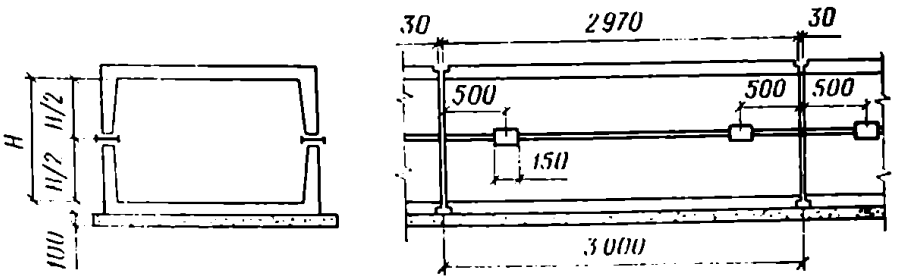



Fig. XIII.6. Low conduits assembled from two U-shaped members

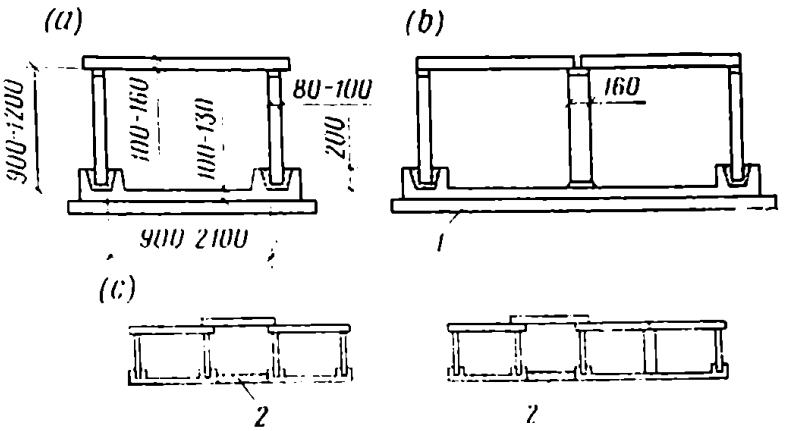



Fig. XIII.7. Low conduits assembled from flat slabs

*1*—sand cushion; *2*—in-situ portion

Another type of low conduits is assembled from precast reinforced concrete flat slabs (Fig. XIII.7). The bottom slabs laid on a layer of sand have grooves 200 mm deep to receive the wall slabs. The gaps are filled with M-300 concrete using fine broken stone as the aggregate. The walls support the roof slabs (Fig. XIII.7*a*). In double-duct conduits, an intermediate wall is installed on the bottom slab (Fig. XIII.7*b*). Multi-duct conduits are assembled from parallel single- and double-duct conduits (Fig. XIII.7*c*). The distribution of reinforcement in the slabs is shown in Fig. XIII.8.

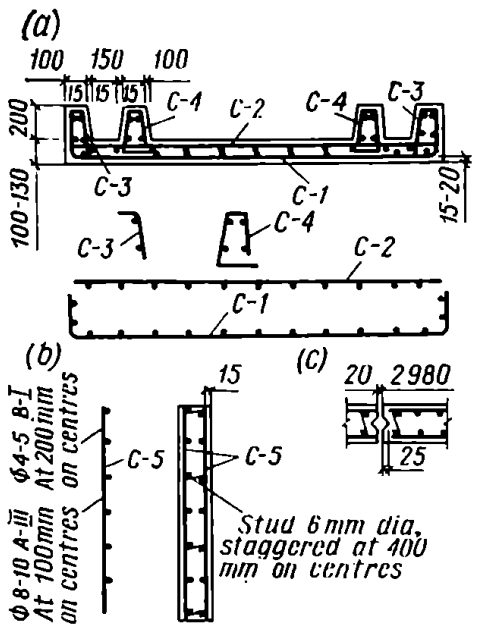



Fig. XIII.8. Distribution of reinforcement in flat slabs

(*a*) floor; (*b*) wall slab; (*c*) wall slab joint

Either type of low conduits may be erected as a half-buried structure with three-layer thermally insulated roof slabs (Fig. XIII.9).

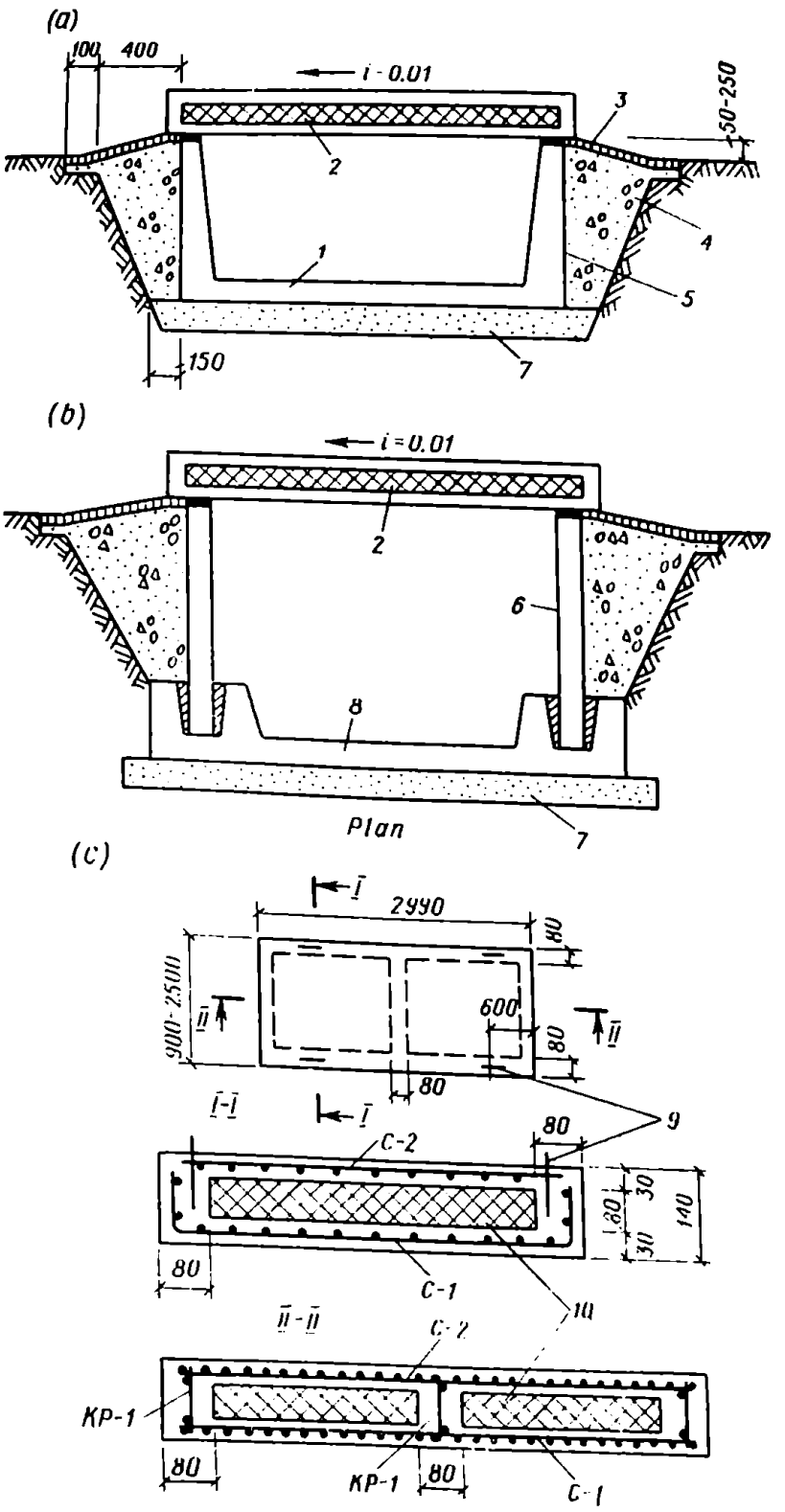
The standard members of low conduits may be installed as deep as 700 to 2 000 mm from the surface and are designed to carry wheel loads.

**Medium-Height Conduits.** Medium-height conduits are intended to carry double-pipe heating lines, process pipelines, cables, and other services.

Figure XIII.10 shows a medium-height conduit having brick walls. Conduits assembled from precast reinforced concrete members may be 1 500, 1 800, and 2 100 mm high and from 1 200 to 4 200 mm wide. They may carry

Fig. XIII.9. Half-buried conduits

(*a*) assembled from U-shaped members; (*b*) assembled from flat slabs; (*c*) thermally insulated roof slab; *1*—U-shaped member; *2*—thermally insulated roof slab; *3*—asphalt; *4*—slag; *5*—water proofing; *6*—wall slab; *7*—sand cushion; *8*—floor slab; *9*—lifting eyes; *10*—foam-concrete inserts


(a)
100
400
i - 0.01
3
50-250
2
4
1
5
150
7
(b)
i = 0.01
2
6
8
Plan
7
(c)
I
2990
80
900-2500
II
600
80
II
80
I-I
I
9
80
C-2
30
180
30
140
80
C-1
10
II-II
C-2
KP-1
80
KP-1
80
C-1

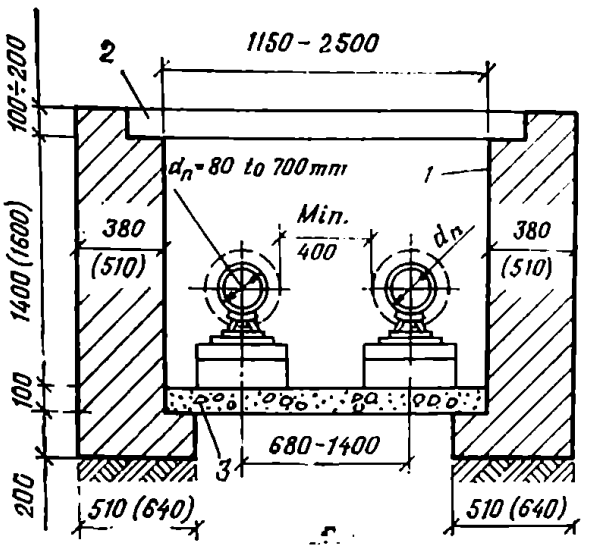



Fig. XIII.10. Medium-height conduit with brick walls (the dimensions in brackets are given for $d_n$ = 600 to 700 mm)
*1*—wall; *2*—reinforced concrete roof slab; *3*—concrete floor

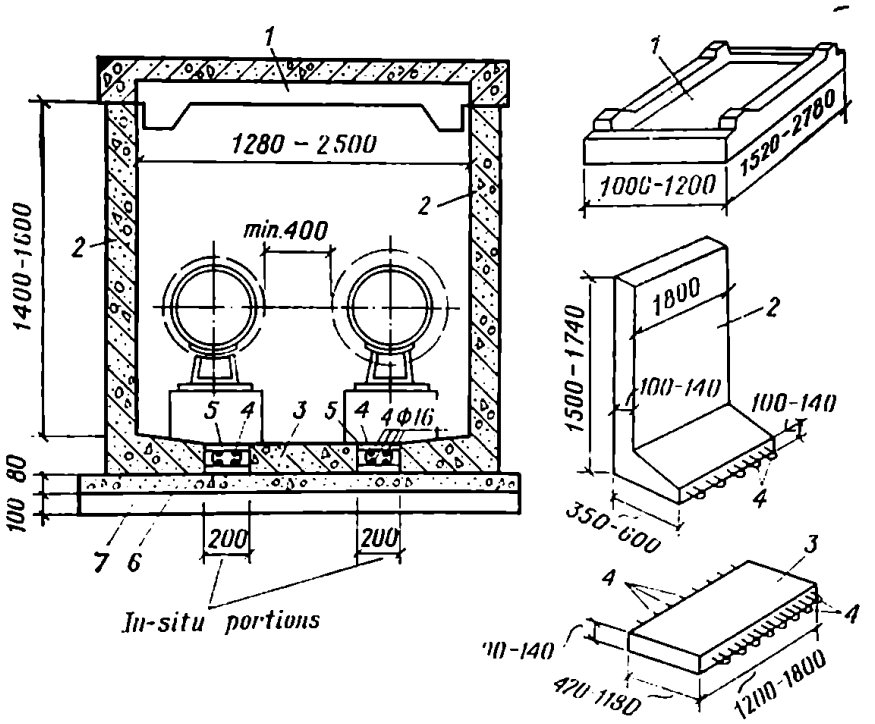



Fig. XIII.11. Medium-height conduit assembled from prefab reinforced concrete blocks
*1*—roof block; *2*—wall block; *3*—floor block; *4*—exposed reinforcement loops; *5*—in-situ concrete; *6*—concrete layer; *7*—sand and broken stone cushion

heat pipelines from 900 to 1 200 mm in diameter. Conduits using U-shaped members similar to those shown in Fig. XIII.6 are of the double-duct type. Each duct is composed of two U-shaped members 1 500 or 1 800 mm wide and, respectively, 900 + 600 mm or 900 + 900 mm high. Medium-height conduits assembled from flat slabs may be either single-duct (similarly to those shown in Fig XIII.7*a*) or double-duct (similarly to those shown in Fig. XIII.7*b*). The former are from 1 200 to 4 200 mm wide and from 1 500 to 2 100 mm high, and the latter are 2 × × 2 100 mm wide and 1 500 or 1 800 mm high.

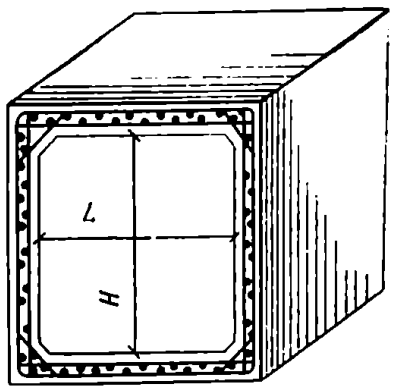


Fig. XIII.12. Closed-frame tunnel member

Medium-height conduits may also use precast reinforced concrete blocks (Fig. XIII.11). The bottom is a flat block and the walls are L-shaped slabs in which one side forms the conduit wall and the other is part of the bottom. The wall slabs are connected to the bottom blocks by castellated joints constituted by loop reinforcement stick-outs locked by tie bars. The gaps between the blocks are filled with M-300 concrete. The wall blocks are stable during erection and need no additional supports. The wall and bottom blocks are the same in thickness.

The roof blocks are equipped with ribs around the periphery. The ribs running across the conduit have protrusions to fix the top of the wall blocks against lateral earth pressure.

**Tunnels (Headers).** In construction, tunnels are similar to medium-height conduits and may use brick walls (as in Fig. XIII.10), precast reinforced concrete wall blocks (as in Fig. XIII.11), or members of a closed-frame cross section (Fig. XIII.12) if a tunnel is small. The last-mentioned construction type is economical of material, but its component members are difficult to fabricate at factories. Also, because the blocks are short (which is dictated by their weight), too many joints, which are extremely labour-consuming

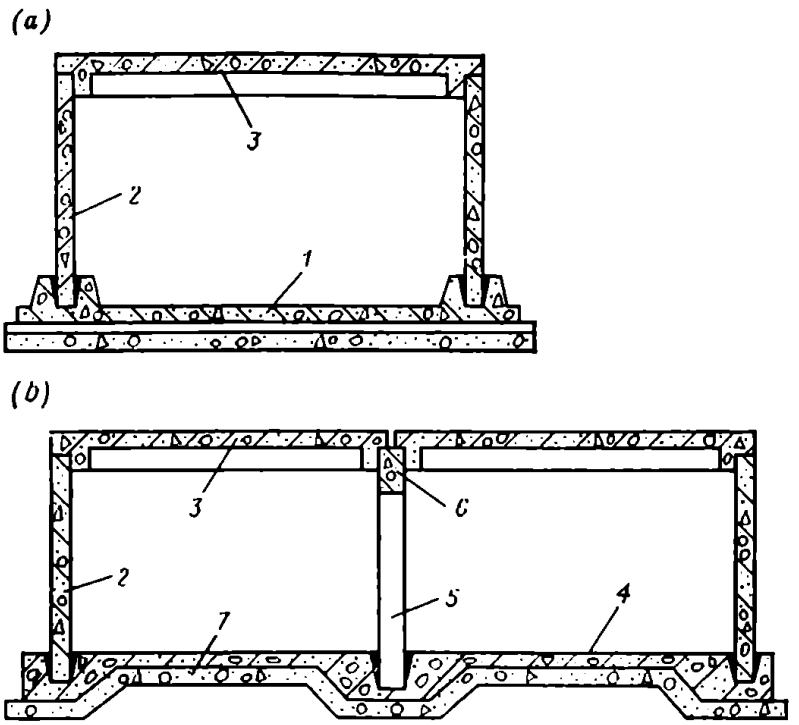



Fig. XIII.13. Tunnels assembled from standardized reinforced concrete members

(*a*) single-duct tunnel; (*b*) double-duct tunnel; *1*—bottom slab; *2*—wall panel; *3*—roof slab; *4*—in-situ floor; *5*—column; *6*—beam; *7*—concrete cushion

to fill up, have to be provided. A better choice is reinforced concrete tunnels cast in-situ in sliding formwork.

In Soviet practice, use is most frequently made of standardized reinforced concrete members (Fig. XIII.13). The width of small-size tunnels (up to 2 400 mm) is taken as a multiple of 300 mm (1 500, 1 800, 2 100, and 2 400 mm), and that of large-size tunnels is a multiple of 600 mm (3 000, 3 600, and 4 200 mm). The tunnel height is 2 100, 2 400, and 3 000 mm.

Single-duct tunnels use three types of precast reinforced concrete slabs, namely bottom, wall, and roof slabs. Double-duct tunnels, which have an in-situ bottom because of their large width, use the same wall and roof slabs and additional prefab columns and beams supporting the roof slabs at the

joints. Along the tunnel length, the centre-to-centre distance between the columns is 3 m.

In each ring element along a tunnel, the wall slab is installed in a groove in the bottom, and the gap is filled with concrete using fine broken stone as the aggregate. The top of the wall slab enters a groove in the roof slab. Thus, under the action of lateral earth pressure, the wall panel works as a beam fixed at the bottom and hinged at the top. Precast columns are installed in sockets made in the in-situ bottom.

All component members may be installed as deep as 700 to 2 000 mm from the surface (as reckoned to the top of a tunnel) and are designed to carry wheel loads.

All the tunnels discussed above are laid in open trenches. If necessary, deeply buried tunnels may be shield-driven. Such tunnels are round in cross section and made of concrete blocks installed as a metal shield moves forward, and are lined with reinforced concrete blocks.

## XIII.3. Servicing Chambers, Compensating Niches, and Supports for Heat Pipelines

**Servicing Chambers.** The servicing chambers of heat pipelines may be rectangular or circular in plan. They are built of brick, in-situ reinforced concrete, or precast plain and reinforced concrete members. Chamber dimensions vary over a wide range from 1.8 m square to 9 × 7.2 m. The minimum depth from the ground surface or paved road is from 0.3 to 0.5 m. The walls of servicing chambers have openings for heat pipelines. Roof slabs at the corners are equipped with manholes 650 to 700 mm in diameter providing access from the ground surface. To ease access, steel steps spaced 300 mm apart are built into the walls or vertical metal flights of stairs are installed. The bottom of a servicing chamber has a pit to collect water.

Figure XIII.14 shows a servicing chamber having brick walls, an in-situ concrete bottom, and a precast reinforced concrete roof. The roof slabs are laid directly on the wall. In large-size chambers, use is additionally made of intermediate prefab beams.

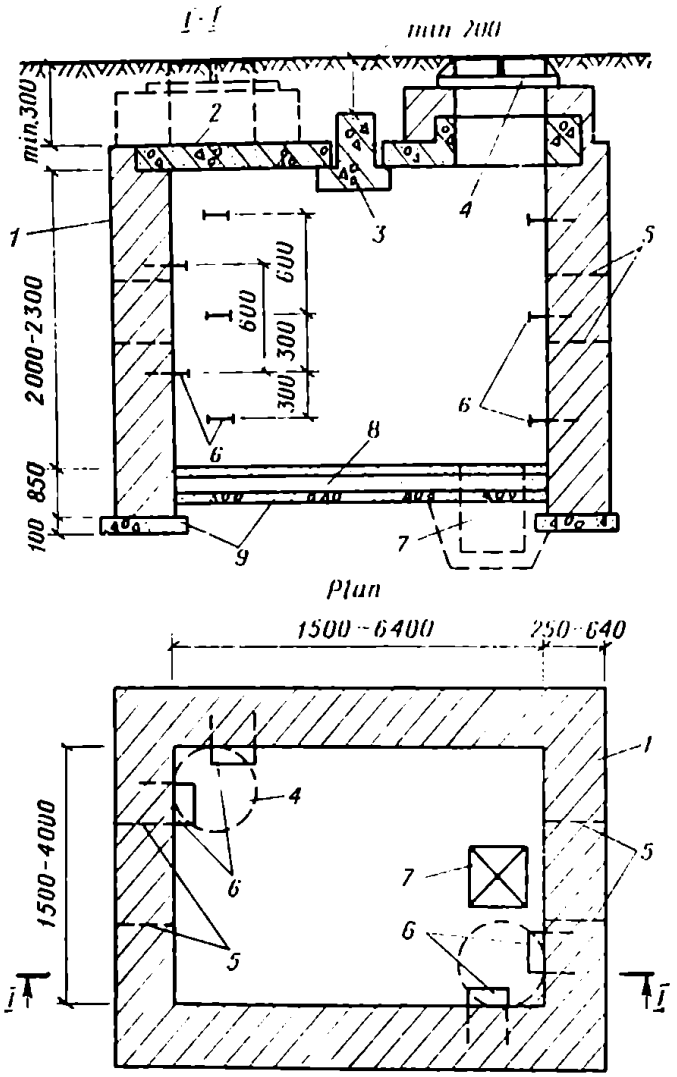



Fig. XIII.14. Servicing chamber

*1*—brick masonry on cement mortar; *2*—precast reinforced concrete s precast reinforced concrete beam; *4*—manholes 650 mm in diameter; *5*—for heat pipelines; *6*—steps; *7*—pit; *8*—concrete floor; *9*—compacte stone cushion

Precast wall members significantly cut down the requirements and time of construction. Rectangular bers may be made of precast concrete blocks (Fig. X and *b*), ribbed and flat slabs (Fig. XIII.15*c*), L-shaped

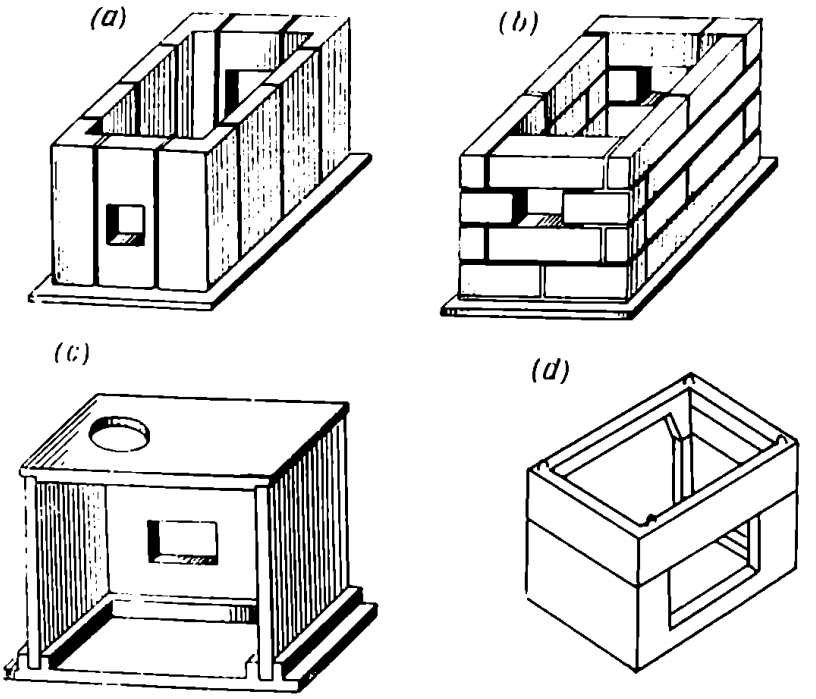



Fig. XIII.15. Servicing chambers assembled from prefab members

similar to those shown in Fig. XIII.11, space blocks (Fig. XIII.15*d*), etc. Circular chambers are assembled from reinforced concrete pipes 2.5 m in diameter (Fig. XIII.16*a*), or vertical flat slabs joined at the job (Fig. XIII.16*b*).

The construction types of servicing chamber walls are many and diverse, so standard reinforced concrete members are only used for roofs. The type and material of walls are chosen from what the contractor can obtain or supply. In plan, the internal dimensions of servicing chambers are taken from 1.8 to 9 m as multiples of 0.6 m. Chamber roofs are assembled from the same types of slabs used in conduits and tunnels. Supplementary members are slabs with round openings 700 mm in diameter, 500 × 250 mm rectangular beams 3.2 to 5.2 m long, and reinforced concrete rings for manholes. In chambers 1.8 m wide, the slabs are placed directly on the walls (Fig. XIII.17*a*). In wider chambers, intermediate beams are installed in the longitudinal direction (Fig. XIII.17*b*). Large chambers use intermediate co-

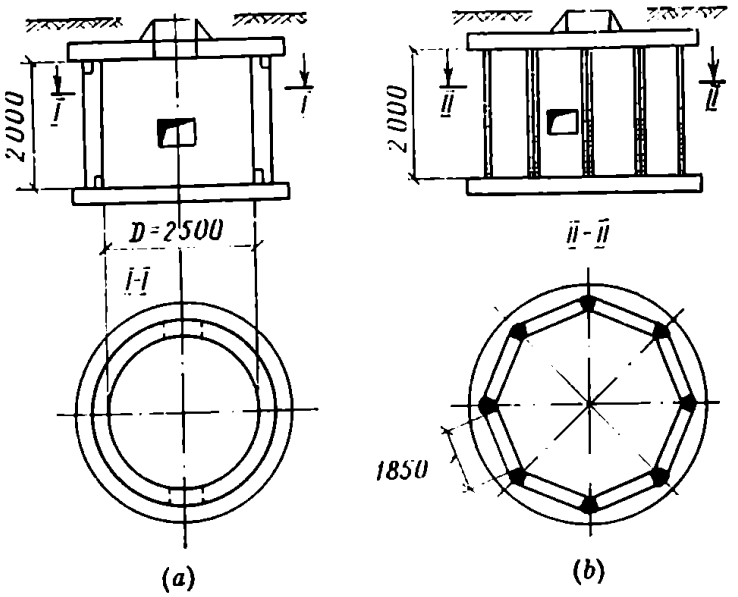



Fig. XIII.16. Circular servicing chambers

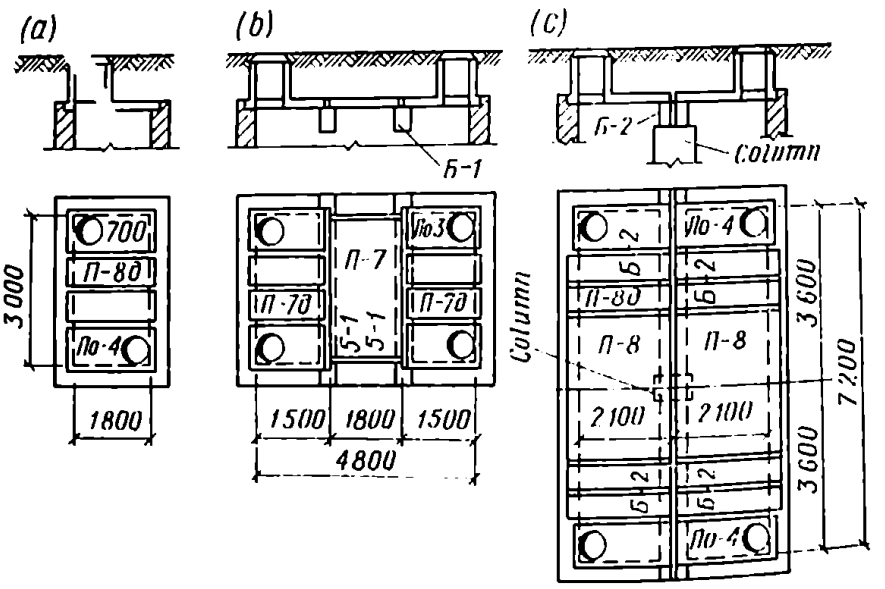



Fig. XIII.17. Construction types of chamber roofs

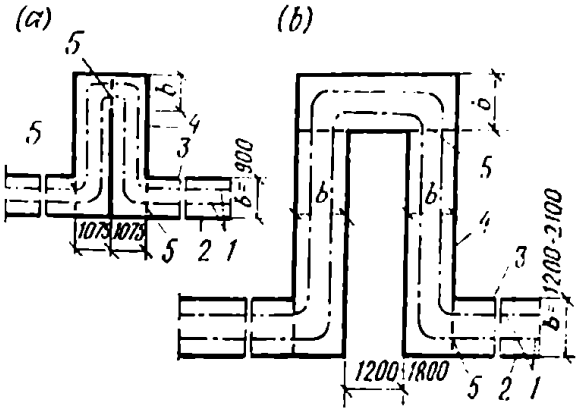



Fig. XIII.18. Construction types of compensating niches
*1*—heat pipelines; *2*—conduit; *3*—movement joint; *4*—compensating niche; *5*—steel beam composed of two angles, supporting roof slabs

lumns which support the beams (Fig. XIII.17*c*). The columns are built of brick or plain or reinforced concrete, whichever may be the case.

**Compensating Niches.** To cancel temperature strain in heat pipelines between immovable supports, use is made of flexible expansion U-bends. They are accommodated along the length of a pipeline conduit in expanded sections known as compensating niches (Fig. XIII.18). A compensating niche has the same configuration in plan as the U-bend it contains. It is separated by movement joints from the adjacent straight portions of the conduit. The U-bend and, as a consequence, the compensating niche dimensions depend on the heat pipeline diameter. If the pipeline diameter does not exceed 150 mm, a niche is arranged as a double-duct conduit (see Fig. XIII.18*a*). For larger pipelines, it consists of two individual single-duct conduits spaced 1.2 to 1.8 m apart (Fig. XIII.18*b*).

The walls of a compensating niche may be built of brick, assembled from precast concrete blocks, or use reinforced concrete cast in-situ. The bottom is made of plain or reinforced concrete. The roof is always assembled from precast reinforced concrete slabs as used in the main body of the conduit. At bends, additional beams are installed to support the roof slabs. The beams are made of twin steel angles so as not to reduce the internal dimensions of the conduit.

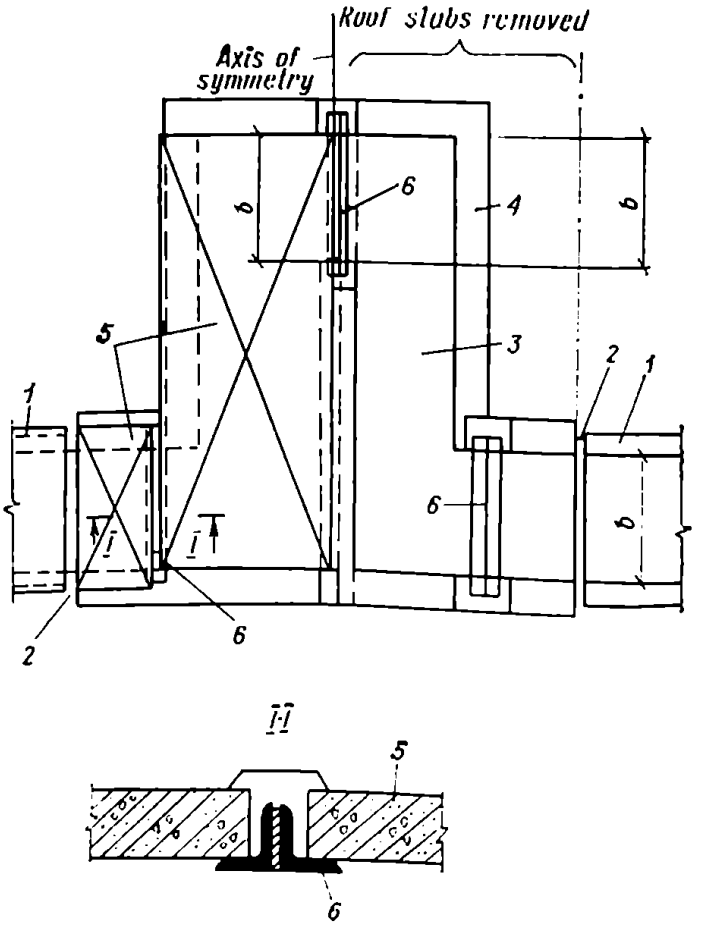



Fig. XIII.19. Compensating niche

*1*—conduit; *2*—movement joint; *3*—in-situ floor; *4*—niche walls (brick, concrete blocks, or in-situ reinforced concrete); *5*—roof slabs; *6*—steel beams made of two angles

The horizontal flanges of the angles support the roof slabs and the vertical flanges are arranged in the gaps between the slabs.

The construction of a compensating niche is shown in Fig. XIII.19.

**Movable and Immovable Supports.** The immovable (fixed or dead) supports of heat pipelines are intended to resist longitudinal forces due to the unbalanced internal pressure in the pipe and also frictional forces appearing in movable (sliding) supports (also known as saddles) forced to

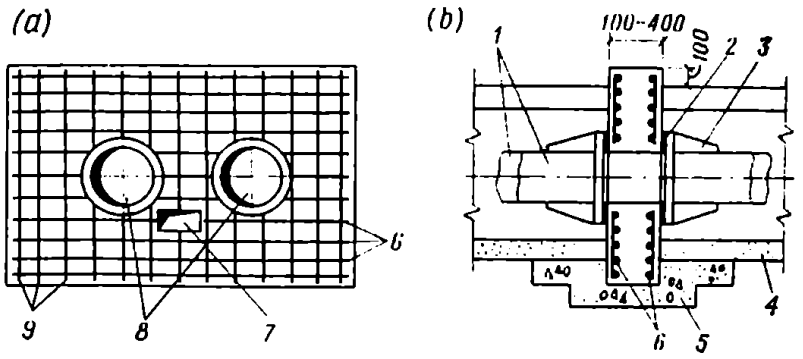



Fig. XIII.20. Wall-type immovable support
(*a*) side view (with reinforcement); (*b*) sectional view; *1*—heat pipelines; *2*—flange; *3*—gusset; *4*—conduit floor; *5*—concrete; *6*—load-bearing reinforcement; *7*—opening for water; *8*—pipeline openings; *9*—distribution reinforcement

move by a temperature strain. The longitudinal forces imposed on the fixed supports depend on the pipe diameter, the length between expansion U-bends, and the type of U-bends, and may vary over a wide range from 1 to 5 tf (10 to 50 kN) to 50 to 70 tf (500 to 700 kN). The immovable supports (Fig. XIII.20) are usually made as a precast or in-situ reinforced concrete wall having openings for pipes. To transfer forces from the pipes, steel flanges reinforced by longitudinal gussets are used. Since forces may be applied in either direction, the flanges are installed on either side of the support which is, therefore, reinforced symmetrically.

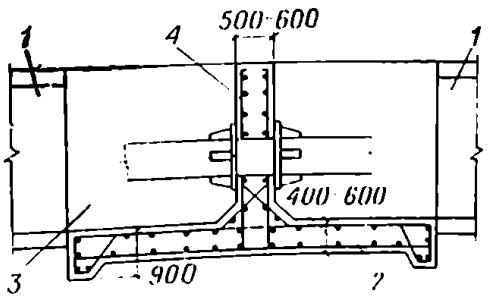



Fig. XIII.21. In-situ concrete T-section immovable support
*1*—conduit; *2*—support bottom; *3*—support side walls; *4*—support transverse wall

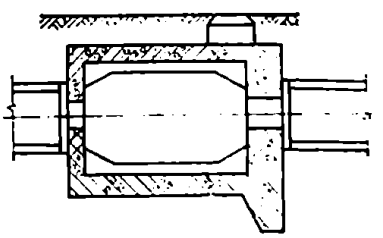


Fig. XIII.22. In-situ concrete servicing chamber used as an immovable support for large-diameter pipelines

The support is 100 mm thick for pipes up to 50 mm in diameter, and 150 mm thick for pipes 80 to 150 mm in diameter. If the pipe diameter is from 200 to 400 mm, the thickness of the support is chosen equal to the pipe diameter. In this case, the support is assumed to transfer forces only to the longitudinal wall of the conduit.

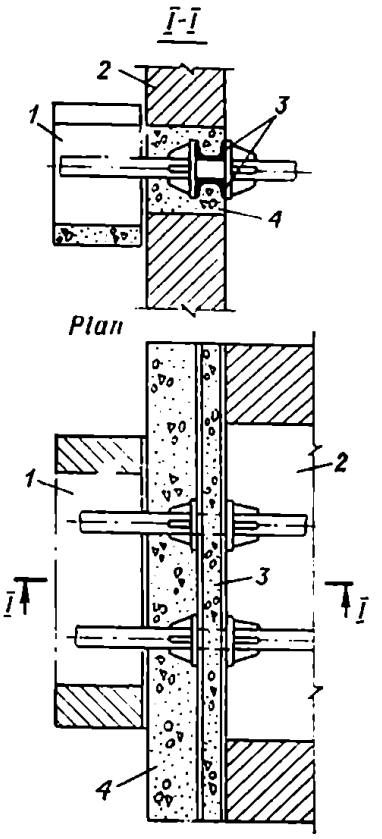



Fig. XIII.23. Immovable support in the brick wall of a servicing chamber
*1*—conduit; *2*—chamber; *3*—steel channels; *4*—concrete

Figure XIII.21 shows a T-section immovable support for large-diameter pipes. The support comprises a portion of a conduit about 4 or 5 m long with in-situ reinforced concrete walls and bottom. The middle portion of the conduit has a transverse wall 500 to 600 mm thick to which forces from the pipes are transferred. At the edges, the bottom has protruding lugs which contribute to the sliding resistance of the entire structure. A similar structure may be assembled from prefabricated members joined by welding together reinforcement stick-outs and concreting the gaps.

Also, immovable supports may use the walls of servicing chambers as shown in Fig. XIII.22. The wall that carries the pipes is made thicker than usual and has a special tooth improving the sliding resistance of the chamber. With small-diameter pipes, the immovable support is arranged in the brick or concrete wall of a chamber. For this purpose, two steel channels are installed at the pipe level, to which pipe support gussets are welded, and the wall portion is concreted (Fig. XIII.23). Another type is an immovable support made of steel channels and installed directly in a

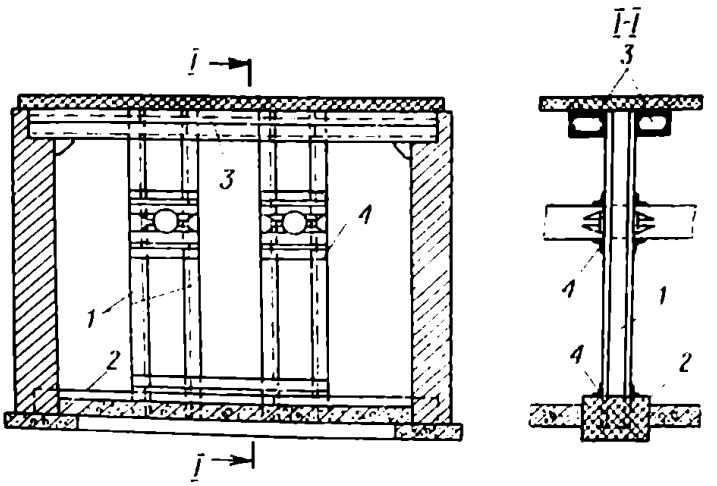



Fig. XIII.24. Steel immovable supports in servicing chambers
1—double-channel strut; 2—reinforced concrete beam; 3—steel beams; 4—strut brace

servicing chamber (Fig. XIII.24). In this support, each pipe is laid between two vertical struts built into a reinforced concrete beam at the bottom and fixed by twin steel beams at the top. Under the action of the forces transferred by the pipes, the struts and beams work in bending, so, when the forces are considerable, they should preferably be made of large-size steel shapes. The support reactions are taken up by the chamber walls.

The movable supports of pipelines consist essentially of metal saddles welded to a pipe itself. The saddles rest on steel sheets built into reinforced concrete support cushions. For pipes 25 to 80 mm in diameter, the saddle is made of a single bent plate (Fig. XIII.25*a*); for larger pipes, it consists of several sheets welded together (Fig. XIII.25*b*). Friction between the saddle and the cushion can be reduced by using roller bearings (Fig. XIII.25*c*).

The distance between supports depends on the pipe diameter, because the pipe works in bending under the action of its self-weight and the weight of the heat-transfer agent as a continuous beam. The reinforced concrete support cushions distribute the pipe pressure over the conduit bottom, so their dimensions likewise depend on the pipe diameter.

Movable supports may also be erected as steel beams assembled from angles or channels which are installed across

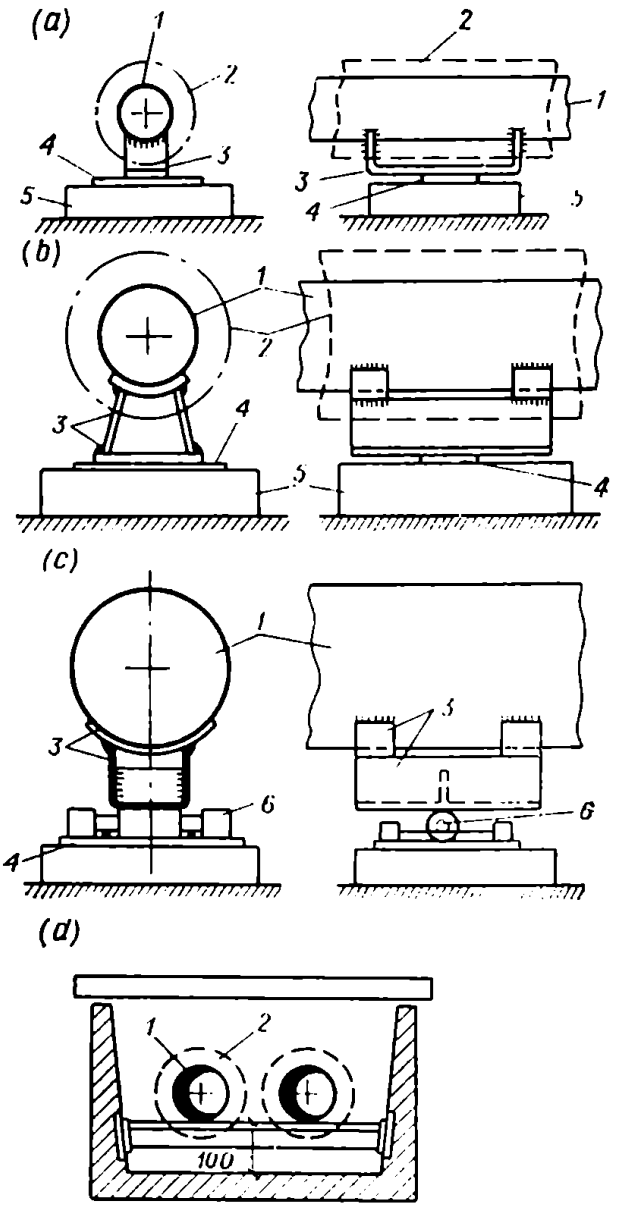



Fig. XIII.25. Construction types of movable supports
*1*—steel pipe; *2*—thermal insulation; *3*—saddle; *4*—base plate; *5*—reinforced concrete cushion; *6*—roller bearing

a conduit and welded to embedded fittings in the conduit walls (Fig. XIII.25*d*).

**Movement Joints.** As already mentioned in Sec. XIII.1, movement joints are intended to facilitate the independent

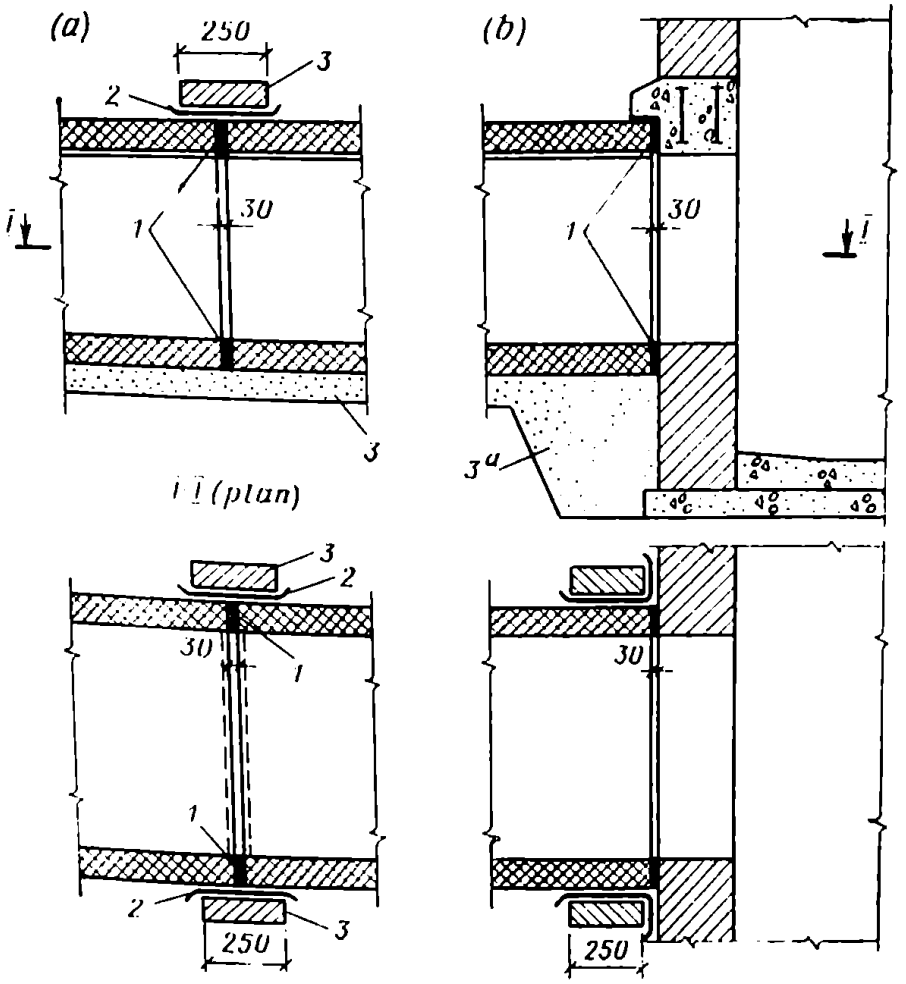



Fig. XIII.26. Movement joints
(*a*) in span; (*b*) where conduit adjoins servicing chamber; *1*—bitumen; *2*—prepared roofing paper; *3*—protective brick wall

settlement of individual heat pipeline portions and reduce temperature strain in the structures. Where a movement joint is to be installed (along the length of a conduit or where a conduit adjoins servicing chambers or compensating niches), the structural members are placed with a 30-mm gap which is afterwards filled with bitumen. On the side surfaces and at the top, movement joints are covered with two layers of prepared roofing paper and protected with brick (Fig. XIII.26).

## XIII.4. Heat Pipelines Laid without Conduits

As previously noted, heat pipelines may be laid directly in the ground without any conduits. This cuts down the cost of construction but requires reliable insulation and also freedom for the pipes to move axially.

Heat pipelines may be laid in an open trench in a protective enclosure made of reinforced concrete pipes. Inside the enclosure, the pipes are fixed by steel bar brackets welded to their surface (Fig. XIII.27*a*). The protective enclosure may also consist of asbestos-cement pipes or, when no open trenches are used (for example under paved roads), steel pipes. Sometimes, protective enclosures are assembled from reinforced concrete segments with pasted thermal insulation (Fig. XIII.27*b*). The lower segments having cylindrical supporting cushions are laid in a trench on a layer of broken stone. After the steel pipes have been installed, the

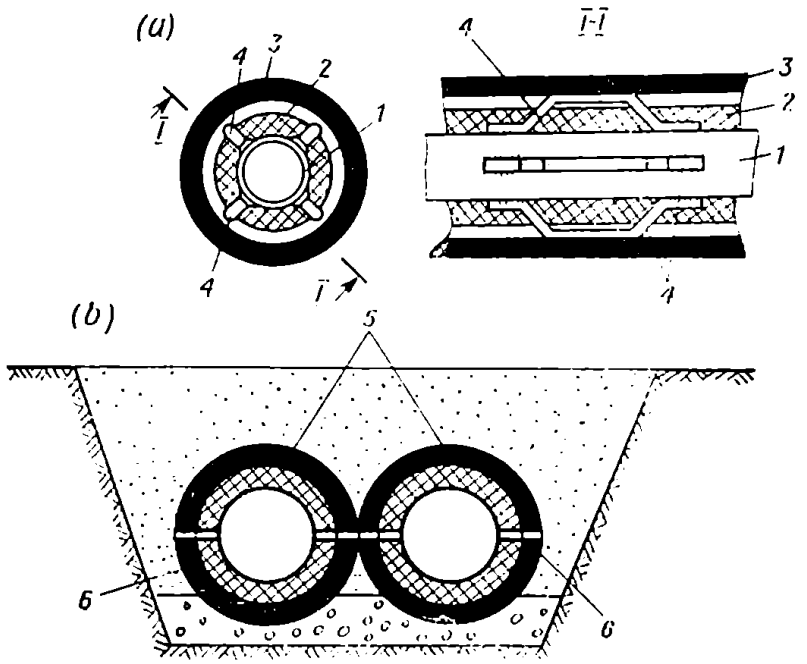



Fig. XIII.27. Heat pipelines laid in an open trench

*1*—steel pipeline; *2*—thermal insulation; *3*—pipe shell; *4*—steel brackets used as pipeline supports; *5*—upper segment; *6*—lower segment

top segments are erected, and the gaps between the top and bottom segments are filled with cement grout.

The labour requirements and cost of construction can markedly be reduced if the pipes are given a coat of effectively water-proofed reinforced foam concrete at a factory. The bond between the steel pipes and enclosure is not disturbed by temperature movements of the pipeline. All the loads induced by the soil pressure and friction of the enclosure against the soil are carried directly by the steel pipes which, naturally, should adequately be strong. Such pipelines are used in relatively dense soils.

Steel pipelines may also be laid in a layer of, say, asphaltic backfill which does not absorb water from the ground and, when heated by the heat-transfer agent, is baked to form a dense anticorrosion sheath around the pipes. Outside the baked sheath, the backfill remains porous, so that it serves as thermal insulation and does not restrain free movement of the pipeline. All the loads that may be imposed on the pipeline are resisted solely by the steel pipes.

## XIII.5. Overhead Heat Pipelines

Overhead heat pipelines are usually mounted on individual piers or scaffold bridges. When a heat pipeline is run close to buildings, the pipes are supported by brackets built into the building walls. To leave enough headroom for traffic, overhead heat pipelines are installed 5 to 6 m above the ground surface.

If pipelines are installed on individual piers (Fig. XIII.28*a*), the steel pipes contribute to the load-bearing capacity of the entire structure, so the distance between the piers increases with increasing pipe diameter. This construction is most economical with large-diameter pipes* (300 mm or more). The distance between the piers is governed not only by the pipe strength, but also by their sag which should not interfere with the free flow of water or condensate at a given longitudinal slope. The maximum safe sag $f$ (in cm) is

$$f \leqslant 0.25il$$

* If the temperature of the heat-transfer agent does not exceed 300°C.

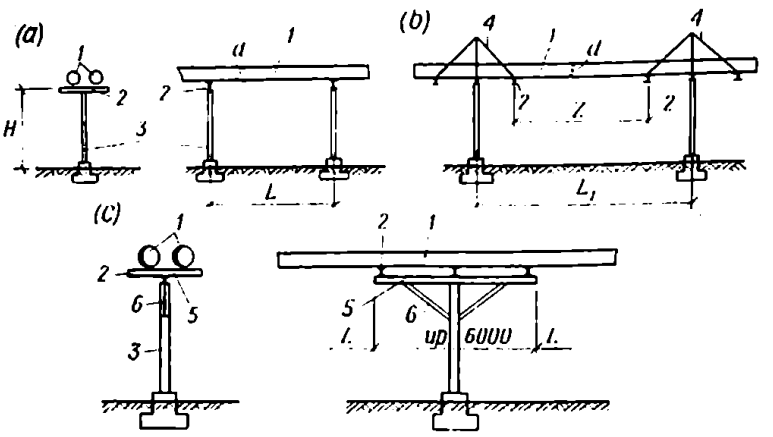



Fig. XIII.28. Pipelines installed on individual piers
*1*—pipelines; *2*—metal cross arms; *3*—reinforced concrete columns; *4*—guys; *5*—longitudinal beam; *6*—braces

where $l$ is the distance between the piers in cm, and $i$ is the longitudinal slope of the pipeline.

Large-diameter pipes may be accompanied by small-diameter pipes which are suspended on (or supported by) the larger pipes at every 2 to 4 m. The load due to the weight of the smaller pipes should be taken into consideration in the design of the main pipes.

To increase the distance between piers, the load-bearing pipes can be strengthened by strand or bar guys (Fig. XIII.28*b*), or the cross arms at the piers can be extended by inserting longitudinal metal beams with braces as shown in Fig. XIII.28*c*.

If the load does not exceed 3 tf (30 kN), the piers are assembled from rectangular reinforced concrete columns (Fig. XIII.29*a*) and metal cross arms 0.6 to 2.4 m long (Fig. XIII.29*b*), or columns, cross arms, and 6-m longitudinal beams strengthened by braces (Fig. XIII.28*c*). The height $H$ of the piers may range between 2.4 and 7.8 m in steps of 0.6 m. If the load is heavier than 3 tf (30 kN) and the pier height is from 5.4 to 7.8 m, use is made of T-shaped or U-shaped columns and cross arms (Fig. XIII.30).

Pipelines are installed directly on the top face of a column or a cross arm supported by a column.

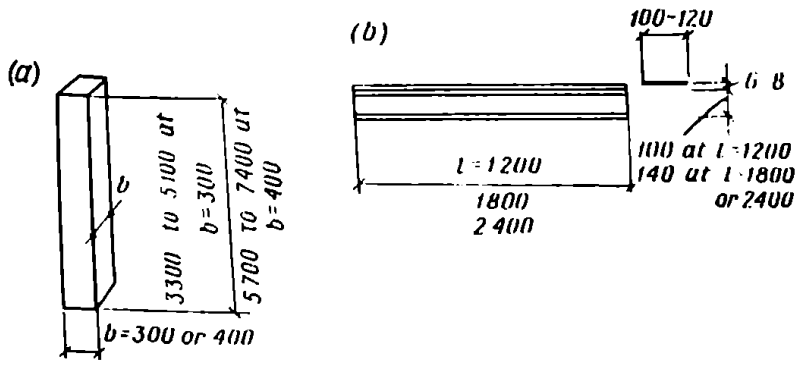



Fig. XIII.29. Reinforced concrete columns and metal cross arms of individual piers

A scaffold bridge consists essentially of reinforced concrete beams or steel trusses spanning between piers (Fig. XIII.31). Pipes are laid on cross arms; the spacing between the cross arms depends on the pipe diameter. A scaffold bridge may have a deck or a footwalk along its length. Scaffold bridges are expensive, so their use is warranted only if a large number of small-diameter pipes are to be laid or the pipes must regularly be inspected.

In the USSR, type-design single-tier scaffold bridges carrying loads up to 0.5 tf $m^{-1}$ (5 kN $m^{-1}$) use reinforced concrete columns and metal trusses 12 or 18 m long. The cross

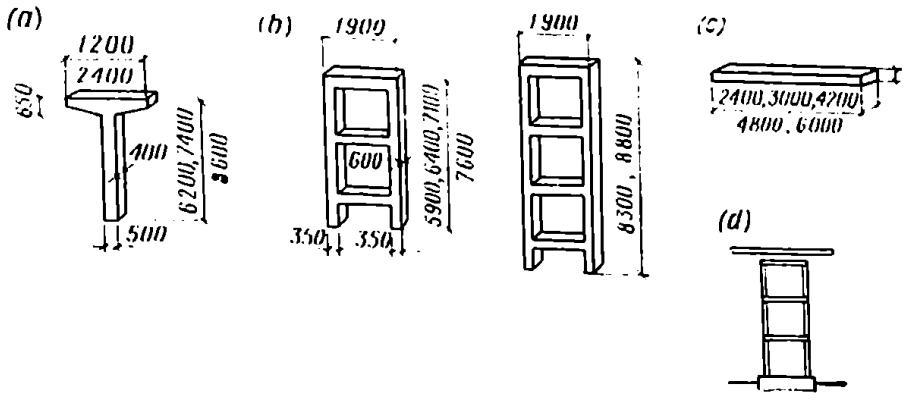



Fig. XIII.30. Reinforced concrete members of individual piers, used when the load exceeds 3 tf

(a) T-shaped columns; (b) U-shaped columns; (c) cross arms; (d) U-shaped column with cross arm

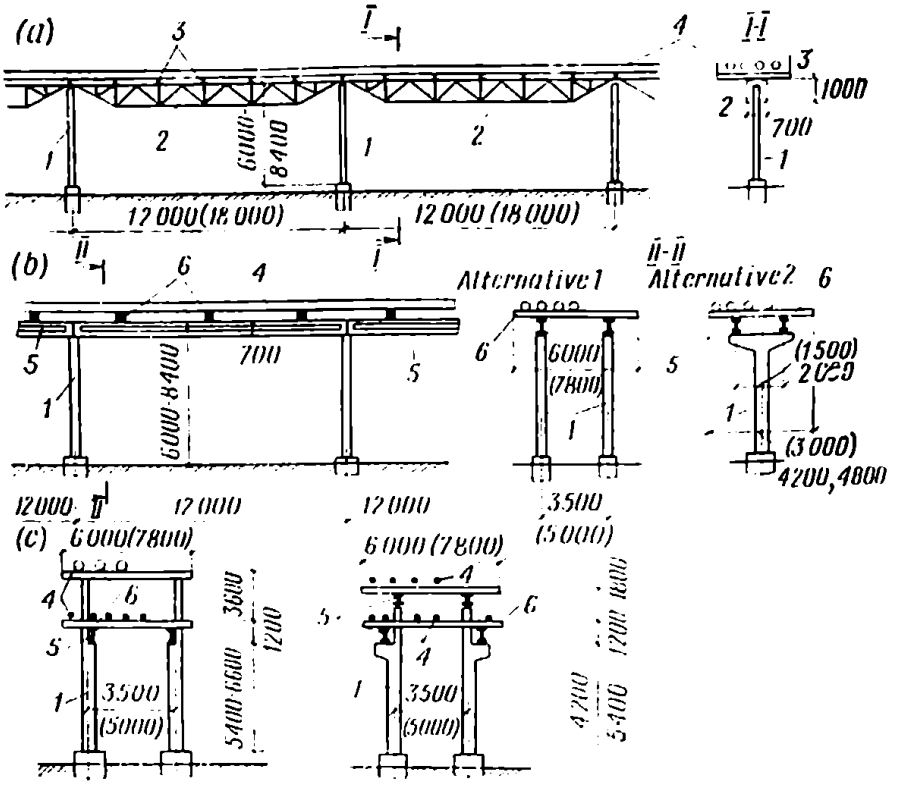
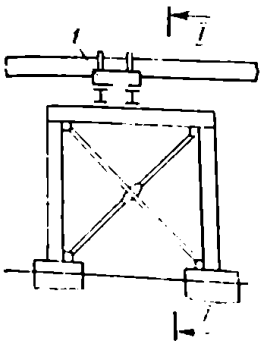



Fig. XIII.31. Construction types of scaffold bridges
*1*—reinforced concrete columns; *2*—metal trusses; *3*—metal cross arms; *4*—pipelines; *5*—reinforced concrete beams; *6*—reinforced concrete cross arms

arms installed on the trusses are likewise made of metal (Fig. XIII.31*a*). If the load is from 0.5 to 4 tf $m^{-1}$ (5 to 40 kN $m^{-1}$) and the bridge height is from 6 to 8.4 m, all members of a single-tier scaffold bridge are built of precast reinforced concrete. The piers may be either single-strut (assembled from straight, T-shaped, or U-shaped columns) or double-strut (assembled from straight columns). They carry longitudinal T-section prestressed reinforced concrete beams 12 m long supporting cross arms 3 to 7.8 m long (Fig. XIII.31*b*).

The piers of double-tier reinforced concrete scaffold bridges (Fig. XIII.31*c*) are likewise spaced 12 m apart. The longitudinal beams may be installed at two levels or only



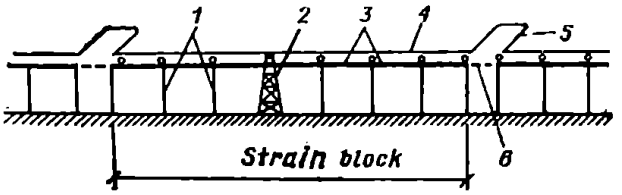




Fig. XIII.33. Arrangement of expansion U-bends in pipelines laid on scaffold bridges
*1*—intermediate pier; *2*—anchor pier; *3*—trusses or beams; *4*—heat pipeline; *5*—expansion U-bend; *6*—insert

Fig. XIII.32. Anchor pier
*1*—immovable pipeline support

at the bottom level if the top-tier pipelines rest on the cross arms only above the piers.

Along its length, a scaffold bridge has two types of piers, namely intermediate and anchor (or dead-end) piers. The former carry the movable supports of pipelines, and the latter, the immovable supports.

An anchor pier is usually installed in the middle of a strain block which may be as long as 72 m. It consists of individual columns joined together by metal braces (Fig. XIII.32). Anchor piers ensure the spatial rigidity of the entire structure.

Expansion U-bends are installed in gaps 3 or 6 m wide between strain blocks (Fig. XIII.33). The gaps are bridged by single-span beams which are free to move when the blocks undergo thermal deformations.

Brackets used to support pipes when heat lines are run on the walls of buildings are made of steel angles or channels (Fig. XIII.34). The brackets which are built into walls are distributed so as not to impair the strength and stability of the walls. In skeleton-type buildings, they are fastened to the embedded steel parts of the reinforced con-

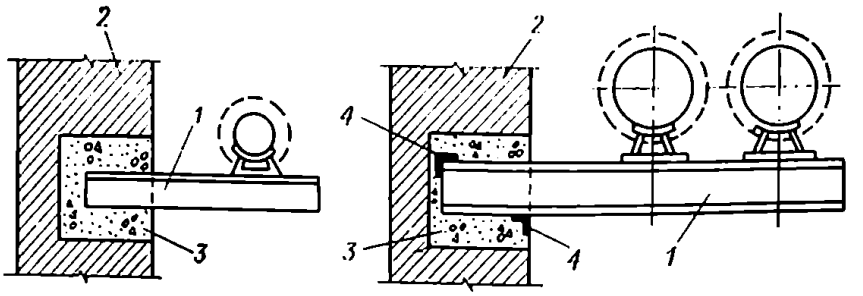



Fig. XIII.34. Brackets attached to a wall
*1*—bracket; *2*—wall; *3*—concrete; *4*—distribution pad

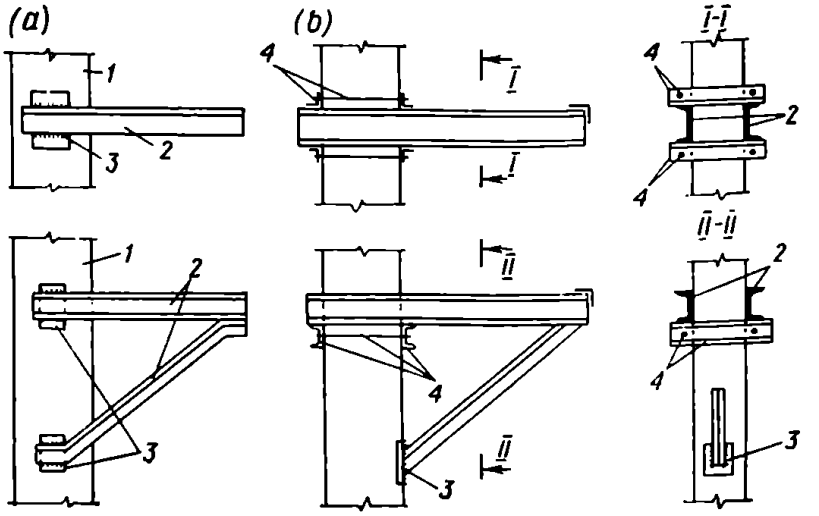



Fig. XIII.35. Brackets attached to reinforced concrete columns
*1*—columns; *2*—bracket; *3*—embedded part; *4*—hoop

crete columns (Fig. XIII.35*a*) or to hoops embracing the columns (Fig. XIII.35*b*).

The movable supports of overhead heat pipelines may be of the sliding or the roller type as shown in Fig. XIII.25, or of the suspension type as shown in Fig. XIII.36. Immovable supports are installed on anchor piers similarly to Fig. XIII.20. On brackets, immovable supports are made as follows. Two angles are welded to a pipe on either side of a bracket, and a hoop embracing the pipe is then faste-

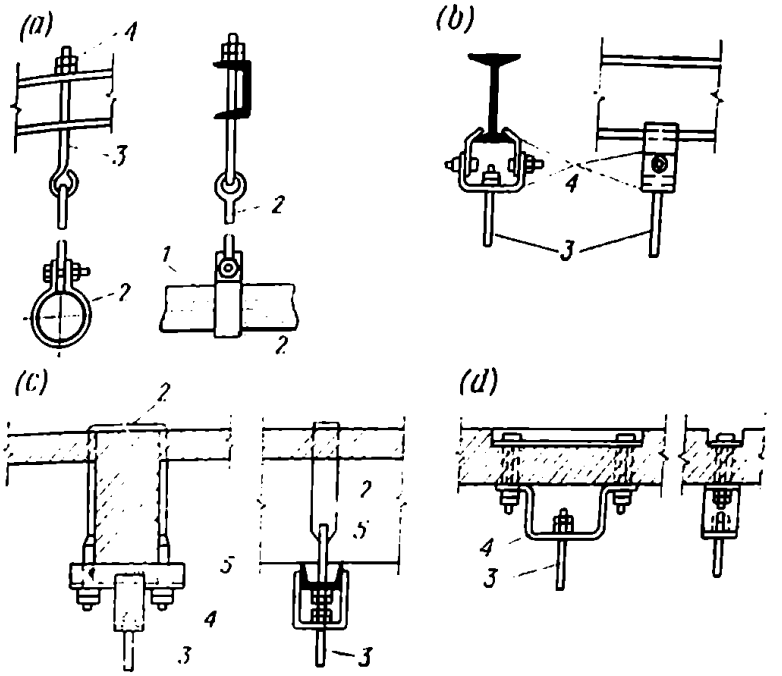



Fig. XIII.36. Suspension-type supports

(a) and (b) fastened to steel beam; (c) fastened to reinforced concrete beam; (d) fastened to slab; 1—heat pipeline; 2—hoop; 3—suspension tie-rod; 4—brackets 5—channel

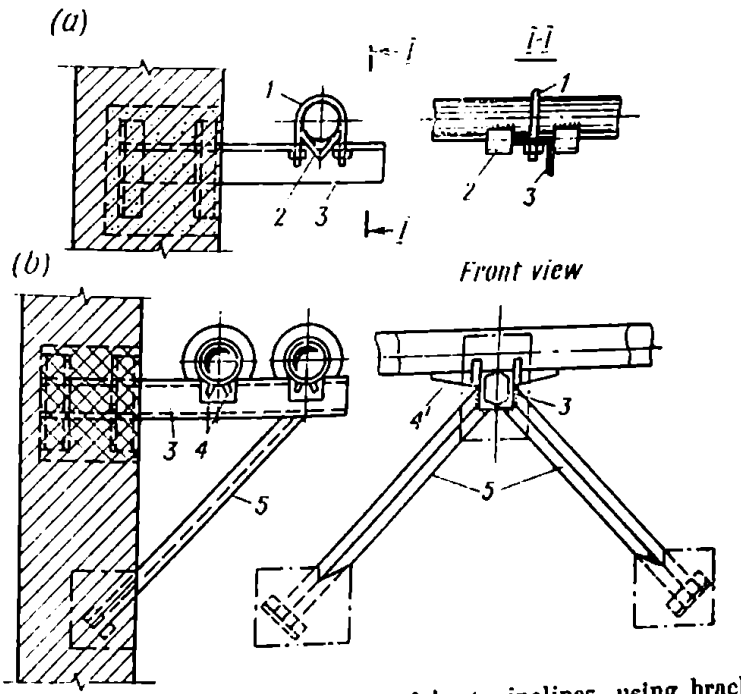



Fig. XIII.37. Immovable supports of heat pipelines, using brackets

1—hoop; 2—angle stops; 3—bracket; 4—strip stops; 5—bracket braces

ned to the bracket (Fig. XIII.37*a*). When they carry pipes larger than 100 mm in diameter, brackets are fastened to walls with additional braces (Fig. XIII.37*b*).

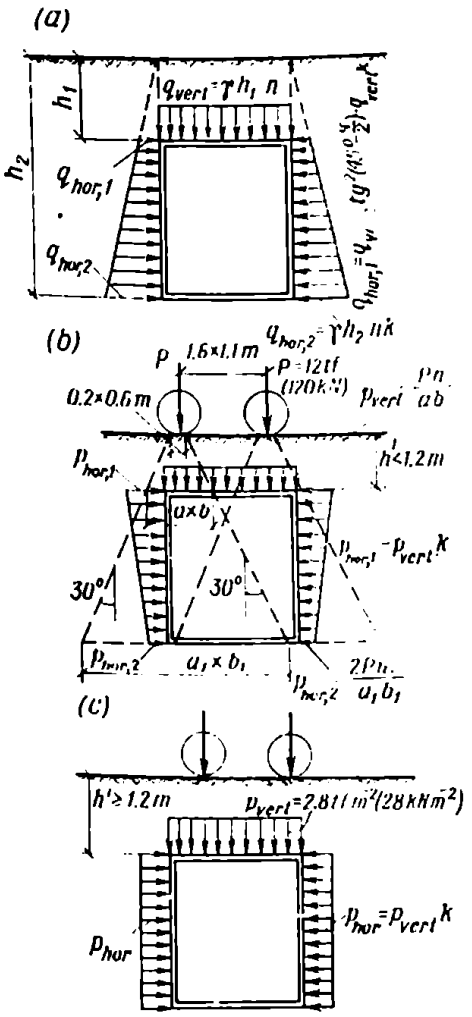



Fig. XIII.38. Loads imposed on a buried conduit

## XIII.6. Design Principles for Heat Pipeline Structures

The conduits, servicing chambers, and compensating niches of buried heat pipelines carry the vertical and lateral earth pressure, the live load induced on the ground surface by vehicles, the weight of the pipelines, and the horizontal forces due to the unbalanced internal pressure and thermal movement of the pipes.

When a heat pipeline is laid in open trenches, the vertical and lateral earth pressure increases linearly with increasing depth (Fig. XIII.38*a*). The respective design vertical load per unit area (in tf m$^{-2}$ or N m$^{-2}$) is given by

$$q_{vert} = \gamma h n$$

where $h$ is the depth in m, $\gamma$ is the bulk unit weight of the soil in t m$^{-3}$ (on the average, $\gamma = 1.8$ t m$^{-3}$), and $n$ is the overload factor taken as 1.2.

The design horizontal load in tf m$^{-2}$ (N m$^{-2}$) is

$$q_{hor} = q_{vert} \operatorname{tg}^2 (45° - \varphi/2) = q_{vert} k$$

where $\varphi$ is the angle of internal friction of the soil (on the average, $\varphi = 30°$).

The wheel load applied at the surface is assumed to propagate at 30° to the vertical in the soil, and at 45° to the vertical in the road pavement. As a consequence, the vertical and horizontal pressure due to the wheel load decreases with increasing depth (Fig. XIII.38*b*). According to relevant standards and specifications, the wheel load should be multiplied by the overload factor $n = 1.4$.

If the top of a structure is 1.2 m or more below the surface (Fig. XIII.38*c*), the characteristic vertical wheel load is assumed to be uniformly distributed and equal to 2 tf $m^{-2}$ (20 kN $m^{-2}$) at any depth $h$ from the surface. The design load is $p_{vert} = 1.4 \times 2 = 2.8$ tf $m^{-2}$ (28 kN $m^{-2}$). The horizontal load is determined similarly to that induced by lateral earth pressure, that is,

$$p_{hor} = p_{vert} \operatorname{tg}^2 (45° - \varphi/2) = p_{vert} k$$

In buried conduits, servicing chambers, and compensating niches the vertical load induced by the weight of the soil above the roof and the self-weight of the structures (including the weight of the pipelines), and also the wheel load is transferred to the foundation material and balanced by the contact soil pressure. The pressure exerted by the total characteristic load should not exceed the design soil strength $R_s$.

In structures having brick or concrete walls and a concrete bottom, the width of the strip through which the load $N^{ch}$ is transferred to the foundation material (Fig. XIII.39*a*) is given by

$$b_f = b + 2h_c$$

where $b$ is the wall thickness at the bottom and $h_c$ is the thickness of the concrete layer which transfers the load at 45° to the vertical. The load $N^{ch}$ is usually calculated per metre run of the wall, so the footing area is equal to $(b_f \times \times 100)$ $cm^2$. As a consequence, the design formula takes the form

$$N^{ch}/100 b_f \leqslant R_s$$

If a structure has a reinforced concrete bottom, the pressure exerted on the soil is distributed as shown in Fig.

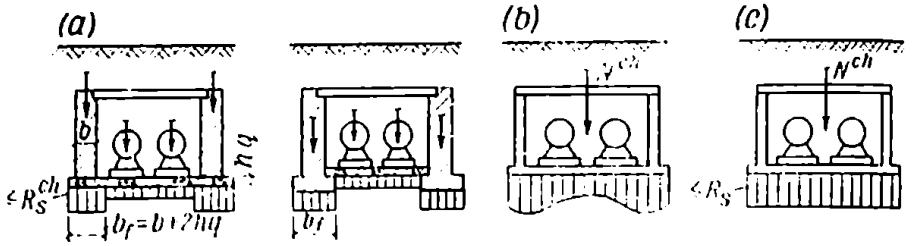



Fig. XIII.39. Pressure beneath the floor and walls of conduits

XIII.39*b*. For design purposes, it is assumed to be uniformly distributed over the entire bottom area (Fig. XIII.39*c*). The bottom is designed to carry the contact soil pressure under the action of the design loads. The design loading system is shown in Fig. XIII.40. The lateral earth pressure is taken to be applied on either side of the structure, because the soil (with each layer properly compacted) is backfilled on either side of the conduit. The horizontal wheel load may be applied on either side or on one side of the structure.

The design system for the individual members depends on their construction.

Since they are mostly hinged to the walls and beams, the roof slabs are analyzed as single-span panels carrying the uniformly distributed loads $q_{vert}$ and $p_{vert}$ and the self-weight $q_{s\text{-}w}$.

The slab is designed to carry the maximum bending moment induced by these loads. The necessary area of its longitudinal load-bearing reinforcement is found from a check on the normal section shear strength.

The brick and concrete block walls are hinged at the roof and bottom. The wall resists the vertical load induced by the roof slabs it carries and its self-weight, and also the horizontal loads $q_{hor}$ and $p_{hor}$. For design purposes, the wall of a long conduit can be treated as a vertical single-span beam 1 m wide hinged to the roof and bottom. The wall is designed to carry the bending moment induced by $q_{hor}$ and $p_{hor}$ and the shearing force due to $q_{vert}$, $p_{vert}$, $q_{s\text{-}w}$, and its self-weight, using the expressions given in Chap. VIII for masonry structures in eccentrical compression.

If the ratio of the wall length $l$ to its height $h$ is smaller than or equal to 2 : 1 (as often happens in servicing cham-

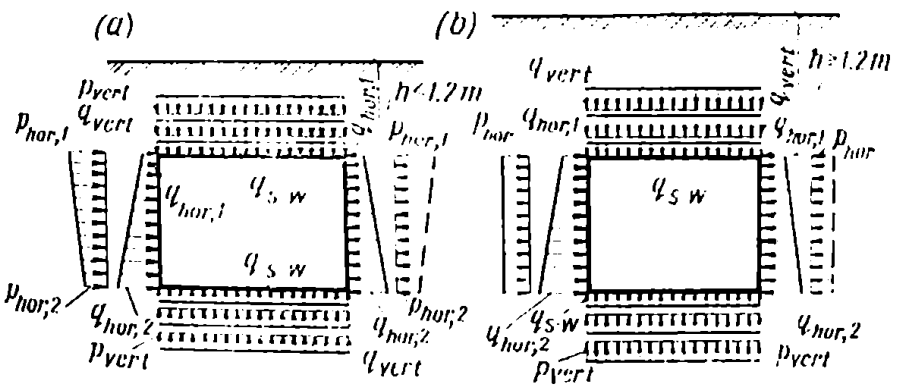



Fig. XIII.40. Design loading system for conduits, tunnels, and servicing chambers with a reinforced concrete floor
(a) embedment depth smaller than 1.2 m; (b) embedment depth equal to or larger than 1.2 m

bers), the wall may be treated as a slab supported on the transverse walls, bottom, and roof. The bending moments in it are determined using the tables for the design of rectangular isotropic slabs (see Appendix XVIII).

The walls should also be checked for strength with the roof slabs removed. In this case, as already mentioned in Sec. XIII.2, the top support for the wall consists of spacers and, as a consequence, its design system remains the same, but the bending moment is assumed to be induced only by the lateral earth pressure (the load $q_{hor}$) and the normal force only by the self-weight of the wall. The wheel load need not be taken into consideration.

In reinforced concrete conduits and tunnels assembled from prefab members, the walls are rigidly fixed at the bottom and hinged at the top to the roof slabs. The design system for such a conduit (tunnel) is a U-shaped frame closed by a hinged rigid spacer (the roof slab) (Fig. XIII.41).

In this frame, we determine the thrust $X$ and the bending moment in any wall or bottom cross section, which is found as the sum of the moments induced by all the forces (including the thrust $X$) applied on one side of the section in question (Fig. XIII.41*b* and *c*).

The load imposed on the roof (and the respective contact pressure from the bottom) gives rise to a compressive force in the walls, and the lateral load imposed on the wall, to

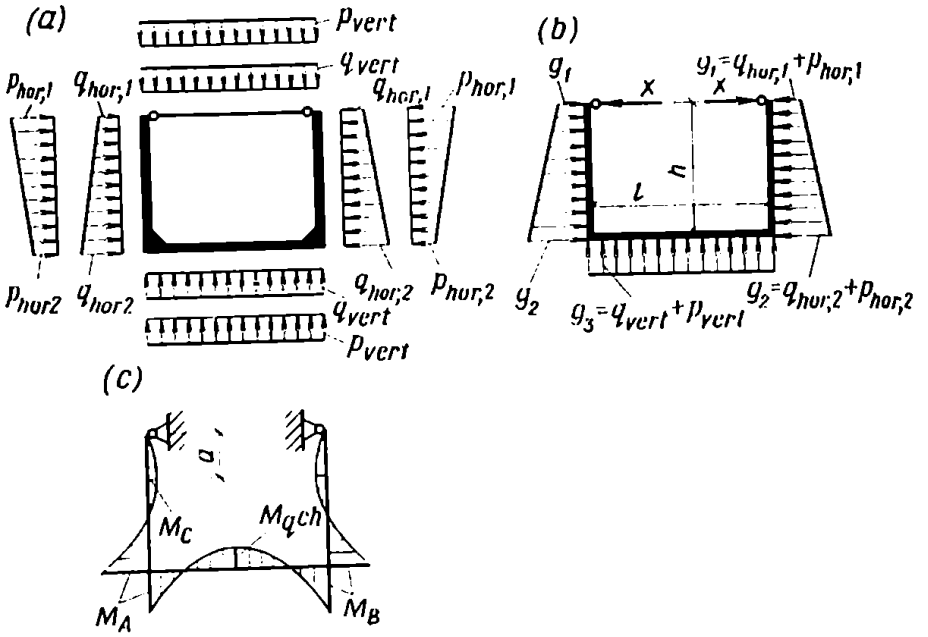



Fig. XIII.41. To the design of precast reinforced concrete conduits and tunnels
(a) loading system; (b) basic design system; (c) bending moment diagram

a compressive force in the bottom. Therefore, the walls and the bottom work in eccentrical compression. Since the wall and bottom are relatively thin, the eccentricity of the longitudinal force, $e = M/N$, is considerably larger than the cross-sectional dimensions. So the necessary steel area may be determined as for bending members (using the moment $M$). When a conduit assembled from U-shaped members has no roof, its walls should resist lateral earth pressure, working as cantilevers fixed at the bottom (Fig. XIII.42). This design system is frequently used to determine the necessary area of the load-bearing reinforcement in U-shaped members. Tunnels are always analyzed with the roof slabs installed (see Fig. XIII.40).

In overhead heat pipelines installed on individual piers, the steel pipes should be checked for strength. Also, it is necessary to determine the pipe sag which should not exceed the maximum safe value given in Sec. XIII.5.

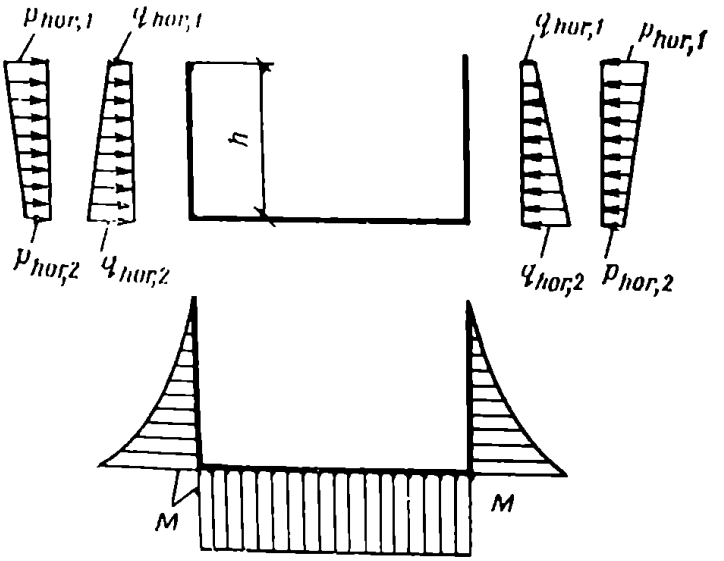



Fig. XIII.42. Design system and bending moment diagram for a precast reinforced concrete conduit with the roof slab removed

In design calculations, a pipe is treated as a multispan continuous beam carrying a uniformly distributed load $q$ due to the self-weight of the pipe and the weight of the thermal insulation and the heat-transfer agent, multiplied by the overload factor 1.2. The snow and ice load is not taken into account. The same design system applies to lateral wind loading, the wind load per metre run of the pipe being

$$q_w = 1.2 \times 1.4Dq_0$$

where 1.2 is the overload factor, 1.4 is the aerodynamic factor, $D$ is the outside diameter of the pipe including the thermal insulation (in cm), and $q_0$ is the characteristic wind velocity (dynamic) head in kgf $m^{-2}$ (N $m^{-2}$) which depends on the locality and height of the structure (see Appendix II).

If a major pipe carries smaller pipes, the loads due to these pipes are assumed to be concentrated. When checking the major pipe for strength, it is necessary to remember that, in addition to the above loads, the pipe walls experience stresses induced by the internal pressure of the heat-transfer agent, axial force appearing because of constrained tempe-

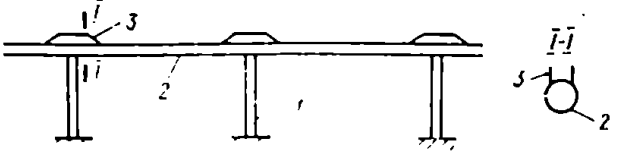



Fig. XIII.43. Pipes reinforced with longitudinal ribs
*1*—pier; *2*—pipe; *3*—ribs

rature deformations (as a result of friction at the supports and in the U-bends), and the like. The design procedure for pipelines can be found in appropriate standards.

All the loads discussed above (except the wind load) apply to both buried and overhead pipelines.

The distance between the piers can be increased by welding longitudinal ribs to the most heavily loaded portions above the supports (Fig. XIII.43).

Piers carrying movable supports are designed as vertical cantilever beams fixed at the footing (Fig. XIII.44*a*). The vertical load consists of the support reaction $N_1$ due to the weight of the pipelines and the self-weight $N_2$ of a pier, which is assumed to be applied from the top. In the plane perpendicular to the pipelines, each pier resists a wind load $N_w$ which is taken as the resultant of the wind pressure exerted on the pipes between the piers being analyzed, and wind load $p_w$ which is applied directly to the pier. Along the axis of the pipelines, a pier carries a horizontal load $N_{fr}$ induced by friction in the pipes. The horizontal loads due to wind and friction may act in either direction.

The effective length of piers, from which their slenderness ratio can be found, is taken equal to twice their height.

The footing of a pier is checked in either direction as an eccentrically loaded structure.

For double-strut piers, the manner of load application is the same (Fig. XIII.44*b*). A cross arm is usually hinged to the struts, therefore it is regarded as a single-span beam carrying the applied loads, and the struts are treated as vertical cantilevers subjected to the support reaction of the cross arm and the wind load.

Intermediate piers in scaffold bridges are designed in a similar way, but in calculating the vertical load, one should

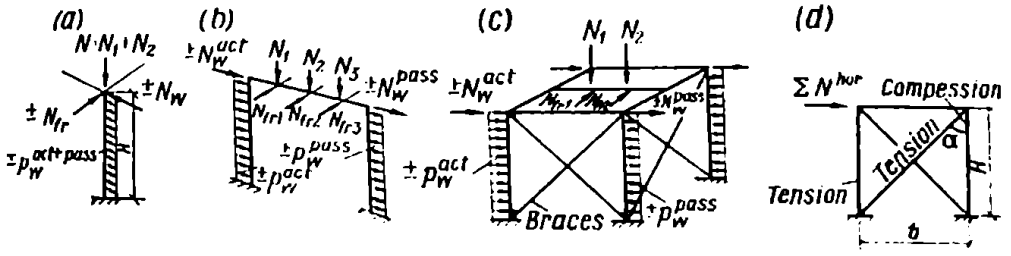



Fig. XIII.44. To the design of piers carrying overhead heat pipelines

take into consideration the weight of the spans (trusses or beams). Anchorage piers are subjected to vertical loads induced by their self-weight and the weight of the pipelines they carry, horizontal loads transferred by the pipelines, and wind load (Fig. XIII.44*c*).

For simplicity, the space system formed by the struts, girders, and cross braces is divided into component members. The girders are designed to carry their share of load as single-span beams. The horizontal load in either direction is assumed to be resisted by a flat system consisting of two struts combined by the cross braces (Fig. XIII.44*d*). Here, the force $\Sigma N_{hor}$ induces the following forces in the struts

$$N_{str} = \Sigma N_{hor} H/b$$

The above force is compressive in one strut and tensile in the other, and is added to the compressive forces due to the vertical loads. If the force $\Sigma N_{hor}$ is large, the compressive forces induced by the vertical loads may prove insufficient to balance the tensile force. This must be kept in mind in the analysis and design of the struts. The force $N_{str}$ decreases with increasing distance between the struts. The forces in the cross braces can approximately be determined if we assume that when one of the braces begins to work in compression, its contribution to the strength of the entire structure is zero (because of its slenderness), and the other brace carries a tensile force

$$\Sigma N_{hor}/\sin \alpha$$

The connection between the braces and struts should also be designed to resist this force.

The spans of scaffold bridges are analyzed as single-span beams or trusses carrying a vertical load due to their self-weight and the weight of the pipelines they support, and a lateral wind load.

*APPENDIX I*

**Units of Measure and Their Conversion**

| Quantity | Unit of measure and its designation | | Conversion |
|---|---|---|---|
| | USSR building code | SI | |
| Force, load, weight | kilogram-force (kgf)<br>tonne-force (tf)<br>(1 tf = 1 000 kgf) | newton (N)<br>kilonewton (kN)<br>(1 kN = 1 000N) | 1 kgf = 9.8 N<br>≈ 10 N<br>1 tf ≈ 10 000 N<br>= 10 kN |
| Linear load, surface load | kilogram-force per metre run (kgf $m^{-1}$)<br>tonne-force per metre run (tf $m^{-1}$)<br>kilogram-force per square metre (kgf $m^{-2}$)<br>tonne-force per square metre (tf $m^{-2}$) | newton per metre run (N $m^{-1}$)<br>kilonewton per metre run (kN $m^{-1}$)<br>newton per square metre (N $m^{-2}$)<br>kilonewton per square metre (kN $m^{-2}$) | 1 kgf $m^{-1}$<br>≈ 10 N $m^{-1}$<br>1 tf $m^{-1}$ ≈ 10 kN $m^{-1}$<br>1 kgf $m^{-2}$<br>≈ 10 N $m^{-2}$<br>1 tf $m^{-2}$<br>≈ 10 kN $m^{-2}$ |
| Stress, pressure, moduli of elasticity | kilogram-force per square millimetre (kgf $mm^{-2}$)<br>kilogram-force per square centimetre (kgf $cm^{-2}$)<br>tonne-force per square metre (tf $m^{-2}$) | pascal (Pa)<br>(1 Pa ≈ 0.1 mm $H_2O$)<br>megapascal (MPa)<br>(1 MPa = $10^6$ Pa) | 1 kgf $mm^{-2}$<br>≈ $10^{-7}$ Pa<br>= 10 MPa<br>1 kgf $cm^{-2}$<br>≈ $10^5$ Pa<br>= 0.1 MPa<br>1 tf $m^{-2}$<br>≈ $10^6$ Pa<br>= 1 MPa |
| Moment of force, moment of couple | kilogram-force × metre (kgf m)<br>tonne-force × metre (tf m) | newton × metre (N m)<br>kilonewton × metre (kN m) | 1 kgf m<br>≈ 10 N m<br>1 tf m<br>≈ 10 kN m |

*APPENDIX II*

**Characteristic Loads and Overload Factors**

| Load | Overload factor |
|---|---|
| Weight of structural members of concrete ($\gamma > 1\,800$ kg $m^{-3}$), reinforced concrete, stone, metal, and wood | 1.1 |

| Load | Overload factor |
|---|---|
| Weight of concrete members ($\gamma \leqslant \leqslant 1\,800$ kg m$^{-3}$) | 1.2 |
| Weight of insulating, levelling, and finishing layers: | |
| —applied at factory | 1.2 |
| —applied at job | 1.3 |
| Natural soils | 1.1 |
| Fill-up soils | 1.2 |
| Self-weight of permanent equipment and weight of its insulation | 1.2 |
| Weight of materials contained in equipment: | |
| —liquids | 1.1 (1 for pipelines) |
| —suspensions, slurries, and particulate materials | 1.2 (1.1 for pipelines) |
| Load due to overhead travelling cranes, overhead-track hoists, loaders, and trucks | 1.2 |
| Uniformly distributed load imposed on floors: | 1.2 for $p^{ch} \geqslant 500$ kgf m$^{-2}$ (5 kN m$^{-2}$) |
| —in apartments, 150 kgf m$^{-2}$ (1.5 kN m$^{-2}$) | 1.3 for $p^{ch} = 200$ to 500 kgf m$^{-2}$ (2 to 5 kN m$^{-2}$) |
| —in offices and reading-halls, 200 kgf m$^{-2}$ (2 kN m$^{-2}$) | 1.4 for $p^{ch} < 200$ kgf m$^{-2}$ (2 kN m$^{-2}$) |
| —in laboratories, min. 200 kgf m$^{-2}$ (2 kN m$^{-2}$) | |
| —auditoriums, gymnasiums, balconies, 400 kgf m$^{-2}$ (4 kN m$^{-2}$) | |
| —in libraries and archives, min. 500 kgf m$^{-2}$ (5 kN m$^{-2}$) | |
| —in attics (in addition to weight of equipment), 75 kgf m$^{-2}$ (0.75 kN m$^{-2}$) | |
| —in lobbies, corridors, and stairs, 300 to 400 kgf m$^{-2}$ (3 to 4 kN m$^{-2}$) | |
| Snow load (depends on region): | |
| —weight of snow $P_0 = 50$ to 250 kgf m$^{-2}$ (0.5 to 2.5 kN m$^{-2}$) | 1.4 to 1.6 depending on relative magnitudes of roof self-weight and weight of snow |
| $P_{sn}^{ch} = P_0\, c$ | |
| where $c$ depends on roof shape | |
| Wind load (depends on region): | |
| —velocity (dynamic) head, $q_0 = 27$ to 100 kgf m$^{-2}$ (0.27 to 1 kN m$^{-2}$) | 1.2 (1.3 for high-rise buildings) |
| $q_w^{ch} = q_0\, kc_1$ | |

*Appendix II (continued*

| Load | Overload factor |
|---|---|
| where $k$ accounts for building height and type of locality and $c_1$ is aerodynamic factor depending on building shape | |

*Note.* When a dead load reduction adversely affects the exposure conditions of a structure (say, in a check on stability against overturning), the overload factor is taken to be 0.9.

*APPENDIX III*

## Characteristic Strength $R_{pr}^{ch}$ and $R_{ten}^{ch}$ and Design Strength $P_{pr\ II}$ and $R_{ten\ II}$ of Heavy Concrete, kgf cm$^{-2}$ (MPa)

| Quantity | Concrete brand number | | | | | | |
|---|---|---|---|---|---|---|---|
| | M-100 | M-150 | M-200 | M-300 | M-400 | M-500 | M-600 |
| Axial compressive (prism crushing) strength, $R_{pr}^{ch}$ and $R_{pr\ II}$ | 60 (6) | 85 (8.5) | 115 (11.5) | 170 (17) | 225 (22.5) | 280 (28) | 340 (34) |
| Axial tensile strength, $R_{ten}^{ch}$ and $R_{ten\ II}$ | 7.5 (0.75) | 9.5 (0.95) | 11.5 (1.15) | 15 (1.5) | 18 (1.8) | 20 (2) | 22 (2.2) |

*Note.* For concretes using alumina cement as the binding agent, the values of $R_{ten}^{ch}$ and $R_{ten\ II}$ should be multiplied by 0.7.

*APPENDIX IV*

**Design Strength $R_{pr}$ and $R_{len}$ of Heavy Concrete in Terms of the First Group of Limit States, kgf cm$^{-2}$ (MPa)**

| Quantity | Concrete brand number | | | | | | |
|---|---|---|---|---|---|---|---|
| | M-100 | M-150 | M-200 | M-300 | M-400 | M-500 | M-600 |
| Axial compressive (prism crushing) strength, $R_{pr}$ | 45 (4.5) | 70 (7) | 90 (9) | 135 (13.5) | 175 (17.5) | 215 (21.5) | 245 (24.5) |
| Axial tensile strength, $R_{len}$ | 4.8 (0.48) | 6.3 (0.63) | 7.5 (0.75) | 10 (1) | 12 (1.2) | 13.5 (1.35) | 14.5 (1.45) |

*Note.* For concretes using alumina cement as the binding agent, the values of $R_{len}$ should be multiplied by 0.7.

*APPENDIX V*

**Tangent Modulus of Elasticity $E_c \times 10^{-3}$ for Heavy Air-Cured Concrete in Compression and Tension, kgf cm$^{-2}$ (MPa)**

| Concrete brand number | M-100 | M-150 | M-200 | M-300 | M-400 | M-500 | M-600 |
|---|---|---|---|---|---|---|---|
| Tangent modulus of elasticity | 170 (17) | 210 (21) | 240 (24) | 290 (29) | 330 (33) | 360 (36) | 380 (38) |

*Note.* For concretes heat-cured under atmospheric pressure, $E_c$ is multiplied by 0.9, for autoclaved concretes, $E_c$ is multiplied by 0.75.

*APPENDIX VI*

## Characteristic and Design Strength and Modulus of Elasticity of Bar Reinforcement, kgf cm$^{-2}$ (MPa)

| Class of steel | Characteristic tensile strength $R_s^{ch}$ and design tensile strength $R_s$ II for second group of limit states | Design strength for first group of limit states | | | Modulus of elasticity, $E_s \times 10^{-6}$ |
|---|---|---|---|---|---|
| | | tensile | | compressive | |
| | | longitudinal, transverse, and bent-up bars in inclined-section analysis under bending moment, $R_s$ | transverse and bent-up bars in inclined-section analysis under shearing force, $R_{s,\,tr}$ | $R_{s,\,com}$ | |
| A-I | 2 400 (240) | 2 100 (210) | 1 700 (170) | 2 100 (210) | 2.1 |
| A-II | 3 000 (300) | 2 700 (270) | 2 150 (215) | 2 700 (270) | 2.1 |
| A-III: | | | | | |
| 6-8 mm in diameter | 4 000 (400) | 3 400 (340) | 2 700* (270) | 3 400 (340) | 2.0 |
| 10-40 mm in diameter | 4 000 (400) | 3 600 (360) | 2 900* (290) | 3 600 (360) | 2.0 |
| A-IV | 6 000 (600) | 5 000 (500) | 4 000 (400) | 4 000 (400) | 2.0 |
| A-V | 8 000 (800) | 6 400 (640) | 5 100 (510) | 4 000 (400) | 1.9 |
| Ат-IV | 6 000 (600) | 5 000 (500) | 4 000 (400) | 4 000 (400) | 1.9 |
| Ат-V | 8 000 (800) | 6 400 (640) | 5 100 (510) | 4 000 (400) | 1.9 |
| Ат-VI | 10 000 (1 000) | 8 000 (800) | 6 400 (640) | 4 000 (400) | 1.9 |

* For class A-III transverse bars in welded-bar mats, whose diameter is smaller than 1/3 of the longitudinal bar diameter, $R_{s,\,tr}$ is taken as 2 400 kgf cm$^{-2}$ (240 MPa) for bars 6 to 8 mm in diameter and as 2 500 kgf cm$^{-2}$ (250 MPa) for bars 10 to 40 mm in diameter.

*APPENDIX VII*

## Characteristic and Design Strength and Modulus of Elasticity of Wire Reinforcement, kgf cm$^{-2}$ (MPa)

| Class of steel | Wire diameter, mm | Characteristic tensile strength $R_s^{ch}$ and design tensile strength $R_s$ II for second group of limit states | Design strength for first group of limit states | | | Modulus of elasticity, $E_s \times 10^{-6}$ |
|---|---|---|---|---|---|---|
| | | | tensile | | compressive | |
| | | | longitudinal, transverse, and bent-up bars in inclined-section analysis under bending moment, $R_s$ | transverse and bent-up bars in inclined-section analysis under shearing force, $R_{s,\,tr}$ | $R_{s,\,com}$ | |
| B-I | 3-5 | 5 500 (550) | 3 150 (315) | 2 200 (220) | 3 150 (315) | ·2 (0.2) |
| Bp-I | 3-4 | 5 500 (550) | 3 500 (350) | 2 600 (260) | 3 500 (350) | 1.7 (0.17) |
| | 5 | 5 250 (525) | 3 400 (340) | 2 500 (250) | 3 400 (340) | 1.7 (0.17) |
| B-II | 3 | 19 000 (1 900) | 12 300 (1 230) | 9 800 (980) | 4 000 (400) | 2 (0.2) |
| | 4 | 18 000 (1 800) | 11 600 (1 160) | 9 300 (930) | 4 000 (400) | 2 (0.2) |
| | 5 | 17 000 (1 700) | 11 000 (1 100) | 8 800 (880) | 4 000 (400) | 2 (0.2) |
| | 6 | 16 000 (1 600) | 10 300 (1 030) | 8 300 (830) | 4 000 (400) | 2 (0.2) |
| | 7 | 15 000 (1 500) | 9 700 (970) | 7 700 (770) | 4 000 (400) | 2 (0.2) |
| | 8 | 14 000 (1 400) | 9 000 (900) | 7 200 (720) | 4 000 (400) | 2 (0.2) |
| Bp-II | 3 | 18 000 (1 800) | 11 600 (1 160) | 9 300 (930) | 4 000 (400) | 2 (0.2) |
| | 4 | 17 000 (1 700) | 11 000 (1 100) | 8 800 (880) | 4 000 (400) | 2 (0.2) |
| | 5 | 16 000 (1 600) | 10 300 (1 030) | 8 300 (830) | 4 000 (400) | 2 (0.2) |
| | 6 | 15 000 (1 500) | 9 700 (970) | 7 700 (770) | 4 000 (400) | 2 (0.2) |
| | 7 | 14 000 (1 400) | 9 000 (900) | 7 200 (720) | 4 000 (400) | 2 (0.2) |
| | 8 | 13 000 (1 300) | 8 400 (840) | 6 700 (670) | 4 000 (400) | 2 (0.2) |
| K-7 | 4.5 | 19 000 (1 900) | 12 300 (1 230) | 9 800 (980) | 4 000 (400) | 1.8 (0.18) |
| | 6 | 18 550 (1 855) | 11 900 (1 190) | 9 500 (950) | 4 000 (400) | 1.8 (0.18) |
| | 7.5 | 18 000 (1 800) | 11 600 (1 160) | 9 300 (930) | 4 000 (400) | 1.8 (0.18) |
| | 9 | 17 500 (1 750) | 11 300 (1 130) | 9 000 (900) | 4 000 (400) | 1.8 (0.18) |
| | 12 | 17 000 (1 700) | 11 000 (1 100) | 8 800 (880) | 4 000 (400) | 1.8 (0.18) |
| | 15 | 16 500 (1 650) | 10 600 (1 060) | 8 500 (850) | 4 000 (400) | 1.8 (0.18) |

APPENDIX VIII

## Design Cross-Sectional Area and Weight of Reinforcement. Available Hot-Rolled Deformed Bars and Ordinary and High-Strength Wire

| Diameter, mm | Design cross-sectional area, cm², number of bars | | | | | | | | | | Weight, kg m⁻¹ | Hot-rolled deformed bars | | | | | | | Wire | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | A-II | A-III | A-IV | At-IV | A-V | At-V | At-VI | ordinary | high-strength |
| 3 | 0.071 | 0.14 | 0.21 | 0.28 | 0.35 | 0.42 | 0.49 | 0.57 | 0.64 | 0.71 | 0.055 | — | — | — | — | — | — | — | × | × |
| 3.5 | 0.096 | 0.19 | 0.29 | 0.38 | 0.48 | 0.58 | 0.67 | 0.77 | 0.86 | 0.96 | 0.075 | — | — | — | — | — | — | — | × | — |
| 4 | 0.126 | 0.25 | 0.38 | 0.5 | 0.63 | 0.76 | 0.88 | 1.01 | 1.13 | 1.26 | 0.098 | — | — | — | — | — | — | — | × | × |
| 4.5 | 0.159 | 0.32 | 0.48 | 0.64 | 0.8 | 0.95 | 1.11 | 1.27 | 1.43 | 1.59 | 0.125 | — | — | — | — | — | — | — | × | × |
| 5 | 0.196 | 0.39 | 0.59 | 0.79 | 0.98 | 1.18 | 1.37 | 1.57 | 1.77 | 1.96 | 0.154 | — | — | — | — | — | — | — | × | × |
| 5.5 | 0.238 | 0.48 | 0.71 | 0.95 | 1.19 | 1.43 | 1.66 | 1.9 | 2.14 | 2.38 | 0.188 | — | — | — | — | — | — | — | × | × |
| 6 | 0.283 | 0.57 | 0.85 | 1.13 | 1.42 | 1.7 | 1.98 | 2.26 | 2.55 | 2.83 | 0.222 | — | × | — | — | — | — | — | × | × |
| 7 | 0.385 | 0.77 | 1.15 | 1.54 | 1.92 | 2.31 | 2.69 | 3.08 | 3.46 | 3.85 | 0.302 | — | × | — | — | — | — | — | × | × |
| 8 | 0.503 | 1.01 | 1.51 | 2.01 | 2.51 | 3.02 | 3.52 | 4.02 | 4.53 | 5.03 | 0.395 | — | × | — | — | — | — | — | × | × |
| 9 | 0.636 | 1.27 | 1.91 | 2.54 | 3.18 | 3.82 | 4.45 | 5.09 | 5.72 | 6.36 | 0.499 | — | × | — | — | — | — | — | × | × |
| 10 | 0.785 | 1.57 | 2.36 | 3.14 | 3.93 | 4.71 | 5.5 | 6.28 | 7.07 | 7.85 | 0.617 | × | × | × | × | × | × | × | × | × |
| 12 | 1.131 | 2.26 | 3.39 | 4.52 | 5.65 | 6.79 | 7.92 | 9.05 | 10.18 | 11.31 | 0.888 | × | × | × | × | × | × | × | × | × |
| 14 | 1.539 | 3.08 | 4.62 | 6.16 | 7.69 | 9.23 | 10.77 | 12.31 | 13.85 | 15.39 | 1.208 | × | × | × | × | × | × | × | × | × |
| 16 | 2.011 | 4.02 | 6.03 | 8.04 | 10.05 | 12.06 | 14.07 | 16.08 | 18.1 | 20.11 | 1.578 | × | × | × | × | × | × | × | × | × |
| 18 | 2.545 | 5.09 | 7.63 | 10.18 | 12.72 | 15.27 | 17.81 | 20.36 | 22.9 | 25.45 | 1.998 | × | × | × | × | × | × | × | × | × |
| 20 | 3.142 | 6.28 | 9.41 | 12.56 | 15.71 | 18.85 | 21.99 | 25.14 | 28.28 | 31.42 | 2.466 | × | × | × | × | × | × | × | × | × |
| 22 | 3.801 | 7.6 | 11.4 | 15.2 | 19 | 22.81 | 26.61 | 30.41 | 34.21 | 38.01 | 2.984 | × | × | × | × | × | × | × | × | × |
| 25 | 4.909 | 9.82 | 14.73 | 19.63 | 24.54 | 29.45 | 34.36 | 39.27 | 44.13 | 49.09 | 3.853 | × | × | × | × | × | × | × | × | × |
| 28 | 6.158 | 12.32 | 18.47 | 24.63 | 30.79 | 36.95 | 43.1 | 49.26 | 55.42 | 61.58 | 4.834 | × | × | × | × | × | × | × | × | × |
| 32 | 8.042 | 16.08 | 24.13 | 32.17 | 40.21 | 48.25 | 56.3 | 64.34 | 72.38 | 80.42 | 6.313 | × | × | × | × | × | × | × | × | × |
| 36 | 10.18 | 20.36 | 30.54 | 40.72 | 50.9 | 61.08 | 71.26 | 81.44 | 91.62 | 101.8 | 7.99 | × | × | × | × | × | × | × | × | × |
| 40 | 12.56 | 25.12 | 37.68 | 50.24 | 62.8 | 75.36 | 87.92 | 100.48 | 113.04 | 125.6 | 9.87 | × | × | × | × | × | × | × | × | × |

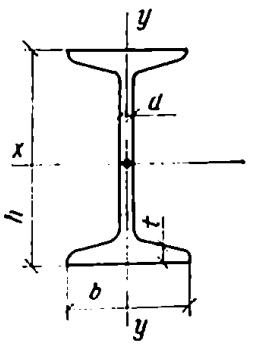
*Note.* The sign '—' stands for diameters unavailable in hot rolling, 'x' stands for diameters available in hot-rolled from.

## APPENDIX IX

### Characteristics of Class K-7 Wire Strands

| Nominal strand diameter, mm | Outer wire diameter, mm | Strand cross-sectional area, cm² | Theoretical weight per metre run, kg |
|---|---|---|---|
| 4.5 | 1.5 | 0.127 | 0.099 |
| 6 | 2 | 0.226 | 0.176 |
| 7.5 | 2.5 | 0.354 | 0.276 |
| 9 | 3 | 0.509 | 0.397 |
| 12 | 4 | 0.908 | 0.703 |
| 15 | 5 | 1.415 | 1.113 |

*Note.* The nominal strand diameter is taken as three times the nominal outer wire diameter.

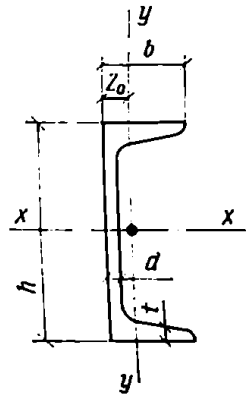



## APPENDIX X

### I-Beams: Dimensions and Properties

| Serial size | Dimensions, mm | | | | Area of section, cm² | Weight per metre, kg | $J_x$, cm⁴ | $W_x$, cm³ | $r_x$, cm | $J_y$, cm⁴ | $W_y$, cm³ | $r_y$, cm | $J_{tor}$, cm⁴ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $h$ | $b$ | $d$ | $t$ | | | | | | | | | |
| 16 | 160 | 81 | 5 | 7.8 | 20.2 | 15.9 | 873 | 109 | 6.57 | 58.6 | 14.5 | 1.7 | 4.46 |
| 20 | 200 | 100 | 5.2 | 8.4 | 26.8 | 21 | 1 840 | 184 | 8.28 | 115 | 23.1 | 2.07 | 6.92 |
| 20[a] | 200 | 110 | 5.2 | 8.6 | 28.9 | 22.7 | 2 030 | 203 | 8.37 | 155 | 28.2 | 2.32 | 7.94 |
| 22 | 220 | 110 | 5.4 | 8.7 | 30.6 | 24 | 2 550 | 232 | 9.13 | 157 | 28.6 | 2.27 | 8.60 |
| 30 | 300 | 135 | 6.5 | 10.2 | 46.5 | 36.5 | 7 080 | 472 | 12.3 | 337 | 49.9 | 2.69 | 17.4 |
| 40 | 400 | 155 | 8.3 | 13 | 72.7 | 57 | 19 062 | 953 | 16.2 | 667 | 86.1 | 3.03 | 40.6 |

*Note.* This table does not include beam sizes 10, 12, 14, 18, 18[a], 22[a], 24, 24[a], 27, 27[a], 30[a], 33, 36, 45, 50, 55, and 60 which are also available in the USSR.




*APPENDIX XI*

## Channels: Dimensions and Properties

| Serial size | Dimensions, mm | | | | Area of section, $cm^2$ | Weight per metre, kg | $J_x$, $cm^4$ | $W_x$, $cm^3$ | $r_x$, cm | $J_y$, $cm^4$ | $W_y$, $cm^3$ | $r_y$, cm | $z_0$, cm |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $h$ | $b$ | $d$ | $t$ | | | | | | | | | |
| 16 | 160 | 64 | 5 | 8.4 | 18.1 | 14.2 | 747 | 93.4 | 6.42 | 63.3 | 13.8 | 1.87 | 1.8 |
| 20 | 200 | 76 | 5.2 | 9 | 23.4 | 18.4 | 1 520 | 152 | 8.07 | 113 | 20.5 | 2.2 | 2.07 |
| 20[a] | 200 | 80 | 5.2 | 9.7 | 25.2 | 19.8 | 1 670 | 167 | 8.15 | 139 | 24.2 | 2.35 | 2.28 |
| 24 | 240 | 90 | 5.6 | 10 | 30.6 | 24 | 2 900 | 242 | 9.73 | 208 | 31.6 | 2.6 | 2.42 |
| 30 | 300 | 100 | 6.5 | 11 | 40.5 | 31.8 | 5 810 | 387 | 12 | 327 | 43.6 | 2.84 | 2.52 |
| 40 | 400 | 115 | 8 | 13.5 | 61.5 | 48.3 | 15 220 | 761 | 15.7 | 642 | 73.4 | 3.23 | 2.75 |

*Note.* This table does not include channel sizes 5, 5.5, 8, 10, 12, 14, 14[a], 16[a], 18, 18[a], 22, 22[a], 24[a], 33, and 36 which are also available in the USSR.

*APPENDIX XII*

## Coefficient $\varphi_b$ for Checking Beams for Stability

For symmetric I-beams,

$$\varphi_b = \psi (J_y/J_x) (h/l)^2 \times 10^3 \qquad (1)$$

where $h$ is the beam depth, $l$ is the distance between the points where the compressive flange is fixed, $J_x$ and $J_y$ are the moments of inertia of the cross section, and $\psi$ is a tabular coefficient depending on $\alpha$:

for rolled I-beams

$$\alpha = 1.54 (J_{tor}/J_y) (l/h)^2$$

Here, $J_{tor}$ is the torsional moment of inertia of the cross section (see Appendix X);

for build-up I-beams welded of three sheets

$$\alpha = 8 (l\delta_1/bh)^2 (1 + h\delta^3/2b\delta_1^3)$$

Here, $h$ is the overall depth of the beam cross section, $b$ and $\delta_1$ are the flange width and thickness, and $\delta$ is the web thickness.

### Coefficient $\psi$ for I-Beams Made of Class C38/23 Steel

| $\alpha$ | Coefficient $\psi$ | | | | |
|---|---|---|---|---|---|
| | no fixation in the span | | | | top flange fixed at two or more points, irrespective of loading |
| | concentrated load applied to | | uniformly distributed load applied to | | |
| | top flange | bottom flange | top flange | bottom flange | |
| 0.4 | 1.77 | 5.03 | 1.60 | 3.85 | 2.20 |
| 1 | 1.85 | 5.11 | 1.67 | 3.90 | 2.27 |
| 8 | 2.63 | 5.91 | 2.35 | 4.59 | 2.90 |
| 24 | 4.03 | 7.31 | 3.55 | 5.79 | 4.00 |
| 48 | 5.60 | 8.88 | 4.90 | 7.13 | 5.23 |
| 80 | 7.31 | 10.59 | 6.30 | 8.58 | 6.51 |
| 160 | 10.59 | 13.83 | 9.04 | 11.30 | 8.95 |
| 240 | 13.21 | 16.36 | 11.21 | 13.48 | 10.86 |
| 400 | 17.24 | 20.48 | 14.57 | 16.80 | 13.91 |

If the value of $\varphi_b$ found from Eq. (1) exceeds 0.85, it is taken equal to $\varphi_b'$ which is given below

| $\varphi_b$ | 0.85 | 0.90 | 0.95 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.55 |
|---|---|---|---|---|---|---|---|---|---|
| $\varphi_b'$ | 0.85 | 0.87 | 0.89 | 0.904 | 0.927 | 0.948 | 0.964 | 0.98 | 1.00 |

*APPENDIX XIII*

## Coefficient $\varphi^{ec}$ for Checking Steel Members Eccentrically Loaded in the Plane of Flexure for Stability

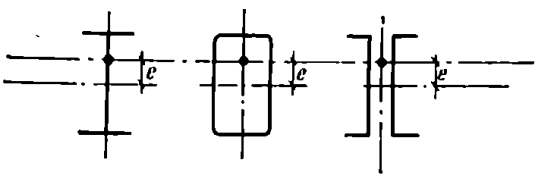
$$\bar{\lambda} = \lambda \sqrt{R/E}$$

$$m = eF/W$$

$$m_1 = \eta m$$

Here, $\lambda = l_0/r$ is the slenderness ratio of a member, $\bar{\lambda}$ is the assumed slenderness ratio, $R$ is the design strength of steel, $E$ is the modulus of elasticity of steel, $e = M/N$ is the eccentricity of the applied force, $F$ is the cross-sectional area, $W$ is the moment of resistance of the section (for the most compressed fibre), $\eta$ is the factor taking care of the section shape (for solid-web members), and $m$ is the relative eccentricity.

Type of cross section




Coefficient $\eta$ at

| $0 \leqslant \bar{\lambda} \leqslant 5$ | | $\bar{\lambda} > 5$ |
|---|---|---|
| $0.1 \leqslant m < 5$ | $5 \leqslant m \leqslant 20$ | $0.1 \leqslant m \leqslant 0.2$ |
| $1.75\text{-}0.13\bar{\lambda}$ | $1.5\text{-}0.08\bar{\lambda}$ | 1.1 |

**Coefficient $\varphi^{ec}$ for Solid-Web Members**

| $\bar{\lambda}$ | Coefficient $\varphi^{ec}$ as a function of $m_1$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.1 | 0.5 | 1.0 | 1.5 | 2.0 | 3.0 | 4.0 | 5.0 | 6.0 | 8.0 | 10.0 | 20.0 |
| 0.5 | 0.97 | 0.85 | 0.72 | 0.62 | 0.54 | 0.42 | 0.34 | 0.28 | 0.24 | 0.18 | 0.15 | 0.07 |
| 1.0 | 0.92 | 0.78 | 0.65 | 0.56 | 0.48 | 0.38 | 0.31 | 0.26 | 0.22 | 0.17 | 0.14 | 0.07 |
| 1.5 | 0.87 | 0.72 | 0.59 | 0.51 | 0.44 | 0.35 | 0.28 | 0.24 | 0.20 | 0.16 | 0.13 | 0.07 |
| 2.0 | 0.81 | 0.65 | 0.54 | 0.46 | 0.40 | 0.31 | 0.26 | 0.22 | 0.19 | 0.15 | 0.12 | 0.06 |
| 3.0 | 0.67 | 0.52 | 0.42 | 0.36 | 0.32 | 0.26 | 0.22 | 0.19 | 0.17 | 0.13 | 0.11 | 0.06 |
| 4.0 | 0.50 | 0.39 | 0.33 | 0.29 | 0.26 | 0.21 | 0.18 | 0.16 | 0.14 | 0.12 | 0.10 | 0.06 |
| 5.0 | 0.35 | 0.29 | 0.25 | 0.22 | 0.20 | 0.17 | 0.15 | 0.13 | 0.12 | 0.10 | 0.09 | 0.05 |
| 6.0 | 0.26 | 0.22 | 0.20 | 0.18 | 0.17 | 0.14 | 0.15 | 0.11 | 0.10 | 0.09 | 0.08 | 0.05 |
| 8.0 | 0.15 | 0.14 | 0.13 | 0.12 | 0.11 | 0.10 | 0.09 | 0.08 | 0.08 | 0.07 | 0.06 | 0.04 |
| 10.0 | 0.10 | 0.09 | 0.09 | 0.08 | 0.08 | 0.07 | 0.07 | 0.06 | 0.06 | 0.05 | 0.05 | 0.03 |
| 14.0 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.03 |

**Coefficient $\varphi^{ec}$ for Open-Work Members**

| $\bar{\lambda}$ | Coefficient $\varphi^{ec}$ as a function of $m$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0.1 | 0.5 | 1,0 | 1.5 | 2.0 | 3.0 | 5.0 | 10.0 | 20.0 |
| 0.5 | 0.91 | 0.67 | 0.50 | 0.40 | 0.33 | 0.25 | 0.17 | 0.09 | 0.05 |
| 1.0 | 0.87 | 0.64 | 0.48 | 0.39 | 0.33 | 0.24 | 0.16 | 0.09 | 0.05 |
| 2.0 | 0.77 | 0.56 | 0.42 | 0.35 | 0.29 | 0.23 | 0.16 | 0.09 | 0.04 |
| 3.0 | 0.64 | 0.46 | 0.36 | 0.30 | 0.25 | 0.20 | 0.14 | 0.08 | 0.04 |
| 5.0 | 0.35 | 0.28 | 0.23 | 0.20 | 0.18 | 0.15 | 0.11 | 0.07 | 0.04 |
| 10.0 | 0.10 | 0.09 | 0.09 | 0.07 | 0.07 | 0.07 | 0.06 | 0.04 | 0.03 |
| 14.0 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.04 | 0.04 | 0.03 | 0.02 |

*APPENDIX XIV*

## Coefficient $c$ for Checking Steel Members Eccentrically Loaded in a Plane Perpendicular to the Plane of Flexure for Stability

For members of class C38/23 steel ($\lambda_y < 100$),

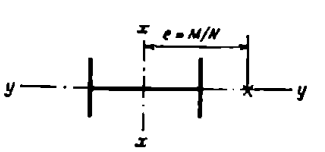
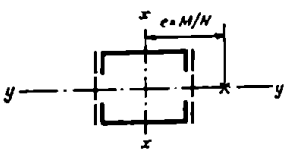
$$c = 1/(1 + \alpha m_x)$$

Here, $m_x$ is the relative eccentricity in the plane of flexure

$$m_x = M_x F/NW$$

where $M_x$ is the maximum moment in the middle third of the span (but not smaller than $M_{max}/2$) for members fixed at the ends, or the moment at the fixed end for cantilevers, $F$ is the cross-sectional area, and $W$ is the moment of resistance of the section (for the most compressed face).

### Coefficient $\alpha$

| Relative eccentricity | Type of cross section | |
|---|---|---|
| |   |   |
| $m_x \leqslant 1$ | 0.7 | 0.6 |
| $1 < m_x \leqslant 5$ | $0.7+0.05\,(m_x-1)$ | $0.6+0.05\,(m_x-1)$ |
| $m_x > 5$ | 0.9 | 0.8 |

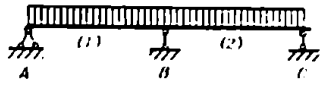
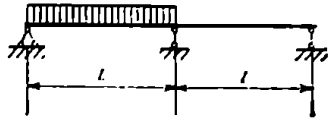
*APPENDIX XV*

## Tables for the Design of Equispan Continuous Beams

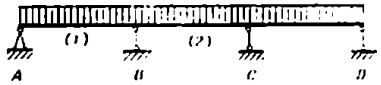
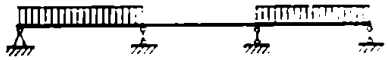
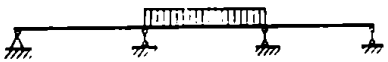
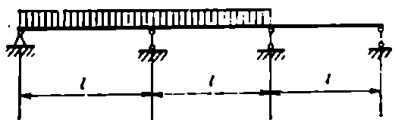
$$M = (\alpha g + \beta p)\, l^2$$

$$Q = (\gamma g + \delta p)\, l$$

## Two-Span Beams

| Loading system | Moments in span | | Moment at support | Shearing forces | | | |
|---|---|---|---|---|---|---|---|
| | $M_1$ | $M_2$ | $M_B$ | $Q_A$ | $Q_B^l$ | $Q_B^r$ | $Q_C$ |
|  | 0.07 | 0.07 | −0.125 | 0.375 | −0.625 | 0.625 | −0.375 |
|  | 0.096 | −0.025 | −0.063 | 0.473 | −0.563 | 0.063 | 0.063 |

## Three-Span Beams

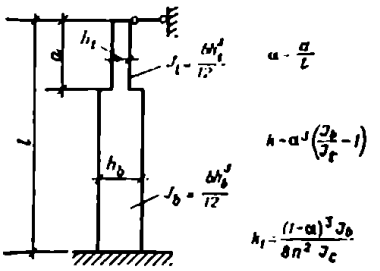
| Loading system | Moments in span | | Moments at supports | | Shearing forces | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $M_1$ | $M_2$ | $M_B$ | $M_C$ | $Q_A$ | $Q_B^l$ | $Q_B^r$ | $Q_C^l$ | $Q_C^r$ | $Q_D$ |
|  | 0.08 | 0.025 | −0.1 | −0.1 | 0.4 | −0.6 | 0.5 | −0.5 | 0.6 | −0.4 |
|  | 0.101 | −0.05 | −0.05 | −0.05 | 0.45 | −0.55 | 0 | 0 | 0.55 | −0.45 |
|  | −0.025 | 0.075 | −0.05 | −0.05 | −0.05 | −0.05 | 0.5 | −0.5 | 0.05 | 0.05 |
|  | | | −0.117 | −0.033 | 0.383 | −0.617 | 0.583 | −0.417 | 0.033 | 0.033 |

**Design Soil Strength $R_0$**

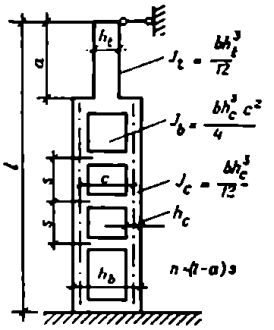
| Soil | $R_0$, kgf cm$^{-2}$ | |
|---|---|---|
| Rudaceous soil | | |
| Pebble (waste rock) soil containing sand | 6 | |
| Crystalline-rock gravel | 5 | |
| Sedimentary-rock gravel | 3 | |
| Sandy soil | Dense | Medium-density |
| Coarse sand, irrespective of moisture content | 6 | 5 |
| Medium sand, irrespective of moisture content | 5 | 4 |
| Fine sand, low moisture content | 4 | 3 |
| Fine sand, wet and water-bearing | 3 | 2 |
| Powdered sand, low moisture content | 3 | 2.5 |
| Powdered sand, wet | 2 | 1.5 |
| Powdered sand, water-bearing | 1.5 | 1 |
| Clay soil | Soil consistency | |
| | 0 | 1 |
| Sandy loam, porosity ratio 0.5 | 3 | 3 |
| Sandy loam, porosity ratio 0.7 | 2.5 | 2 |
| Loam, porosity ratio 0.5 | 3 | 2.5 |
| Loam, porosity ratio 0.7 | 2.5 | 1.8 |
| Loam, porosity ratio, 1.0 | 2 | 1 |
| Clay, porosity ratio 0.5 | 6 | 4 |
| Clay, porosity ratio 0.6 | 5 | 3 |
| Clay, porosity ratio 0.8 | 3 | 2 |

## Support Reactions in Transverse-Frame Fixed-Top Columns in a Single-Storey Industrial Building

Single-piece column



Two-piece column

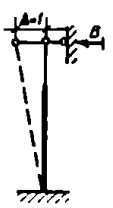
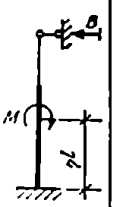
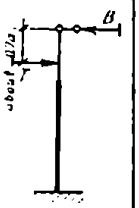
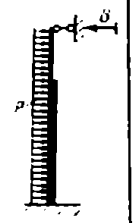


$\alpha = a/l$

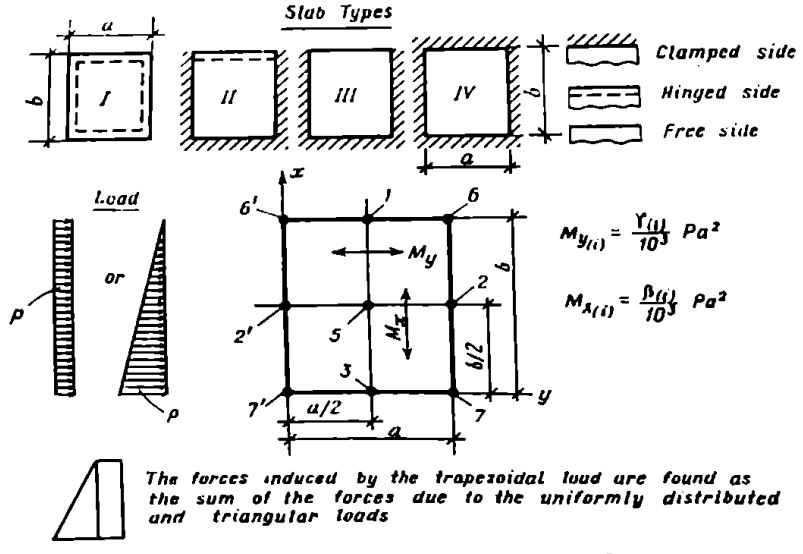
$k = \alpha^3 (J_b/J_t - 1)$

$k_1 = (1 - \alpha)^3 J_b/8n^2 J_c$

For a single-piece column, $k_1 = 0$

| Loading system | Support reaction $B$ | Loading system | Support reaction $B$ |
|---|---|---|---|
|  | $B = \dfrac{3EJ_b}{l^3(1+k+k_1)}$ |  | $B = \dfrac{3M\eta(2-\eta)}{2l(1+k+k_1)}$ |
| | $B = \dfrac{3M(1+k/\alpha)}{2l(1+k+k_1)}$ |  | $B = \dfrac{T(1-\alpha+k_1)}{1+k+k_1}$ |
| | $B = \dfrac{3M(1-\alpha^2)}{2l(1+k+k_1)}$ |  | $B = \dfrac{3pl[1+\alpha k+1.33(1+\alpha)k_1]}{8(1+k+k_1)}$ |

## To the Design of Slabs Supported on Four Sides






*The forces induced by the trapezoidal load are found as the sum of the forces due to the uniformly distributed and triangular loads*

## Uniformly Distributed Load

| Type I | | | b/a | Type II | | | | Type III | | | | | | Type IV | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a/b | $\beta_5$ | $\gamma_5$ | | $\beta_5$ | $\gamma_5$ | $-\beta_3$ | $-\gamma_2$ | $\beta_5$ | $\gamma_5$ | $-\beta_3$ | $-\gamma_2$ | $-\gamma_1$ | $-\gamma_0$ | $\beta_5$ | $\gamma_5$ | $\frac{-\beta_1}{-\beta_3}$ | $\frac{-\gamma_2}{-\gamma_2'}$ |
| 0.5 | 17 | 96 | 0.5 | 3 | 41 | 56 | 84 | 2 | 12 | 50 | 34 | 27 | 71 | 4 | 40 | 50 | 82 |
| 0.6 | 24 | 82 | 0.6 | 6 | 38 | 56 | 81 | 4 | 17 | 53 | 42 | 33 | 80 | 8 | 37 | 56 | 78 |
| 0.7 | 30 | 68 | 0.7 | 9 | 35 | 56 | 77 | 6 | 21 | 54 | 49 | 38 | 84 | 12 | 32 | 50 | 72 |
| 0.8 | 33 | 56 | 0.8 | 12 | 31 | 56 | 72 | 8 | 24 | 55 | 50 | 41 | 85 | 14 | 27 | 55 | 65 |
| 0.9 | 36 | 46 | 0.9 | 15 | 27 | 55 | 68 | 9 | 26 | 56 | 62 | 43 | 85 | 16 | 22 | 53 | 58 |
| 1.0 | 37 | 37 | 1.0 | 17 | 23 | 54 | 60 | 8 | 28 | 56 | 66 | 44 | 85 | 18 | 18 | 51 | 51 |
| | | | 1.2 | | | | | 8 | 32 | 56 | 73 | 44 | 86 | | | | |
| | | | 1.5 | | | | | 5 | 36 | 56 | 79 | 45 | 85 | | | | |
| | | | 2.0 | | | | | 3 | 40 | 56 | 83 | 45 | 84 | | | | |

## Triangular Load Pattern

| $a/b$ | Type I | | | | Type II | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\beta_5$ | $\gamma_5$ | max $\beta_{1-3}$ | max $\gamma_{1-3}$ | $\beta_5$ | $\gamma_5$ | max $\beta_{1-3}$ | max $\gamma_{1-3}$ | $-\beta_3$ | $\gamma_2$ | max $\gamma_{6-7}$ |
| 0.5 | 9 | 50 | 21 | 51 | 2 | 20 | 9 | 23 | 45 | 41 | —50 |
| 0.6 | 12 | 41 | 22 | 42 | 3 | 19 | 9 | 20 | 43 | 40 | —46 |
| 0.8 | 17 | 28 | 23 | 28 | 7 | 15 | 9 | 15 | 38 | 35 | —37 |
| 1.0 | 18 | 18 | 22 | 18 | 9 | 11 | 10 | 11 | 34 | 29 | —29 |

| $b/a$ | Type III | | | | | | $a/b$ | Type IV | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\beta_5$ | $\gamma_5$ | $-\beta_3$ | $-\gamma_2$ | $\gamma_1$ | $-\gamma_6$ | | $\beta_5$ | $\gamma_5$ | max $\beta_{1-3}$ | max $\gamma_{1-3}$ | $-\beta_3$ | $-\beta_1$ | $-\gamma_2$ | max $\gamma_{6-7}$ |
| 0.5 | 2 | 4 | 23 | 3 | 7 | 4 | 0.5 | 2 | 20 | 9 | 22 | 45 | 11 | 41 | —50 |
| 0.6 | 3 | 5 | 26 | 6 | 8 | 6 | 0.6 | 4 | 18 | 9 | 20 | 43 | 14 | 39 | —46 |
| 0.8 | 6 | 9 | 29 | 15 | 10 | 10 | 0.8 | 7 | 14 | 10 | 14 | 38 | 17 | 33 | —36 |
| 1.0 | 7 | 12 | 34 | 30 | 10 | 12 | 1.0 | 9 | 9 | 10 | 9 | 33 | 18 | 25 | —27 |
| 1.5 | 4 | 16 | 41 | 86 | 6 | 16 | | | | | | | | | |
| 2.0 | 2 | 19 | 45 | 165 | 4 | 18 | | | | | | | | | |

# INDEX


Aluminium shapes, 157
Anchor, Freyssinet, 51
  socket-type, 51
Anchor pier, 383
Anchorage zone, 39

Bar mat, splicing of, 26, 50
Bars, bent-up, 105
  deformed, 26
  inclined, 77
  nonprestressed, anchorage length of, 48
  plain, 26
Beam, reinforced concrete, defined, 72
  reinforced concrete, types of, 74
  steel, design of, 164
Bell, 296
Bolt, black, 162
  bright, 162
  high-strength, 163
Buckling coefficient, 140
Building, bay of, 227
  bearing-wall type, 193
  for water supply systems, 237
  prefabrication of structural members, 194
  skeleton-type, 193
  span of, 227
  unified modular system, 195

Caisson, 238
Capital, 196
Channels, dimensions and properties, 403
Chute, 284
Cement, alumina, 22
  Portland, 22
Cold brittleness, 48
Cold drawing, 30
Column, steel, design and proportioning of, 175
  steel, in axial compression, 175
  steel, in eccentrical compression, 184
  stub, 57

Column base, 175
Column head, 175
Compression, axial, 55
  eccentrical, 55, 120
Compression zone, relative depth of, 85, 87
Compensating niche, 355, 371
Concrete, 19
  axial tensile strength of, 21
  brands of, 20
  cellular, 25
  compressive strength of, 19
  creep in, 22
  cube crushing strength of, 20
  gypsum, 25
  heat-resistant, 24
  heavy, 20, 25
  light-weight, 25
  lime, 25
  medium-heavy, 25
  porous-aggregate, 20
  prism crushing strength of, 21
  reinforced, 16
  shrinkage of, 36
  slag-binding agent, 25
  superheavy, 25
  superlight, 25
Concrete aggregates, 19
Concrete cover, 56
Conduit, 355
  high, 356
  low, 356
  medium-height, 356
Connections of beams and girders, 165
Contact soil pressure, 221
Crack-opening analysis, 118
Crack width, limiting, 15
Cross arm, 380
Crushing pyramid, 216
Cutoff point, actual, 106
  location of, 107
  theoretical, 106

Design, in terms of serviceability, 14
  in terms of ultimate strength, 13
Dimension categories, 196
Dimensions, actual, 196
  nominal, 196

structural, 196
Drop panel, 215
Duct former, flexible, 44
rigid, 44

Eccentricity, accidental, 55, 120
Elastic limit, 31
proof, 31
Expansion U-bend, 355, 371

Factor, combination, 12
overload, 10
partial safety, 10
safety, 10
service, 10
tensioning accuracy, 46
Flexural rigidity, 114
Flux, 157
Frame, 193

Gas shielded welding, MAG, 158
MIG, 158
TIG, 158
Girder, 164, 200
Grillage, 164

Header, 355
Heat pipeline, laid in open trench, 378
overhead, 355, 379
Heating system, distributing, 355
structures for, 355
trunk, 355

I-beams, dimensions and properties, 402
Inclined-section shear strength analysis, 98

Joint, bolted, 162
castellated, 365
flexible, 284
movement, 36, 278, 376
riveted, 162
welded, butt, 159
welded, lap, 160
welded, T-, 160

Kern, 111
Kern moment method, 110

Lateral bracing, 189
Launder, 284
Limit of sag, 118
Limit state design, 9
Load, characteristic, 11
dead, 12
design, 11
live, 12
special, 12
Load combinations, 12
basic, 12
special, 12
Loleyt principle, 81
Loss of stability, 167
Losses in prestress, early, 42
late, 42

Manual metal arc, 157
Masonry, 135
average ultimate strength of, 136
elastic characteristic of, 138
reinforced, 142
reinforced, elastic characteristic of, 144
structural clay tile, 143
Masonry materials, 135
durability of, 136
Masonry members in compression, 138
Metal structures, 152
Modulus of elasticity, 23
initial tangent, 23
secant, 23
Modulus of subgrade reaction, 262
Mortar, 135
brand numbers of, 135
cement, 135
heavy, 135
light, 135
lime, 135
mixed, 135

Neutral axis, 96
Normal-section strength analysis, 85

Overreinforced section, 81

panel, beam-type, 279
cantilever-type, 278
flat-convex, 254
shell-type, 254
percent elongation at rupture, 153
percentage of elongation, total, 31
uniform, 31
percentage of reinforcement, 56
volumetric, 144
pipe, reinforced concrete, flush-connected, 297
reinforced concrete, non-pressure, 296

reinforced concrete, pressure, 296
reinforced concrete, stepped-bell, 297
reinforced concrete, tapered-bell, 297
Plastic hinge, 168, 198
Prestress, steady-state, 70
Prestressed concrete members, incipient-cracking resistance of, 108
Prestressed concrete structures, design of, 40
Prestressing, 17
by electric heating, 41
mechanical, 41
posttensioning, 17
pretensioning, 17
Punching pyramid, 223

Redistribution of moments, 198
Reinforced concrete, categories of crack resistance, 37
properties of, 35
Reinforced concrete members, in axial compression, 55
in axial tension, 62
in bending, 72
in bending, condition for shear strength, 100
in bending, curvature of the deflected axis, 114
in bending, state of stress, 80
in bending, transverse bar design, 101
in eccentrical compression, 120
in eccentrical tension, 132
Reinforced concrete floor, beam and girder construction, 196
beam and girder construction, in-situ, 208
flat-slab construction, 196, 215
slab and girder construction, precast, 200
with slabs spanning in two directions, 212
Reinforced concrete footing, axially loaded, design of, 222
eccentrically loaded, design of, 225
isolated (independent), 219
mat (raft), 219
strip (continuous), 219
Reinforcement, classes of, 28
erection, 25
fabric, continuous, 73
fabric, sectionalized, 73
load-bearing, 25
local, 80
prestressed, 26
shear, 75
types of joints, 33
Reinforcement ratio, 56
Reinforcing cable, multi-layer, 29
single-layer, 29
Reinforcing cage, 26
Reinforcing steel, ductility of, 31
Reinforcing strand, 29
grades of, 28
Resistance to floating-up, 245
Rigidity characteristic, 270
Rivet, 162
Rubber key, 278

Saddle, 372
Sag analysis, 114
Scaffold bridge, construction types of, 382
Servicing chamber, 355, 367
Shear wall, 193
Shortness, cold, 152
red, 152
Slab, defined, 72
floor, hollow-core, 201
floor, ribbed, 201
floor, solid, 201
one-way, 72, 197
roof, barrel, 233
roof, large-size, 232
roof, low-pitched ribbed, 233
two-way, 197
Slenderness ratio, 121
reduced, 141
Soil, bearing capacity of, 221
Spatial rigidity, 193
Spigot, 296
Steel, brittleness of, 153
carbon, 152
class designation, 154
classes of strength, 154
grade designation, 154
groups of, 154
killed, 152
low-alloyed, 152
low-carbon, 152
rimmed, 152
semi-killed, 152
Steel expansion compensator, 278
Stirrup, closed, 76
multiple, 77
U-type, 76
Strain, elastic, 22
plastic, 22
total, 22
Strain block, 36
Strength, characteristic, 12
design, 13
impact, 153
mean, 11
proof, 12
proof yield, 30
punching, 223
transfer, 40
ultimate, 13
Strength of materials, frequency distribution curve for, 11
Stress-strain curve, 22
Stress transmission zone, 39
Structural system, elastic, 146
rigid, 145
Structure, prestressed steel, 191
reinforced concrete, in-situ, 18
reinforced concrete, precast, 18
reinforced concrete, precast/in-situ, 19
Submerged arc, 157

Substructure, 238
Superstructure, 238
Support, immovable, 355, 372
immovable, T-section, 373
immovable, wall-type, 373
movable, 355, 372
movable, roller, 384
movable, sliding, 384
movable, suspension-type, 384

Tank, circular, 246
circular, clarification, 253
circular, design of, 260
circular, methane, 254
circular, radial settling, 252
circular, steel, 292
circular, vertical settling, 252
circular, water, 249
rectangular, 246
rectangular, aeration, 278
rectangular, design of, 285
rectangular, horizontal settling, 276
rectangular, water, 273
Tension, axial, 62
eccentrical, 132
Tension zone, 86
Thickener, 253
Transformed section, 41
Truss, defined, 188
Howe, 188
reinforced concrete, camel-back, 220
reinforced concrete, quadrangular, 229
steel, design and proportioning of, 188
Warren, flat-top, 188
Warren, quadrangular, 188
Tube drainage, 356
Tunnel, 356
construction types of, 365
double-duct, 366
shield-driven, 367
single-duct, 366

Units of measure, 15
conversion of, 395

Water-cement ratio, 20
Water tower, construction types and design of, 300
Weld, strength of, 160
types of, 158
Well, 299
Wire fabric, 26
splicing of, 50
Wobble coefficient, 44

Yield point, 30